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OIL PRESSURE DISTRIBUTION IN VARIABLE HEIGHT GAPS

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Summary. The paper deals with problems connected with oil flow in variable height gaps in hydraulic machines. On the basis of the Navier-Stokes equations and the continuity equation, equations describing pressure in the gap were determined. The results of calculating pressure distribution in variable height gaps (both confusor and diffuser types) are presented, depending on oil viscosity and relative motion of one of the gap walls.

Key words: variable-height gap, hydraulic oil, pressure distribution.

INTRODUCTION

In pumps and engines within hydraulic systems pairs of surfaces can be found separated by gaps filled with oil [Ivantysyn and Ivantysynova 2001, Stryczek 1984]. Phenomena occurring in the gaps affect the operation of hydraulic systems, for instance energy loss affects the overall efficiency. Thus, taking such factors into consideration is essential in designing hydraulic machines [Lasaar 2000, Osiecki 2004, Podolski 1981, Szydelski and Olechowicz 1986].

A gap in hydraulic machines is an oil-filled space between the two surfaces of neighbouring parts. The distance between the surfaces, referred to as height, is typically of a few micrometers size and of various shapes [Baszta 1966, 1971].

The liquid flow in the gap is a leakage, which can occur in the following cases:

- when there is a pressure difference between the two ends of the gap, in which case it is a pressure flow;
- when one of the surfaces moves with respect to the other, in which case it is a friction flow;
- when pressure difference and surface motion co-occur, in which case it is a pressure-friction flow [Nikitin 1982].

Fig. 1 presents a division of flat gaps. A parallel positioning of the surfaces is an ideal case, but in practice, due to manufacturing imprecision or load asymmetry, variable-height gaps are much more frequent. Among the variable-height gaps, confusor gaps can be distinguished, in which the height decreases with the flow direction, and diffuser gaps, in which the height increases with the flow direction [Trifonow 1974].

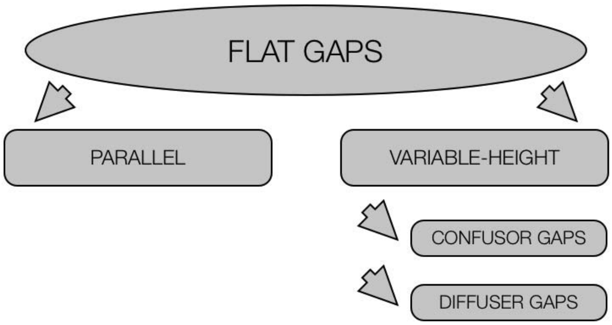

Fig. 1. Division of flat gaps

APPLYING THE NAVIER-STOKES EQUATIONS FOR DETERMINING OIL PRESSURE DISTRIBUTION IN A GAP

Fluid motion is described by means of the Navier-Stokes equations, also known as the motion equations, or the momentum equations. Usually, the flow continuity equation is also applied. For an oil-filled flat gap, it is most convenient to represent those equations in the Cartesian coordinate system x, y, z as [Kondakow 1975, Osipow 1966]:

$$\frac{\partial \mathrm{v}_x}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_x}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_x}{\partial y}+\mathrm{v}_z\frac{\partial \mathrm{v}_x}{\partial z}=\nu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2}+\frac{\partial^2 \mathrm{v}_x}{\partial y^2}+\frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right)-\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{1}$$

$$\frac{\partial \mathrm{v}_y}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_y}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_y}{\partial y}+\mathrm{v}_z\frac{\partial \mathrm{v}_y}{\partial z}=\nu\left(\frac{\partial^2 \mathrm{v}_y}{\partial x^2}+\frac{\partial^2 \mathrm{v}_y}{\partial y^2}+\frac{\partial^2 \mathrm{v}_y}{\partial z^2}\right)-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2}$$

$$\frac{\partial \mathrm{v}_z}{\partial t}+\mathrm{v}_x\frac{\partial \mathrm{v}_z}{\partial x}+\mathrm{v}_y\frac{\partial \mathrm{v}_z}{\partial y}+\mathrm{v}_z\frac{\partial \mathrm{v}_z}{\partial z}=\nu\left(\frac{\partial^2 \mathrm{v}_z}{\partial x^2}+\frac{\partial^2 \mathrm{v}_z}{\partial y^2}+\frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right)-\frac{1}{\rho}\frac{\partial p}{\partial z}. \tag{3}$$

The left-hand sides of equations (1 ÷ 3) represent the inertia forces of the operating agent whereas the right-hand sides represent viscosity forces and pressure of oil [Bukowski 1975, Walczak 2006].

