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DESCRIPTION AND PRICING OF SELECTED TWO-ASSET OPTIONS AND SUGGESTIONS CONCERNING THEIR USE ON THE GRAIN MARKET IN POLAND

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Abstract. Multi-asset options are options with payoffs affected by at least two underlying instruments. These instruments can be assets such as stocks, indices, currencies or commodities. An important factor influencing values of multi-asset options is correlation among log-returns of underlying assets. Thus the options are also called correlation options. In the paper there are presented description and models for pricing selected correlation options: quotient, product, spread and two-color rainbow options. There are also given some examples of their use on the grain market in Poland.

Key words: multi-asset options, correlation, Black-Scholes model modifications, grain market

INTRODUCTION

Option contracts have been known and used for a long time now. A story about Thales presented in the works by Aristotle is believed to be the first mention of practical use of options. The story is about Thales' idea to exercise option on olive presses [Ong 1996]. However, the origin of contemporary option markets dates back to 1790 when options on agricultural commodities were introduced into the United States of America in order to hedge against price fluctuation of cereals supplied only after harvest. First financial options, i.e. stock options, were introduced in the 19th century. Originally, they were sold on the over-the-counter market. It was in 1973, when the Chicago Board Options Exchange was established, that these options became subject to public trading. Shortly after, other financial assets became underlying assets for options: in 1981 – debt instruments, in 1982 – currencies and futures contracts, and finally in 1983 – stock indexes [Dębski 2005]. Nowadays, the most important world exchanges, such as the Chicago Board of Trade,

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Chicago Mercantile Exchange, New York Board of Trade, Euronext.Liffe, Borsa de Mercado e Futuros or South African Futures Exchange, offer both commodity and financial options[1].

As far as exchange traded commodity options are concerned, options on futures contracts constitute the majority. Nevertheless, wider range of commodity options is available on the over-the-counter market. These are as a rule non-standard contracts (so-called exotic options) for which underlying assets are mainly spot prices and not futures prices of goods. The range of non-standard options is virtually unlimited. If an investor needs innovative construction, financial engineering methods enable him/her to invent a family of new exotic products meeting his/her preferences. Nevertheless, in practice certain types of options are popular and belong to the following classes: path-dependent options, time-dependent options, binary options and correlation options. According to Alexander and Venkatramanan [2008], in the case of commodity options, hybrid options (caps, floors, corridors) are generally used, and then: Asian options, barrier options and spread options. However, Geman [2007] mentions multi-asset options, namely quanto, exchange and the aforesaid spread options, apart from Asian and barrier options. She is inclined to believe that spread options are the most popular on energy markets together with forward start options belonging to the group of time-dependent options.

Apart from exchange and spread options, other popular examples of particularly interesting group of multi-asset options are quotient options, product options, basket options and rainbow options. Description, pricing methods and examples of using basket options on agricultural products can be found in the book by Krawiec and Krawiec [2002] and exchange options – in the paper by Krawiec [2010]. The present paper is aimed at describing and discussing methods for pricing as well as presenting exemplary uses of selected two-asset options on the grain market in Poland.

METHODS FOR PRICING SELECTED TWO-ASSET OPTIONS

Multi-asset, multivariate or correlation options are instruments whose payment depends on at least two underlying assets not necessarily belonging to the same class. These include stock prices, exchange rates, index values or prices of commodities. Options that are written on various types of assets are referred to as cross-asset options [Pruchnicka-Grabias 2006]. Although multi-asset options may cover any number of assets, in practice the majority are two-asset options (basket options, usually written on a great number of underlying assets, are exception to the rule).

In the case of every multi-asset option, there are more variables that have effect on the value of such an option (compared to one-asset options). For two-asset options, these variables are underlying assets prices S_1 and S_2, exercise price or prices X (in the case of multi-asset options there may be one or two exercise prices), volatilities of underlying assets σ_1 and σ_2, risk-free rate r, dividend yields of underlying assets q_1 and q_2, time to expiration T and correlation in the form of coefficient of correlation between log-returns

[1] A detailed discussion on derivatives offered by separate exchanges was provided by Banks [2003].

of underlying assets ρ. Since the number of variables determining the value of option is greater, the number of Greek parameters[2] to be analyzed is greater as well. In the case of two-asset options, one should calculate and interpret two deltas, gammas and vegas as well as additional Greek parameter – chi that determines sensitivity of option premium with respect to the correlation between asset values [Zahng 2006].

