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Quadratic nonlinear model of grain mixture movement in cylindrical vibratory centrifugal sifter

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Summary. The hydrodynamic theory describes the standard vertical movement of pseudo rarefied mixture on the inner surface of the cylindrical vibratory centrifugal sifter, which rotates at a constant angular velocity around a vertical axis. It is a quadratic dependence of rheological tangent stresses in a mixture on the speed of its shift. We did nonlinear differential equations of motion of the grain mixture and corresponding extremum conditions. An analytical solution for extremum problem in the form of squaring was obtained, which can be calculated on a computer by numerical methods. Two variants of approximate analytical calculation of integral are proposed, with help of which the analytical solution of the problem of motion in the first and second approximations was obtained. It essentially uses a small ratio of the thickness of the layer of the mixture to a rolling radius of cylindrical sifter that normally takes place in the practice of separation. As a result, compact approximate formulas to calculate the flow rate and volumetric productivity of the sifter were figured out. Comparison of the results was conducted, which is led to by different approximations. For specific numerical data values we estimated the influence of rheological constants on kinematic characteristics of the flow. It is shown that the two constants crucial to the theoretical value of the mixture flow rate and volumetric productivity of the sifter. Therefore, the proposed nonlinear model of motion has advantages over the adequacy of its linear analogues, which it is summarizing. Compact formulas were derived for assessing the upper limit values of both rheological constants. A method of identifying values of constants was worked out it is based on measurements of flow velocity on the free surface of the movable layer of mixture. An example of identifying values of rheological constants was considered.

Key words: mathematical model, pseudo rarified separated mixture, quadratic rheological law, vibratory centrifugal sifter, speed/velocity of motion, productivity of sifter, identification.

INTRODUCTION

In common hydrodynamic models of separating mixtures traffic on cylindrical centrifugal sifter we typically use linear dependence of tangent stress on shift velocity [1-13]. The obtained theoretical values of flow rate are consistent with experiment at relatively high values of the coefficient vibro viscosity of the mixture, even for larger than dynamic viscosity of glycerol [14]. Therefore it is advisable to perfect linear models of motion by transition to more general nonlinear theories. One way of this transition is to use the quadratic dependence of tangential stresses in the mixture on the speed of its shift. This dependence we describe in [16-20]. Having two constants in rheological dependence makes it possible to improve the adequacy of the mathematical model to appropriate choice of the values of these constants.

ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

In recent years, the movement pseudo rarified layer of separated mixture as a viscous incompressible fluid was considered in many publications. Most of them are included in the list of references in monographs [1-5], where their analytical review was held. Not to repeat it, we just note that in the end it was described the movement both homogeneous and heterogeneous, granular media, which vibro viscous factor is differently dependent on one of the spatial coordinates. It is investigated the stationary harmonic vibrations of flow rate caused by fluctuations of sifters. It is considered the mixtures motion through the holes of sifters that means passing through and dropping fractions of the mixture. Formulas for calculating theoretical values of vibro viscosity of the mixture depending on its physical and mechanical properties and parameters of fluctuations were also calculated. Methods of vibro viscosity factor authentication based on measurements of kinematic parameters of flow were worked out. The influence of extremum effect on the movement of the mixture through a sifter on an inclined plane of finite width was found out. In reference list we present the monographies available not only in Ukraine and Russia but also in the other countries. It is also about dissertation researches in recent years [12, 13]. So, this study is helped with the well-known achievements on this topic.

The aim is to figure out and test the formulas to calculate the flow rate and volumetric productivity of vertical cylindrical vibratory centrifugal sifter at the quadratic dependence of tangent tension in the separated mixtures on the speed of their shift.

THE MAIN RESULTS OF RESEARCHES

Steady axially symmetric flow of the grain mixture in the direction of *oz* axle we describe by differential equation:

$$\frac{d}{dr}(r\tau) = -\rho g r \, . \tag{1}$$

where: $\tau = \tau(r)$ – tangential stresses in the mixture, which possess a specific gravity ρ; g – free fall acceleration; r – radial coordinate, shown in the (Fig. 1).

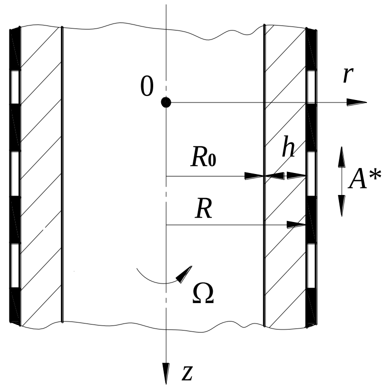

Fig. 1. Computational scheme of vertical cylindrical sifter with separating mixture.