The continuity equation is:

$$\frac{\partial \mathrm{v}_x}{\partial x}+\frac{\partial \mathrm{v}_y}{\partial y}+\frac{\partial \mathrm{v}_z}{\partial z}=0. \tag{4}$$

On the basis of Fig. 2 the current height h of the gap was determined at the distance x from the beginning of the coordinate system:

$$h=\left(h_2-h_1\right)\frac{x}{l}+h_1. \tag{5}$$

Assuming that the oil flows in the direction of the *x* axis, the gap type depends on the heights h_1 and h_2. If $h_1 > h_2$, it is a confusor gap, if $h_1 < h_2$, it is a diffuser gap. If h_1 and h_2 are equal, it is a parallel gap.

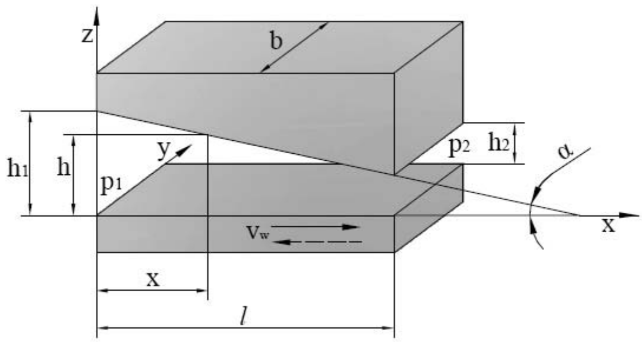

Fig. 2. Diagram of a flat confusor gap

It is assumed that oil, whose viscosity is determined by the dynamic viscosity coefficient μ being under the constant pressure gradient, flows through a flat cline gap confined by impenetrable, perfectly smooth and inelastic walls. The upper wall is stationary and the lower wall, being subject to external forces, counteracts resistance due to oil viscosity, and moves with constant horizontal velocity v_w with respect to the stationary wall. The motion of the particles of incompressible oil with the turn of the *x* axis) is steady, constant and rectilinear. After adopting the assumptions: $v_x = v_x(x,z)$, $v_y = 0$ and $v_z = 0$, equations (1 ÷ 4) become simplified:

$$\mathrm{v}_x \frac{\partial \mathrm{v}_x}{\partial x} = \nu \left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2} + \frac{\partial^2 \mathrm{v}_x}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial p}{\partial x} . \tag{6}$$

$$0 = \frac{\partial p}{\partial y} , \tag{7}$$

$$0 = \frac{\partial p}{\partial z} , \tag{8}$$

$$\frac{\partial \mathrm{v}_x}{\partial x} = 0 . \tag{9}$$

It follows from (6 ÷ 9) that $v_x = v_x(z)$ and $p = p(x)$.

Thus, the motion of oil in the gap is represented by the following differential equation:

$$0 = \nu \frac{\partial^2 \mathrm{v}_x}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} . \tag{10}$$

With the dynamic viscosity coefficient μ:

$$\mu = \nu\rho . \tag{11}$$

Equation (10) becomes:

$$\frac{\partial^2 \mathrm{v}_x}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}. \tag{12}$$

And after integrating equation (12) twice:

$$\mathrm{v}_x = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{z^2}{2} + C_1 z + C_2. \tag{13}$$

The integration constants C_1 and C_2 were determined on the basis of the following boundary conditions:

$v_x = v_w$ for $z = 0$, and $v_x = 0$ for $z = h$.

Ultimately, the formula for flow velocity in the gap is:

$$\mathrm{v}_x = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(z^2 - hz\right) - \frac{\mathrm{v}_\mathrm{w}}{h} z + \mathrm{v}_\mathrm{w}. \tag{14}$$

The oil flow intensity occurring in a gap of the height b can be represented as:

$$Q = b\int_0^h \mathrm{v}_x dz. \tag{15}$$

Then, substituting (14) into (15):

$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial x} bh^3 + \mathrm{v}_\mathrm{w}\frac{h}{2} b. \tag{16}$$

