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THREE-FACTORIAL EXPERIMENTS IN CERTAIN INCOMPLETE SPLIT-SPLIT PLOT DESIGNS

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Summary

Some constructions for incomplete split-split-plot designs in which the levels of factors occur as treatments in BIB designs are given. The formulae for the efficiency factors for the main effects of factors and their interaction effects in the four strata: the inter-block stratum, the whole plots stratum (within the blocks), the subplots (within the whole plots), the sub-subplots (within the sub-plots) are presented.

Key words and phrases: balanced incomplete block designs, efficiency factors, Kronecker product of matrices, main and interaction effects of factors, split-split plot designs, three-factorial experiments

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1. Introduction

Let us consider a three-factorial experiment of split-split-plot type in which factor A occurs on v_1 levels: A_1, A_2, ..., A_{v_1}, factor B on v_2 levels: B_1, B_2, ..., B_{v_2} and factor C occurs on v_3 levels: C_1, C_2, ..., C_{v_3}. In this paper, levels are distributed in the same way as treatments in balanced incomplete block (BIB) designs and the levels of factor A are distributed on the whole plots, the levels of factor B are distributed on the subplots with the whole plots and the levels of factor C are distributed on the sub-subplots with the subplots.

Let a population of experimental units in an environment be divided into b blocks and let each block be additionally divided into: k_1 whole plots whereas each whole plot be divided into k_2 subplots and each subplot be divided into k_3 sub-subplots.

We need to remember that the notions: whole plots, subplots and sub-subplots are given by Gomez and Gomez (1984).

In the next section, we are going to give some basic information about BIB designs and about block designs with incidence matrices equal to the Kronecker product of three incidence matrices of BIB designs. In the third section, the method of the construction of these split-split-plot designs and the formulae for the efficiency factors for the main effects and interaction effects are given and a relevant example is contained in the fourth section.

It is worth mentioning that split-plot designs for three-factorial experiments generated also by BIB designs are presented by Brzeskwiniewicz and Krzyszkowska (2006a, 2006b).

2. Preliminaries

We are going to use the notion of BIB designs as in

Definition 2.1. (see e.g. Raghavarao, 1971). A balanced incomplete block design (BIB) is an arrangement of v_* treatments in b_* blocks of sizes k_* hence every treatment occurs r_* times and every pair of distinct treatments is contained in exactly λ_* blocks. The numbers v_*, b_*, r_*, k_* and λ_* are called the parameters of BIB design accordingly.

Let $d_* = \dfrac{r_* - \lambda_*}{r_* k_*}$ and let $\mathbf{N}_*$ be the incidence matrix of the above BIB design. Then, $(\mathbf{N}_*, \mathbf{N}_*)$ is also incidence matrix of BIB design with parameters $v_{**} = v_*$, $b_{**} = 2b_*$, $r_{**} = 2r_*$, $k_{**} = k_*$ and $\lambda_{**} = 2\lambda_*$ and

$$d_{**} = \frac{r_{**} - \lambda_{**}}{r_{**}k_{**}} = \frac{r_* - \lambda_*}{r_* k_*} = d_* . \tag{2.1}$$

Note that: for classical BIB design (i.e. design with $r_* > \lambda_* > 0$) we have $0 < d_* < 1$, for design in which $v_* = b_*$, $r_* = k_* = 1$, $\lambda_* = 0$ (i.e. $\mathbf{N}_* = \mathbf{I}_{v_*}$) we have $d_* = 1$ and for design in which $v_* = k_*$, $r_* = b_* = \lambda_*$, (i.e. $\mathbf{N}_* = \mathbf{J}_{v_* b_*}$) or $v_* = k_*$, $r_* = b_* = \lambda_* = 1$, (i.e. $\mathbf{N}_* = \mathbf{1}_{v_*}$) we have $d_* = 0$.