On the free surface of a moving layer $r = R_0$ there is no tangential stress. That is why the equation (1) we integrate with extremum conditions:

$\tau(R_0) = 0$, giving:

$$\tau(r) = \frac{\rho g}{2}\left(\frac{R_0^2}{r} - r\right). \tag{2}$$

Following [16-20], the link between τ and $\frac{du}{dr}$ we present as:

$$\tau = \left(\mu + \mu_*\left|\frac{du}{dr}\right|\right)\frac{du}{dr}, \tag{3}$$

where: μ, μ_* – rheological constants; $u(r)$ – speed of mixture flow in the direction of the *oz* axle.

If $\mu_* = 0$ relation (3) becomes the known Newton formula [14].

Equating the right parts of expressions (2) and (3), given the fact that $\frac{du}{dr} < 0$, we get nonlinear differential equation:

$$\left(\frac{du}{dr}\right)^2 - \frac{\mu}{\mu_*}\frac{du}{dr} - \frac{\rho g}{2\mu_*}\left(r - \frac{R_0^2}{r}\right) = 0 .$$

From it follows that:

$$\frac{du}{dr} = \frac{\mu}{2\mu_*} - \sqrt{\frac{\mu^2}{4\mu_*^2} + \frac{\rho g}{2\mu_*}\left(r - \frac{R_0^2}{r}\right)}. \tag{4}$$

Having integrated the equation (4), with the extremum conditions $u(R) = 0$, we find that:

$$u(r) = \int_r^R \sqrt{\frac{\mu^2}{4\mu_*^2} + \frac{\rho g}{2\mu_*}\left(r - \frac{R_0^2}{r}\right)}dr + \frac{\mu}{2\mu_*}(r - R).$$

This integral is quite simple to calculate on the computer. However, to calculate the sifter performance it is desirable to have explicit analytical expression *u(r)*. With this in mind, we move to a new variable of integration by the formula $r = R_0 + \xi$,

where: $\xi \in [0; h]$; $h = R - R_0$. Then:

$$u(\xi) = \int_\xi^h \sqrt{\frac{\mu^2}{4\mu_*^2} + \frac{\rho g}{2\mu_*}\left(R_0 + \xi - \frac{R_0^2}{R_0 + \xi}\right)}d\xi + \frac{\mu}{2\mu_*}(\xi - h). \tag{5}$$

Whereas in the practice of separation of grain mixtures $h \ll R_0$, let us consider two further approximation with analytical determination $u(\xi)$ and the sifter performance *Q*.

In the first approximation we accept:

$$R_0 + \xi - \frac{R_0^2}{R_0 + \xi} \approx 2\xi .$$

For it, integration in (5) makes:

$$u(\xi) = \frac{\mu}{3\beta\mu_*}\left[(1 + \beta h)^{3/2} - (1 + \beta\xi)^{3/2}\right] + \frac{\mu}{2\mu_*}(\xi - h). \tag{6}$$

Here: $\beta = \frac{4\rho g \mu_*}{\mu^2}$.

The maximum flow rate of the mixture is presented by a compact expression:

$$\max u = u(0) = \frac{\mu}{3\beta\,\mu_*}\left[(1 + \beta\,h)^{3/2} - 1\right] - \frac{\mu\,h}{2\mu_*} .$$

Distribution speed (6) meets the next volume performance of the sifter:

$$Q \approx 2\pi\left(R_0 + \frac{h}{2}\right)\int_0^h u(\xi)d\xi =$$

$$= 2\pi\left(R_0 + \frac{h}{2}\right)\cdot\left\{\frac{\mu}{15\beta\mu_*}\left[(1 + \beta h)^{3/2} \times \right.\right.$$

$$\left.\left.\times\left[\left(3h - \frac{2}{\beta}\right) + \frac{2}{\beta}\right] - \frac{\mu h^2}{4\mu_*}\right\}. \tag{7}$$

In the protracted approximation the speed $u(\xi)$ is independent of the radius of the cylindrical sifter. So let us find more accurate solution.