The pressure distribution $p(x)$ of oil in a gap can be found from the following equation [Ivantysynova 2009]:

$$p(x) = \int \frac{\partial p}{\partial x} dx. \tag{17}$$

The differential $\partial p/\partial x$ was obtained from (16):

$$\frac{\partial p}{\partial x} = \frac{6\mu \mathrm{v}_\mathrm{w}}{h^2} - \frac{12\mu Q}{bh^3}. \tag{18}$$

Substituting (18) into equation (17):

$$p(x) = \int \left(\frac{6\mu \mathrm{v}_\mathrm{w}}{h^2} - \frac{12\mu Q}{bh^3} \right) dx. \tag{19}$$

When equation (5) is taken into account, equation (19) becomes:

$$p(x) = \int \left[\frac{6\mu \mathrm{v}_\mathrm{w}}{\left(\left(h_2 - h_1\right)\frac{x}{l} + h_1 \right)^2} - \frac{12\mu Q}{b\left(\left(h_2 - h_1\right)\frac{x}{l} + h_1 \right)^3} \right] dx. \tag{20}$$

Then, after integrating equation (20):

$$p(x)=\frac{6\mu v_w}{\frac{(h_2-h_1)}{l}(-1)}\cdot\frac{1}{\frac{(h_2-h_1)}{l}x+h_1}-\frac{12\mu Q}{b\cdot\frac{(h_2-h_1)}{l}(-2)}\cdot\frac{1}{\left[\frac{(h_2-h_1)}{l}x+h_1\right]^2}+C. \tag{21}$$

The integration constant C was obtained on the assumption that pressure at the entrance to the gap is p_1 (when $x = 0$ $p = p_1$). Thus, the integration constant C is:

$$C=p_1+\frac{6\mu v_w l}{(h_2-h_1)h_1}-\frac{6\mu Ql}{b(h_2-h_1)h_1^2}, \tag{22}$$

and the pressure distribution $p(x)$ of oil in the gap is:

$$p(x)=\frac{6\mu l}{h_2-h_1}\left(-\frac{v_w}{h}+\frac{Q}{bh^2}+\frac{v_w}{h_1}-\frac{Q}{bh_1^2}\right)+p_1. \tag{23}$$

The difference $h_2 - h_1$ can be obtained by transforming equation (5):

$$h_2-h_1=(h-h_1)\frac{l}{x}. \tag{24}$$

Then, substituting (24) into equation (23):

$$p(x)=\frac{6\mu x}{h-h_1}\cdot\left(v_w\frac{h-h_1}{h_1h}+\frac{Q}{b}\cdot\frac{h_1^2-h^2}{h_1^2h^2}\right)+p_1. \tag{25}$$

After transformations, formula (25) becomes:

$$p(x)=\frac{6\mu x}{h_1h}\cdot\left(v_w-\frac{Q}{b}\frac{h+h_1}{hh_1}\right)+p_1. \tag{26}$$

After substituting the boundary conditions $x = l$, $p = p_2$ and $h = h_2$ into equation (26) a formula for the oil flow intensity in the gap was obtained:

$$Q=\frac{(p_1-p_2)\cdot b\cdot(h_1\cdot h_2)^2}{6\mu l\cdot(h_1+h_2)}+\frac{v_w\cdot b\cdot h_1\cdot h_2}{h_1+h_2}. \tag{27}$$

When equations (27) and (26) are taken into consideration, the equation of oil pressure distribution in a variable-height gap finally becomes:

$$p(x)=\frac{6\mu x}{h_1h}\cdot\left[v_w-\left(\frac{(p_1-p_2)(h_1\cdot h_2)^2}{6\mu l\cdot(h_1+h_2)}+v_w\frac{h_1\cdot h_2}{h_1+h_2}\right)\frac{h+h_1}{h\cdot h_1}\right]+p_1. \tag{28}$$

RESULTS OF SIMULATION EXPERIMENTS ON OIL PRESSURE DISTRIBUTION IN A VARIABLE-HEIGHT GAP

The simulation experiments on oil pressure distribution in a variable-height gap were conducted by means of the available computer programs.

The following input data were assumed in the computations:

- pressure at the gap entrance p_1 = 16 [MPa],
- pressure at the gap exit p_2 = 0 [MPa],
- gap length l = 0,030 [m],
- gap height b = 0,015 [m],
- dynamic viscosity coefficient within the range of 0,0122 to 0,0616 [Pas],
- relative velocity of one of the walls – 5 to 5 [m/s].

The entrance and exit heights for the confusor gap are h_1 = 36 [μm] and h_2 = 12 [μm], respectively, and for the diffuser gap h_1 = 12 [μm] and h_2 = 36 [μm].