Theorem 2.1. Let $\mathbf{N}_A$, $\mathbf{N}_B$, $\mathbf{N}_C$ be incidence matrices of BIB designs with parameters v_1, b_1, r_1, k_1, λ_1; v_2, b_2, r_2, k_2, λ_2; and v_3, b_3, r_3, k_3, λ_3, respectively. Then, block design with incidence matrix

$$\mathbf{N}_1 = \mathbf{N}_A \otimes \mathbf{N}_B \otimes \mathbf{N}_C \tag{2.2}$$

has the following parameters: $v = v_1 v_2 v_3$, $b = b_1 b_2 b_3$, $r = r_1 r_2 r_3$, $k = k_1 k_2 k_3$ and their association matrix $\mathbf{N}_1\mathbf{N}_1' = \mathbf{N}_A\mathbf{N}_A' \otimes \mathbf{N}_B\mathbf{N}_B' \otimes \mathbf{N}_C\mathbf{N}_C'$ has eigenvalues equal to: $\rho_0 = rk$; $\rho_1 = r_1k_1r_2k_2(r_3 - \lambda_3)$;
$\rho_2 = r_1k_1(r_2 - \lambda_2)r_3k_3$; $\rho_3 = r_1k_1(r_2 - \lambda_2)(r_3 - \lambda_3)$;
$\rho_4 = (r_1 - \lambda_1)r_2k_2r_3k_3$; $\rho_5 = (r_1 - \lambda_1)r_2k_2(r_3 - \lambda_3)$;
$\rho_6 = (r_1 - \lambda_1)(r_2 - \lambda_2)r_3k_3$; $\rho_7 = (r_1 - \lambda_1)(r_2 - \lambda_2)(r_3 - \lambda_3)$ with multiplicities: 1, $v_3 - 1$, $v_2 - 1$, $(v_2 - 1)(v_3 - 1)$, $v_1 - 1$, $(v_1 - 1)(v_3 - 1)$, $(v_1 - 1)(v_2 - 1)$, $(v_1 - 1)(v_2 - 1)(v_3 - 1)$, respectively.

3. Results

In the planning of experiments carried out in some split-split-plot designs, three incidence matrices: $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_3$ are of great importance. Let $\mathbf{N}_1$ be the ($v \times b$) incidence matrix with respect to blocks, then its (i, j)-element indicates how many times the i-th (i = 1,2, ..., v) combination of levels of three factors occurs in j-th block (j = 1, ..., b). In this paper, $\mathbf{N}_1$ has the form of (2.2), and consequently, $v = v_1 v_2 v_3$ and $b = b_1 b_2 b_3$. Let $\mathbf{N}_2$ be the ($v \times k_1 b$) incidence matrix for whole plots (inside each block), then, its (i, l)-element indicates how many times the i-th combination of levels of three factors (i = 1,2, ..., v) occurs in l-th whole plots (l = 1,2, ..., $k_1 b$) and let $\mathbf{N}_3$ be the ($v \times k_1 k_2 b$) incidence matrix for subplots, then its (i, l_1)-element indicates how many times the i-th combination of levels of three factors (i = 1,2, ..., v) occurs in l_1-th subplot (l_1 = 1,2, ..., $k_1 k_2 b$).

For the purpose of observation, we assume after Mejza (1997a, 1997b) the randomization linear mixed model. The overall analysis of variance for this experiment is split into so-called stratum analyses. These strata are connected with the variability among blocks inside the total experiments, among the whole plots inside blocks among the subplots inside whole plots and among sub-subplots inside subplots.

Information matrices in these strata are, respectively:

$$\mathbf{C}_1 = \frac{1}{k}\mathbf{N}_1\mathbf{N}_1' - \frac{r}{v}\mathbf{J}_v,$$

$$\mathbf{C}_2 = \frac{1}{k_2 k_3}\mathbf{N}_2\mathbf{N}_2' - \frac{1}{k}\mathbf{N}_1\mathbf{N}_1', \tag{3.1}$$

$$\mathbf{C}_3 = \frac{1}{k_3}\mathbf{N}_3\mathbf{N}_3' - \frac{1}{k_2 k_3}\mathbf{N}_2\mathbf{N}_2',$$

$$\mathbf{C}_4 = r\mathbf{I}_v - \frac{1}{k_3}\mathbf{N}_3\mathbf{N}_3'.$$

The efficiency factors for the contrasts connected with main effects of A, B, C and interaction effects A × B, A × C, B × C and A × B × C in t-th stratum (t = 1,2,3,4) are equal to

$$\varepsilon_{it} = \frac{\mu_{it}}{r}, \tag{3.2}$$

respectively, where μ_{it} is i-th non-zero eigenvalue of $\mathbf{C}_t$. The order of i = 1,…,7 corresponds to: main effect of the factor C, main effect of the factor B, interaction effect of B×C, main effect of the factor A, interaction effect of