In the second approximation we accept:

$$R_0 + \xi - \frac{R_0^2}{R_0 + \xi} \approx 2\xi - \frac{\xi^2}{R_0} .$$

Then:

$$u(\xi) \approx \int_\xi^h \sqrt{\frac{\mu^2}{4\mu_*^2} + \frac{\rho g}{2\mu_*}\left(\xi - \frac{\xi^2}{2R_0}\right)}d\xi + \frac{\mu}{2\mu_*}(\xi - h). \tag{8}$$

The integral in (8) is expressed through elementary functions, for [15]:

$$\int\sqrt{a^2 - \eta^2}\,d\eta = \frac{\eta}{2}\sqrt{a^2 - \eta^2} + \frac{a^2}{2}\arcsin\frac{|\eta|}{a} . \tag{9}$$

Therefore, according to (8) and (9):

$$u(\xi) = \frac{\sqrt{\rho g}}{2\sqrt{2\mu_* R_0}}\left\{(R_0 - \xi)\sqrt{a^2 - (R_0 - \xi)^2} - \right.$$

$$- (R_0 - h)\sqrt{a^2 - (R_0 - h)^2} + a^2\arcsin\frac{R_0 - \xi}{a} -$$

$$-a^2 \arcsin \frac{R_0 - h}{a} \Bigg\} + \frac{\mu}{2\mu_*}(\xi - h). \qquad (10)$$

Here: $a^2 = R_0^2 + \dfrac{\mu^2 R_0}{2\mu_* \rho g}$.

Taking into account that [15]:

$$\int \arcsin \frac{x}{a} dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2},$$

by integrating we obtain volumetric productivity of the sifter, which corresponds to the speed (10):

$$Q = \pi \left(R_0 + \frac{h}{2} \right) \Bigg\{ \frac{\sqrt{\rho g}}{3\sqrt{2\mu_* R_0}} \times$$

$$\times \Big[\left(2a^2 + R_0^2\right)\sqrt{a^2 - R_0^2} -$$

$$-\left(2a^2 + R_0^2 + R_0 h - 2h^2\right)\sqrt{a^2 - (R_0 - h)^2} +$$

$$+3a^2 R_0 \left(\arcsin \frac{R_0}{a} - \arcsin \frac{R_0 - h}{a} \right) \Big] - \frac{\mu h^2}{2\mu_*} \Bigg\}. \qquad (11)$$

As you can see, the second approach of R_0 depend on $u(\xi)$ and Q.

THE MAIN RESULTS OF THE RESEARCH

In order to verify the adequacy of the derived formulas let us calculate *u(0) and Q* if ρ = 750 kg / m^3; R=0,3075m; h=0,012m, and different μ and μ_* / μ. The obtained, using three formulas values, are recorded in the table 1. In table 2 we calculated the value Q in two different formulas.

Table 1. Speed *u(0)* calculated in three formulas

μ, Pa*s	μ_* / μ, s	Form. (5)	Form. (6)	Form. (10)
		Value $u(0)$, m/s		
0,4	0,01	0,744	0,751	0,744
0,4	0,05	0,425	0,428	0,425
0,4	0,10	0,320	0,322	0,320
0,5	0,01	0,634	0,640	0,634
0,5	0,05	0,371	0,374	0,371
0,5	0,10	0,281	0,283	0,281
0,6	0,01	0,555	0,560	0,555
0,6	0,05	0,331	0,334	0,331
0,6	0,10	0,252	0,254	0,252
0,7	0,01	0,494	0,499	0,494
0,7	0,05	0,300	0,303	0,300
0,7	0,10	0,230	0,232	0,230

Table 2. Productivity of sifter Q calculated in two formulas

μ, Pa·s	μ_* / μ, s	Form. (7)	Form. (11)
		$10^3 Q$, m^3/c	
0,4	0,01	10,83	10,72
0,4	0,05	6,02	5,98
0,4	0,10	4,49	4,46
0,5	0,01	9,27	9,18
0,5	0,05	5,27	5,22
0,5	0,10	3,96	3,93
0,6	0,01	8,14	8,05
0,6	0,05	4,72	4,67
0,6	0,10	3,56	3,53
0,7	0,01	7,27	7,18
0,7	0,05	4,29	4,25
0,7	0,10	3,26	3,23

As in action [1], the movement in the sifter can be characterized by an average flow speed, which is expressed by the integral:

$$u_{av} = \frac{1}{h} \int_0^h u(\xi) d\xi.$$

Knowing u_{av}, it is convenient to calculate volumetric productivity of sifter, because:

$$Q = 2\pi \left(R_0 + \frac{h}{2} \right) h u_{av}.$$

It turns out that the values u_{av} calculated using the first and second approximations differ slightly. This is proved by the results of calculations set out in table 3 and table 4.