Fig. 3 presents oil pressure distributions for flow in a confusor gap (the convex curve), a parallel gap (the linear pressure) and a diffuser gap (the concave curve).

Fig. 4 presents oil pressure distributions for friction flows caused by a relative motion of one of the walls in a confusor and diffuser gaps, depending on the velocity.

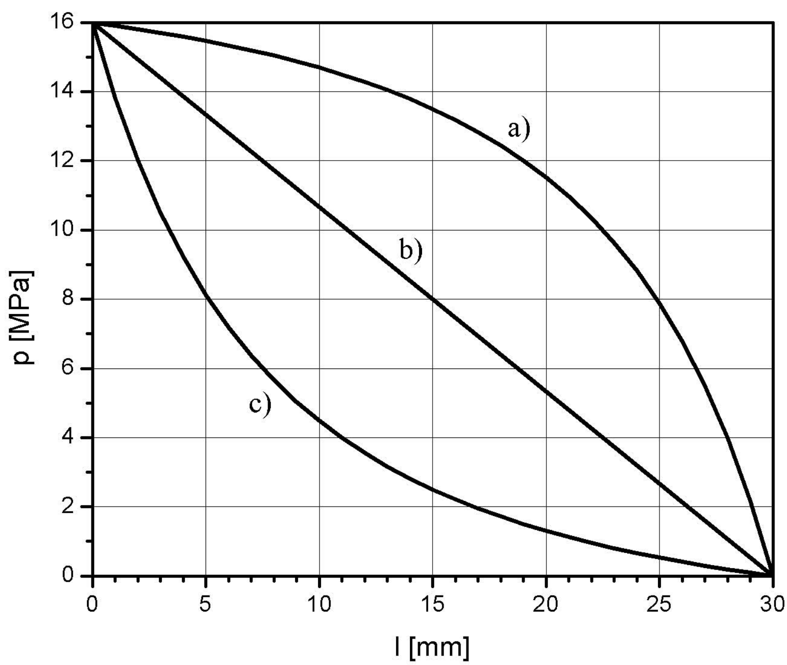

Fig. 3. Oil pressure distributions for pressure flows in a flat gap
a) confusor gap, b) parallel gap, c) diffuser gap

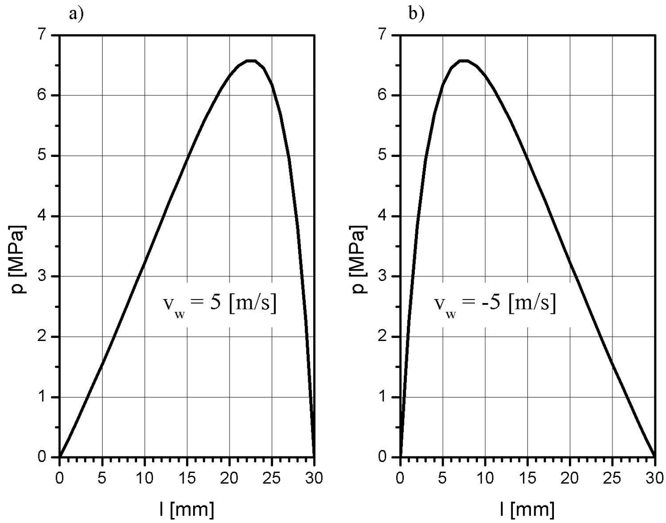

Fig. 4. Oil pressure distributions in a variable-height gap for friction flows depending on the velocity of the lower wall for a) confusor gap, b) diffuser gap

An interesting case of flow in a variable-height gap is the pressure-friction flow caused both by the pressure difference and the relative motion of one of the walls.

Fig. 5 presents oil pressure distributions in a gap depending on the dynamic viscosity coefficient, with relative motion of one of the walls in the direction of the flow, for confusor and diffuser gaps.