A×C, interaction effect of A×B, interaction effect of A×B×C, respectively. Therefore, $\mu_{i2} = \frac{1}{k_2 k_3}\rho_{i2} - \frac{1}{k}\rho_{i1} = \frac{1}{k_2 k_3}\rho_{i2} - \mu_{i1}$, $\mu_{i3} = \frac{1}{k_3}\rho_{i3} - \frac{1}{k_2 k_3}\rho_{i2}$ and $\mu_{i4} = r - \frac{1}{k_3}\rho_{i3}$.

It can be seen that $\mathbf{N}_2 = \mathbf{I}_{v_1} \otimes \mathbf{1}'_{r_1} \otimes \mathbf{N}_B \otimes \mathbf{N}_C$, $\mathbf{N}_2\mathbf{N}'_2 = r_1\mathbf{I}_{v_1} \otimes \mathbf{N}_B\mathbf{N}'_B \otimes \mathbf{N}_C\mathbf{N}'_C$, $\mathbf{N}_3 = \mathbf{I}_{v_1 v_2} \otimes \mathbf{1}'_{r_1 r_2} \otimes \mathbf{N}_C$ and $\mathbf{N}_3\mathbf{N}'_3 = r_1 r_2 \mathbf{I}_{v_1 v_2} \otimes \mathbf{N}_C\mathbf{N}'_C$. Then, eigenvalues of $\mathbf{N}_2\mathbf{N}'_2$ are equal to: $\rho_{12} = \rho_{52} = r_1 r_2 k_2 (r_3 - \lambda_3)$, $\rho_{22} = \rho_{62} = r_1(r_2 - \lambda_2) r_3 k_3$, $\rho_{32} = \rho_{72} = r_1(r_2 - \lambda_2)(r_3 - \lambda_3)$; $\rho_{42} = r_1 r_2 k_2 r_3 k_3$, eigenvalues of $\mathbf{N}_3\mathbf{N}'_3$ are equal to: $\rho_{13} = \rho_{33} = \rho_{53} = \rho_{73} = r_1 r_2 (r_3 - \lambda_3)$; $\rho_{23} = \rho_{43} = \rho_{63} = r_1 r_2 r_3 k_3$, with multiplicities as in Theorem 2.1.

From the formulae for μ_{it}, we obtain efficiency factors ε_{it} for respective stratum effects which satisfy the inequality $0 \le \varepsilon_{it} \le 1$. They are important, given the fact that they show which contrasts are estimable (if $\mu_{it} \ne 0$) and with how high efficiency factors in two or three strata. Then, for the analysis of variance the stratum, with the highest corresponding ε_{it} can be selected.

Note that the efficiency factors are equal:
in the first stratum:

$$\varepsilon_{11} = d_3,\ \varepsilon_{21} = d_2,\ \varepsilon_{31} = d_2 d_3,\ \varepsilon_{41} = d_1,\ \varepsilon_{51} = d_1 d_3,\ \varepsilon_{61} = d_1 d_2,$$

$$\varepsilon_{71} = d_1 d_2 d_3,$$

in the second stratum:

$$\varepsilon_{12} = \varepsilon_{22} = \varepsilon_{32} = 0,\ \varepsilon_{42} = 1 - d_1,\ \varepsilon_{52} = d_3(1 - d_1),$$

$$\varepsilon_{62} = d_2(1 - d_1),\ \varepsilon_{72} = d_2 d_3 (1 - d_1), \tag{3.3}$$

in the third stratum:

$$\varepsilon_{13} = \varepsilon_{43} = \varepsilon_{53} = 0\,,\ \varepsilon_{23} = \varepsilon_{63} = 1 - d_2\,,\ \varepsilon_{33} = \varepsilon_{73} = d_3(1 - d_2),$$

in the fourth stratum:

$$\varepsilon_{14} = \varepsilon_{34} = \varepsilon_{54} = \varepsilon_{74} = 1 - d_3\,,\ \varepsilon_{24} = \varepsilon_{44} = \varepsilon_{64} = 0\,,$$

where $d_i = \dfrac{r_i - \lambda_i}{r_i k_i}$, i = 1,2,3.