Table 3. Calculation of u_{av} if *h=0,012* m

μ, Pa·s	μ_* / μ, s	First approximation	Second approximation
		$10 \cdot u_{av}$, m/s	
0,4	0,01	4,76	4,72
0,4	0,05	2,65	2,63
0,4	0,10	1,98	1,96
0,5	0,01	4,08	4,04
0,5	0,05	2,32	2,30
0,5	0,10	1,74	1,73
0,6	0,01	3,58	3,54
0,6	0,05	2,08	2,06
0,6	0,10	1,57	1,55
0,7	0,01	3,20	3,16
0,7	0,05	1,89	1,87
0,7	0,10	1,43	1,42

Table 4. Calculation of u_{av} if $h=0{,}008$ m

μ, Pa·s	μ_* / μ, s	First approximation	Second approximation
		$10 \cdot u_{av}$, m/s	
0,4	0,01	2,39	2,36
0,4	0,05	1,38	1,37
0,4	0,10	1,04	1,04
0,5	0,01	2,03	2,01
0,5	0,05	1,21	1,20
0,5	0,10	0,92	0,91
0,6	0,01	1,77	1,76
0,6	0,05	1,08	1,07
0,6	0,10	0,82	0,81
0,7	0,01	1,57	1,56
0,7	0,05	0,98	0,97
0,7	0,10	0,75	0,75

The value u_{av} depends on the thickness of the rolling bed *h*. With the growth of *h* occurs the growth of u_{av}.

Average speed value is less than its value on the free surface. In action [1] we found that in the case of the linear model when $\mu_* = 0$, $u_{av}/u(0) = 2/3$. Comparing the results in table 1 and table 3 shows that the nonlinear model when $\mu_* > 0$, $u_{av}/u(0) = 2/3$

Calculations show that the difference between the results of the first and second approximations for the numerical data is insignificant. The first approximation gives slightly inflated values of kinematic flow characteristics. So simple formulas of first approximation can be used first of all to estimate values of kinematic characteristics.

From table 1 and table 2 shows that from μ and μ_* substantially depend on *u(0)* and *Q*. This means that the adequacy of the mathematical model is closely linked with the values of the rheological constants. Therefore, to achieve good coordination between theory and experiment, we must properly present μ and μ_*. For this let us determine of rheological constants based on measurements flow rate. Let us set the thresholds of the constants μ and μ_*, ie intervals: $\mu \in [0; \mu_\Gamma]$; $\mu_* \in [0; \mu_{*\Gamma}]$.

Suppose that $\mu_* = 0$. Then, to determine *u(r)*, instead of (4), we obtain the equation:

$$\frac{du}{dr} = \frac{\rho g}{2\mu_\Gamma}\left(\frac{R_0^2}{r} - r\right).$$

Having integrated it with the boundary condition *u(r)=0* we find that:

$$u(r) = \frac{\rho g}{2\mu_\Gamma}\left(\frac{R^2 - r^2}{2} - R_0^2 \ln\frac{R}{r}\right).$$

On the inner surface of the mixture we have:

$$u(R_0) = \frac{\rho g}{2\mu_\Gamma}\left(\frac{R^2 - R_0^2}{2} - R_0^2 \ln\frac{R}{R_0}\right). \qquad (12)$$

where: $S(R_1) = \frac{1}{3}\left[\sqrt{R\left(R^2 - R_1^2\right)} - \sqrt{2}R_1^{3/2}F\left(\varphi_1, \frac{1}{\sqrt{2}}\right)\right]$;

If according to the data of flow rate measurement, with the thickness of the movable layer $h = h_1$ on the inner surface speed is $u_1 = u(R_1)$, and according to (12):

$$\mu_\Gamma = \frac{\rho g}{2u_1}\left(\frac{R^2 - R_1^2}{2} - R_0^2 \ln\frac{R}{R_1}\right), \qquad (13)$$

where $R_1 = R - h_1$.

Using the formula (13), when $\rho = 750$ kg/m^3; $h = 0{,}012$ m; $R = 0{,}3075$ m; $R_1 = 0{,}2955$ m; $u_1 = 0{,}499$ m/s;; we find that $\mu_\Gamma = 1{,}048$ Pa.s. The corresponding values μ for these calculation data in the table 1 is 0.7 Pa.s that is given in the table 1 μ satisfies the inequality $\mu < \mu_\Gamma$, as was the second constant makes $\mu_* = 0{,}01\mu > 0$.