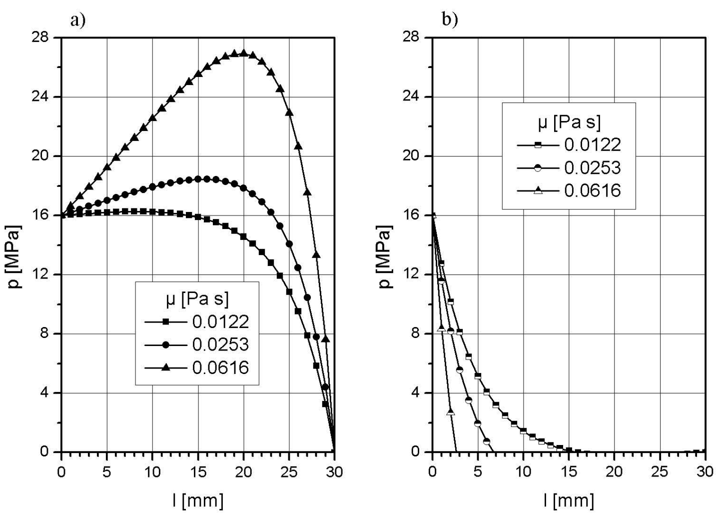

Fig. 5. Oil pressure distributions in the gap depending on the dynamic viscosity coefficient with the relative motion of the lower wall in the direction of the flow (p_1 = 16 MPa, p_2 = 0 MPa, v_w = 5 m/s) for a) confusor gap, b) diffuser gap

Fig. 6 shows oil pressure distributions in a gap with relative motion of one of the walls in the opposite direction to the flow, depending on the dynamic viscosity coefficient for confusor and diffuser gaps.

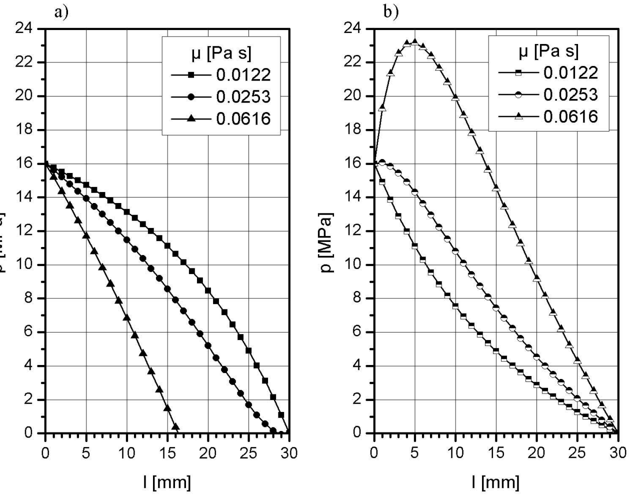

Fig. 6. Oil pressure distributions in a gap depending on the dynamic viscosity coefficient with the lower wall moving in the direction opposite to the flow (p_1 = 16 MPa, p_2 = 0 MPa, v_w = -5 m/s) for a) confusor gap, b) diffuser gap

Figs. 7 and 8 present oil pressure distributions in gaps depending on the relative velocity of one of the walls for confusor and diffuser gaps, respectively.

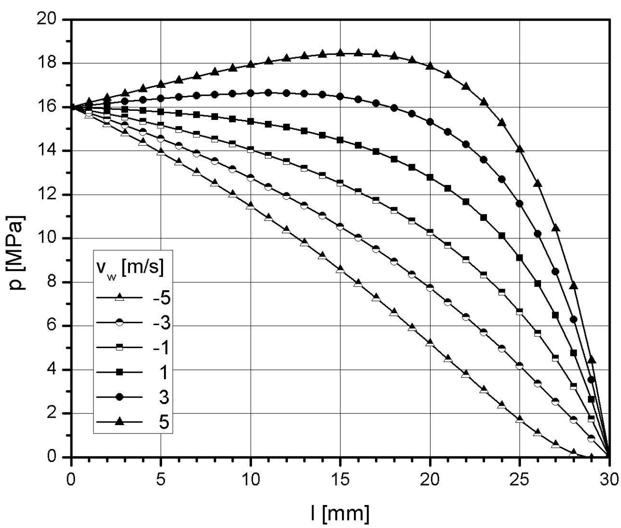

Fig. 7. Oil pressure distributions in a confusor gap depending on the relative velocity v_w of the lower wall (p_1 = 16 MPa, p_2 = 0 MPa)