The above efficiencies are presented in Table 1.

Table 1. Efficiency factors for effects in the strata

t	ε_{1t} (C)	ε_{2t} (B)	ε_{3t} (B×C)	ε_{4t} (A)	ε_{5t} (A×C)	ε_{6t} (A×B)	ε_{7t} (A×B×C)
1	d_3	d_2	$d_2\ d_3$	d_1	$d_1\ d_3$	$d_1\ d_2$	$d_1\ d_2\ d_3$
2	0	0	0	$1-d_1$	$d_3(1-d_1)$	$d_2\ (1-d_1)$	$d_2\ d_3(1-d_1)$
3	0	$1-d_2$	$d_3(1-d_2)$	0	0	$1-d_2$	$d_3(1-d_2)$
4	$1-d_3$	0	$1-d_3$	0	$1-d_3$	0	$1-d_3$
$\sum$	1	1	1	1	1	1	1

4. Discussion

From the above formulae for ε_{ij} follows that if $d_1 = 1$, i.e. if $\mathbf{N}_A = \mathbf{I}_{v_1}$ (and/or $d_2 = 1$, i.e. $\mathbf{N}_B = \mathbf{I}_{v_2}$ and/or $d_3 = 1$, i.e. $\mathbf{N}_C = \mathbf{I}_{v_3}$), then in the second stratum (or in the third stratum or in the fourth stratum) all contrasts are non-estimable, which is not a favourable situation. On the other hand, these designs have few blocks and plots, and we find it a very favourable situation. If $d_1 = 0$, i.e. if $\mathbf{N}_A = \mathbf{J}_{v_1,b_1}$ or $\mathbf{N}_A = \mathbf{1}_{v_1}$ (and/or $d_2 = 0$, i.e. $\mathbf{N}_B = \mathbf{J}_{v_2,b_2}$ or

$\mathbf{N}_B = \mathbf{1}_{v_2}$; and/or $d_3 = 0$, i.e. $\mathbf{N}_C = \mathbf{J}_{v_3,b_3}$ or $\mathbf{N}_C = \mathbf{1}_{v_3}$) then in the second stratum the main effects of A (or in the third stratum the effects of B and A×B or in the fourth stratum effects of C, B×C, A×C and A×B×C) are estimable with full efficiencies, i.e. $\varepsilon_{42} = 1$. This is a favourable situation but, on the other hand, these designs have large plots what is not that favourable.

In Table 2 are presented numbers of stratum, in which main effects of A, B, C and interaction effects of A×B, B×C, A×C and A×B×C are estimable. If these effects are estimable only in one stratum, then efficiency factor is equal to 1. In Table 2 are presented only designs in which d_i = 0 and/or 1 appear at least once.

Table 2. Numbers of strata, in which main and interaction effects are estimable in designs with d_i = 0 or 1 appear at least once (* denotes the design with 0< d_i <1)