Let us suppose that further in (4) $\mu = 0$. Then:

$$\frac{du}{dr} = -\frac{\sqrt{\rho g}}{\sqrt{2\mu_{*\Gamma}}}\sqrt{r - \frac{R_0^2}{r}}.$$

If the boundary conditions $u(R_0) = 0$, this equation has a solution:

$$u(r) = \frac{\sqrt{\rho g}}{\sqrt{2\mu_{*\Gamma}}}\int_r^R \sqrt{y - \frac{R_0^2}{y}}\,dy,$$

which implies that:

$$u(R_0) = \frac{\sqrt{\rho g}}{\sqrt{2\mu_{*\Gamma}}}\int_{R_0}^R \sqrt{y - \frac{R_0^2}{y}}\,dy.$$

This is expressed through the integral tabulated special functions. To check this, let us move on to a new variable of integration: $y = t^2$; $dy = 2t\,dt$. After this change:

$$u(R_0) = \frac{\sqrt{2\rho g}}{\sqrt{\mu_{*\Gamma}}}\int_{\sqrt{R_0}}^{\sqrt{R}} \sqrt{\left(t^2 - R_0\right)\cdot\left(t^2 + R_0\right)}\,dt.$$

Since [15]^

$$\int_b^x \sqrt{\left(x^2 - b^2\right)\cdot\left(x^2 + b^2\right)}\,dx =$$

$$= \frac{x}{3}\sqrt{\left(x^2 - b^2\right)\cdot\left(x^2 + b^2\right)} - \frac{\sqrt{2}b^3}{3}F(\varphi, k),$$

where: $\varphi = \arccos\frac{b}{x}$, $k = \frac{1}{\sqrt{2}}$, $F(\varphi, k)$– elliptical integral of the first type, then:

$$u(R_0) = \frac{\sqrt{2\rho g}}{\sqrt{\mu_{*\Gamma}}}S(R_0);$$

$$S(R_0) = \frac{1}{3}\left[\sqrt{R\left(R^2 - R_0^2\right)} - \sqrt{2}R_0^{3/2}F\left(\varphi_0, \frac{1}{\sqrt{2}}\right)\right];$$

$$\varphi_0 = \arccos\frac{\sqrt{R_0}}{\sqrt{R}}.$$

Then:

$$\mu_{*\Gamma} = \frac{2\rho g}{u^2(R_1)}S^2(R_1), \qquad (14)$$

$R_1 = R - h_1$; $\varphi_1 = \arccos \frac{\sqrt{R_1}}{\sqrt{R}}$; $u(R_1)$ – the velocity on the free surface of the mixture with the inside thickness of the layer h_1.

The values of elliptic integrals of the first type can be found from the interpolation of tabular data that is published in [21, 22] and other publications with special functions.

In the absence of tables in the calculation $\mu_{*\Gamma}$ instead of (14), you can use an approximate formula:

$$\mu_{*\Gamma} \approx \frac{\rho g R_1^3}{8u^2(R_1)} \times$$
$$\times \left[\arccos \frac{R_1 - h}{R_1} - \frac{R_1 - h}{R_1} \sqrt{1 - \left(\frac{R_1 - h}{R_1} \right)^2} \right]^2, \quad (15)$$

which follows from (10) at $\mu = 0$.

Using (14), we compute $\mu_{*\Gamma}$ where $\rho = 750$ kg/m³; $h = 0{,}012$ m; $R = 0{,}3075$ m; $R_1 = 0{,}2955$ m; $u(R_1) = 0{,}322$ m/s. For these numerical data: $\varphi_1 = 11{,}393^0$. Referring the table [22] by interpolation we find $F\left(\varphi_1, \frac{1}{\sqrt{2}}\right) \approx 0{,}200$. As a result, we obtain $\mu_{*\Gamma} = 0{,}048$ Pa.s² that is more than appropriate μ_* in the table 1, because there $\mu = 0{,}4$ Pa·s > 0. If we calculate $\mu_{*\Gamma}$ by the formula (15), $\mu_{*\Gamma} \approx 0{,}054$ Pa.s², this approximate formula is also suitable for evaluation $\mu_{*\Gamma}$.