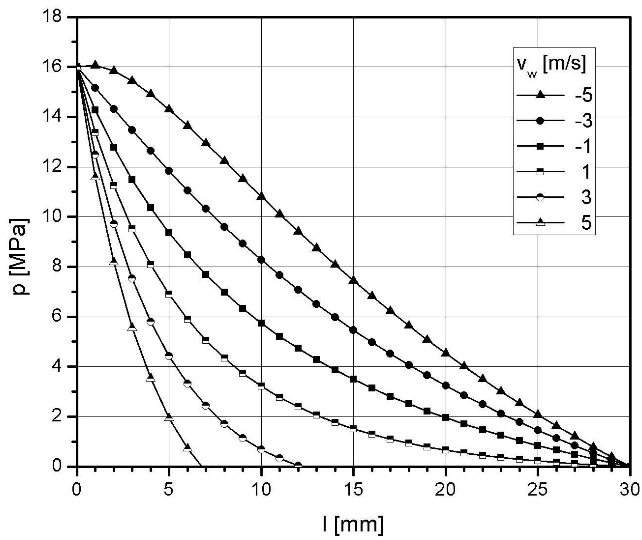

Fig. 8. Pressure distributions in a diffuser gap depending on the relative velocity v_w of the lower wall (p_1 = 16 MPa, p_2 = 0 MPa)

The analysis of oil pressure distributions in variable-height gaps demonstrates that in confusor type gaps the pressure increases along the gap, whereas in diffuser type gaps the pressure drops, compared to the pressure in a parallel gap.

CONCLUSIONS

The conducted analyses lead to the following conclusions:

1. The computation model assumed in the study makes it possible to find pressure distributions in flat variable-height gaps.
2. The pressure distribution depends by and large on the dynamic viscosity of oil and relative motion of one of the walls.
3. As compared to a parallel gap, confusor type gaps show increase in oil pressure, whereas diffuser type gaps show decrease in oil pressure.
4. Oil flow in variable-height gaps will be subject to further investigation.

REFERENCES

1. Baszta T.M., 1966.: Hydraulika w budowie maszyn. Poradnik. WNT, Warszawa.
2. Baszta T.M., 1971.: Maszinostroitielnaja gidrawlika. Maszinostrojenie, Moskwa.
3. Bukowski J., 1975.: Mechanika płynów. PWN, Warszawa.
4. Ivantysyn J., Ivantysynova M., 2001: Hydrostatic Pumps and Motors. Academia Books International. New Delhi.
5. Ivantysynova M., 2009.: Design and Modeling of Fluid Power Systems. Purdue University.
6. Kondakow L. A., 1975.: Uszczelnienia układów hydraulicznych. WNT, Warszawa.

7. Lasaar R., 2000.: The Influence of the Microscopic and Macroscopic Gap Geometry on the Energy Dissipation in the Lubricating Gaps of Displacement Machines. Proc. of 1-st FPNI-PhD Symposium. Hamburg.
8. Nikitin G.A., 1982.: Szczielewyje i łabirintnyje upłotnienia gidroagregatow. Maszinostrojenie, Moskwa.
9. Osiecki A., 2004.: Hydrostatyczny napęd maszyn. WNT, Warszawa.
10. Osipow A.F., 1966.: Objemnyje gidrowliczieskie masziny. Maszinostrojenie, Moskwa.
11. Podolski M.E., 1981.: Upornyje podszipniki skolżenia. Leningrad.
12. Stryczek S., 1984: Napęd hydrostatyczny. Elementy i układy. WNT, Warszawa.
13. Szydelski Z.: Olechowicz J., 1986.: Elementy napędu i sterowania hydraulicznego i pneumatycznego. PWN, Warszawa.
14. Trifonow O.N., 1974.: Gidrawliczieskie sistemy mietałorieżuszczych stankow. MSI, Moskwa.
15. Walczak J., 2006.: Inżynierska mechanika płynów. Wyd. Politechniki Poznańskiej, Poznań.

ROZKŁADY CIŚNIENIA OLEJU W SZCZELINACH PŁASKICH O ZMIENNEJ WYSOKOŚCI

Streszczenie. W opracowaniu przedstawiono problematykę związaną z przepływami oleju przez szczeliny płaskie o zmiennej wysokości występujące w różnego rodzaju maszynach hydraulicznych. Na podstawie równań Naviera-Stokesa i równania ciągłości wyznaczono zależności określające ciśnienie panujące wzdłuż szczeliny. Przedstawiono rezultaty obliczeń rozkładów ciśnienia w szczelinach o zmiennej wysokości konfuzorowych i dyfuzorowych w zależności od lepkości oleju i ruchu względnego jednej ze ścianki tworzącej szczelinę.

Słowa kluczowe: szczelina zmiennej wysokości, olej hydrauliczny, rozkłady ciśnienia.