$d_1d_2d_3$	C	B	B×C	A	A×C	A×B	A×B×C
0 0 0	4	3	4	2	4	3	4
0 0 1	1	3	3	2	2	3	3
0 1 0	4	1	4	2	4	2	4
0 1 1	1	1	1	2	2	2	2
1 0 0	4	3	4	1	4	3	4
1 0 1	1	3	3	1	1	3	3
1 1 0	4	1	4	1	4	1	4
1 1 1	1	1	1	1	1	1	1
0 0 *	3,4	3	3, 4	2	2, 4	3	3, 4
0 1 *	3, 4	1	1, 4	2	2, 4	2	2, 4
1 0 *	1, 4	3	3, 4	1	1, 4	3	3, 4
1 1 *	1, 4	1	1, 4	2	1, 4	1	1, 4
0 * 0	1	1, 3	4	2	4	2, 3	4
0 * 1	4	1, 3	1, 3	2	2	2, 3	2, 3
1 * 0	1	1, 3	4	1	4	1, 3	4
1 * 1	1	1, 3	1, 3	1	1	1, 3	1, 3
* 0 0	4	3	4	1, 2	4	3	4
* 0 1	1	3	3	1, 2	1, 2	1, 2	3
* 1 0	4	1	4	1, 2	4	1,2	4
* 1 1	1	1	1	1, 2	1, 2	1, 2	1, 2
0 * *	1, 4	1, 3	1, 3, 4	2	2, 4	2, 3	2, 3, 4
1 * *	1, 4	1, 3	1, 3, 4	1	1, 4	1, 3	1, 3, 4
* 0 *	1, 4	3	1, 3, 4	1, 2	1, 2, 4	3	3, 4
* 1 *	1, 4	1	1, 3, 4	1, 2	1, 2, 4	1, 2	1, 2, 4
* * 0	4	1, 3	1, 3, 4	1, 2	4	1, 2, 3	4
* * 1	1	1, 3	1, 3, 4	1, 2	1, 2	1, 2, 3	1, 2, 3

Tables of split-plot designs for $v \le 50$, $2 \le r \le 10$, $2 \le k \le 10$ or $v \le 30$, $2 \le r \le 4$, $2 \le k \le 8$ with efficiency factors ε_{ij} in only three strata are presented by Brzeskwiniewicz and Krzyszkowska 2006a and 2007, respectively. Data from the above tables can be used in this paper, but efficiency factors ε_{ij} are calculated by formulae (3.3) in four strata.

From (2.1), it follows that many designs can have the same parameters and efficiency factors. For example, the design with $\mathbf{N}_A = \mathbf{I}_5$ $(v_1 = b_1 = 5,$ $r_1 = k_1 = 1,\ \lambda_1 = 0)$, $\mathbf{N}_B = \mathbf{J}_2$ $(v_2 = b_2 = r_2 = k_2 = 2)$ and $\mathbf{N}_C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ $(v_2 = b_2 = 3, r_2 = k_2 = 2,\ \lambda_3 = 1)$ has the same parameters and efficiency factors as split-split plot design with $\mathbf{N}_A = \mathbf{I}_5$ $(v_1 = b_1 = 5,\ r_1 = k_1 = 1,\ \lambda_1 = 0)$, $\mathbf{N}_B = \mathbf{1}_2$ $(v_2 = k_2 = 2,\ b_2 = r_2 = \lambda_2 = 1)$ and $\mathbf{N}_C = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$ $(v_3 = 3,\ b_3 = 6,\ r_3 = 4,\ k_3 = 2,\ \lambda_3 = 2)$.

Note, for example, that from $\mathbf{N}_A = \mathbf{I}_5$ we have $d_1 = 1$ and therefore (see Table 2) in second stratum all contrasts i.e. contrasts with main effects of A, B, C and interaction effects of A×B,A×C, B×C, A×B×C are non-estimable. Main effect of A is also non-estimable in the third and the fourth stratum but estimable in first stratum only, this situation not being favourable. On the other hand, design with $\mathbf{N}_A = \mathbf{I}_5$ has few block and few plots which is a favourable situation.

5. Example

A design with parameters $v = 18$, $b = 9$, $r = 4$, $k = 8$ is considered (see design no. 9 in Brzeskwiniewicz and Krzyszkowska, 2007). To construct the incidence matrix of this design, we use matrix $\mathbf{1}_2$ and the incidence matrix of the design BIB with parameters $v_* = 3$, $b_* = 3$, $r_* = 2$, $k_* = 2$ and $\lambda_* = 1$. Incidence matrices of these designs have the following form: $\mathbf{N}_A = \mathbf{1}_2$ $(v_1 = k_1 = 2,$

$b_1 = r_1 = \lambda_1 = 1$), $\quad \mathbf{N}_B = \mathbf{N}_C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad$ ($v_2 = b_2 = v_3 = b_3 = 3$, $r_2 = k_2 = r_3 = k_3 = 2$, $\lambda_2 = \lambda_3 = 1$). In this design $d_1 = 0$, $d_2 = 0.25$, $d_3 = 0.25$. From $\mathbf{N}_1 = \mathbf{N}_A \otimes \mathbf{N}_B \otimes \mathbf{N}_C$ we can obtain the arrangement of levels A, B and C in blocks.