For the identification of rheological constants μ and μ_* we need to know the velocity on the free surface with two layer thicknesses of the mixture. Let in thickness h_1 and h_2 the speeds will be u_1 i u_2. Substituting these values in (6), we come to a system of two equations:

$$u_1 = \frac{\mu}{3\beta \mu_*} \left[(1 + \beta h_1)^{3/2} - 1 \right] - \frac{\mu h_1}{2\mu_*};$$
$$u_2 = \frac{\mu}{3\beta \mu_*} \left[(1 + \beta h_2)^{3/2} - 1 \right] - \frac{\mu h_2}{2\mu_*}. \quad (16)$$

Dividing the first of them by the second we get a nonlinear equation with one unknown:

$$\frac{u_1}{u_2} = \frac{(1 + \beta h_1)^{3/2} - 1 - 1{,}5\beta h_1}{(1 + \beta h_2)^{3/2} - 1 - 1{,}5\beta h_2},$$

which we provide to look like:

$$\lambda \left[(1 + \delta x)^{3/2} - 1 - 1{,}5\delta x \right] -$$
$$- (1 + x)^{3/2} + 1 + 1{,}5x = 0. \quad (17)$$

Here $\lambda = \frac{u_1}{u_2}$; $\delta = \frac{h_2}{h_1}$; $x = \beta h_1$.

Transcendental equation (17) we have to solve by numerical methods [23, 24], knowing that $x > 0$. After determining the root ratio $x = x_*$ we can calculate μ / μ_* by the formula:

$$\gamma = \frac{\mu}{\mu_*} =$$
$$= \frac{u_1}{h_1 \left\{ \frac{1}{3x_*} \left[(1 + x_*)^{3/2} - 1 \right] - \frac{1}{2} \right\}},$$

and then the very constants:

$$\mu = \frac{4\rho g h_1}{\gamma x_*}; \quad \mu_* = \frac{\mu}{\gamma}.$$

Consider an example. If the results of measurements are $h_1 = 0{,}008$ m; $h_2 = 0{,}012$ m; $u_1 = 0{,}17$ m/s; $u_2 = 0{,}33$ m/s. For these initial data: $\lambda = 0{,}515$; $\delta = 1{,}5$; . Calculated by dichotomy method [23, 24], the root of the equation (17) is approximately 19.56. Then $\gamma = 19{,}83$ 1/s; $\mu = 0{,}607$ Pa·s; $\mu_* = 0{,}031$ Pa·s². To test the accuracy of the identification substitute value μ and μ_* derived in (16). If $\rho = 750$ kg/m³; $R = 0{,}3075$ m; we obtain: $u_1 = 0{,}168$ m/s; $u_2 = 0{,}327$ m/s. These values of speeds are close to those used for identification confirming acceptable accuracy to solve the inverse problem.

It is easy to check that to satisfy the inequalities identified μ and μ_* and $\mu < \mu_\Gamma$, $\mu_* < \mu_{*\Gamma}$, if μ_Γ and $\mu_{*\Gamma}$ are calculated by formulas (13), (15).

Note that identification can be experimentally measured, in addition to speed, and other characteristics of the kinematic flow. This is stated in [25 - 28], but only for linear motion model.

CONCLUSIONS

1. These formulas allow calculating the speed of a steady flow of mixture and productivity of the sifter.
2. The above kinematic characteristics significantly depend on the values of both rheological constants of quadratic nonlinear model. Therefore, the opportunity to set up proper choice of the values of these constants theory improves accuracy compared with the known linear hydrodynamic analogues.

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КВАДРАТИЧНО-НЕЛИНЕЙНАЯ МОДЕЛЬ ДВИЖЕНИЯ ЗЕРНОВОЙ СМЕСИ ПО ЦИЛИНДРИЧЕСКОМ ВИБРОЦЕНТРОБЕЖНОМ РЕШЕТЕ

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Аннотация. С использованием гидродинамической теории, описано устоявшиеся вертикальное движение псевдорозжижженой смеси по внутренней поверхности виброцентробежного цилиндрического решета, которое вращается с постоянной угловой скоростью вокруг вертикальной оси. Принято квадратичную реологическую зависимость касательного напряжения в смеси от скорости в ней смещения. Составлено нелинейное дифференциальное уравнение движения зерновой смеси и соответствующие ему краевые условия. Построено аналитическое решение краевой задачи в виде квадратуры, которую можно вычислять на компьютере численными методами. Предложено два варианта приближенного аналитического вычисления интеграла, в результате чего получены аналитические решения задачи движения в первом и втором приближениях.

Ключевые слова: математическая модель, псевдорозжижжена сепарированная смесь, квадратичный реологический закон, цилиндрическое видровидцетровое решето, скорость движения, производительность решета, идентификация.