The distribution (before randomization) is schematically illustrated by:

Table 3. The arrangement of levels A, B and C in blocks for example

block I			block II			block III			block IV				block VIII			block IX		
A_1	B_1	C_1	A_1	B_1	C_1	A_1	B_1	C_2	A_1	B_1	C_1	. . .	A_1	B_2	C_1	A_1	B_2	C_2
		C_2			C_3			C_3			C_2				C_3			C_3
	B_2	C_1		B_2	C_1		B_2	C_2		B_3	C_1			B_3	C_1		B_3	C_2
		C_2			C_3			C_3			C_2				C_3			C_3
A_2	B_1	C_1	A_2	B_1	C_1	A_2	B_1	C_2	A_2	B_1	C_1		A_2	B_2	C_1	A_2	B_2	C_2
		C_2			C_3			C_3			C_2				C_3			C_3
	B_2	C_1		B_2	C_1		B_2	C_2		B_3	C_1			B_3	C_1		B_3	C_2
		C_2			C_3			C_3			C_2				C_3			C_3

where A_1, A_2 are levels of A; B_1, B_2, B_3 are levels of B; and C_1, C_2, C_3 are levels of C. The efficiency factors for estimation of effects A, B, C, A×B, A×C, B×C and A×B×C in four strata are the following (see Table 1).

Table 4. The efficiency factors for effects in strata for example

t	ε_{1t} (C)	ε_{2t} (B)	ε_{3t} (B×C)	ε_{4t} (A)	ε_{5t} (A×C)	ε_{6t} (A×B)	ε_{7t} (A×B×C)
1	0.25	0.25	0.06	0	0	0	0
2	0	0	0	1	0.25	0.25	0.06
3	0	0.75	0.19	0	0	0.75	0.19
4	0.75	0	0.75	0	0.75	0	0.75

It can be seen that for the purpose of analysis and estimation of: C, B×C, A×C, A×B×C effects, the fourth stratum; of B, A×C – the third stratum, and of A – the second stratum are adequate respectively.

It should be emphasized that this design is use as split-plot design of type (*i*) and (*ii*) by Brzeskwiniewicz, Krzyszkowska (2007). In type (*i*) efficiency factors for $t = 1$ and $t = 2$ are the same as above and for $t = 3$ are equal to sum for $t = 3$ and $t = 4$ in split-split plot design. In type (ii) efficiency factors for $t = 1$ are equal, for $t = 2$ are the same as for $t = 3$ in split-split-plot design, and for $t = 3$ are equal to sum for $t = 2$ and $t = 3$ in split-split-plot design.

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TRÓJCZYNNIKOWE DOŚWIADCZENIA ZAKŁADANE W PEWNYCH UKŁADACH SPLIT-SPLIT PLOT

Streszczenie

Praca dotyczy konstrukcji niekompletnych układów split-split-plot dla doświadczeń trójczynnikowych za pomocą iloczynu Kroneckera trzech macierzy incydencji układów zrównoważonych o blokach niekompletnych BIB. Podane są wzory na współczynniki efektywności dla efektów głównych i interakcyjnych w czterech warstwach: międzyblokowej, między poletkami I rzędu (wewnątrz bloków), między poletkami II rzędu (wewnątrz poletek I rzędu) oraz między poletkami III rzędu (wewnątrz poletek II rzędu).

Słowa kluczowe: zrównoważone układy o blokach niekompletnych, współczynniki efektywności, iloczyn Kroneckera macierzy, główne i interakcyjne efekty czynników, układy split-split-plot, doświadczenia trójczynnikowe

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