Quotient options

As the name suggests, quotient, ratio or relative outperformance options are written on the quotient of prices of two underlying assets. They may be used for deriving benefits from relative change in two securities, markets or portfolios. Although similar objectives may be accomplished by means of spread options, quotient options have certain advantage over them, namely for the purpose of pricing one may easily adopt analytical approach in the form of simple modification of Black-Scholes model. The holder of quotient call option captures profit if the quotient is subject to increase, whereas the buyer of quotient put option – when the quotient is subject to decrease. On the day of expiration a quotient call option gives the following payoff:

$$\max\left[\frac{S_1}{S_2} - X,\ 0\right]$$

and a quotient put option pays off:

$$\max\left[X - \frac{S_1}{S_2},\ 0\right],$$

where: S_1 and S_2 – prices of the first and the second underlying asset respectively,
X – exercise price.

The value of quotient call option may be evaluated in a following way [Haug 2007]:

$$c = e^{-rT}[FN(d_2) - XN(d_1)] \tag{1}$$

and the value of quotient put option:

$$p = e^{-rT}[XN(d_1) - FN(d_2)] \tag{2}$$

[2] Greek parameters measure the sensitivity of option value to small changes in given underlying parameters. Basic Greeks are: delta (it measures the rate of change of option value with respect to changes in underlying asset price), gamma (it measures the rate of change in the delta with respect to changes in the underlying asset price), vega (it measures the sensitivity to the volatility of underlying asset), theta (it measures the sensitivity of the value of the option to the passage of time) and rho (it measures the sensitivity to the interest rate). For more detailed information on traditional and modern Greeks, see Haug [2007], Kolb and Overdahl [2007], Zahng [2006] or Hull [2012].

where:

$$F = \frac{S_1}{S_2} e^{(b_1 - b_2 + \sigma_2^2 - \rho\sigma_1\sigma_2)T} \quad (3)$$

$$d_1 = \frac{\ln(F/X) + T\hat{\sigma}^2/2}{\hat{\sigma}\sqrt{T}} \quad (4)$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{T} \quad (5)$$

$$\hat{\sigma} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (6)$$

and σ_1, σ_2 – historical volatilities of assets 1 and 2 respectively,
r – risk-free rate,
T – time to expiration,
b_1 and b_2 – costs of carry[3] of assets 1 and 2,
$N(d)$ – the cumulative probability distribution function for a standardized normal distribution.

The value of quotient option decreases monotonically with the correlation coefficient. Chi, measuring the sensitivity of premium to change in the correlation between the continuously compounded returns of the two underlying assets, takes negative value both for call option and put option. The higher the positive (negative) correlation between underlying assets, the greater the probability that prices will change in the same (opposite) direction and that the value of ratio will be lower (higher). For the owner of quotient call option, increase in the price of the first asset and decrease in the price of the second asset is advantageous. Whereas for the owner of quotient put option decrease in the price of the first asset and increase in the price of the second asset is favorable. Figure 1 presents quotient call and put prices depending on the value of correlation between underlying assets.

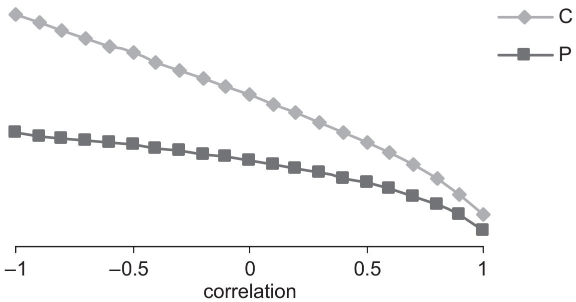

Fig. 1. Quotient call and put prices as a function of the correlation between underlying assets log-returns

Source: Own elaboration.

[3] For a non-dividend-paying stock, the cost of carry is r, for a stock index, it is $r - q$, for a currency, it is $r - r_f$, for a commodity that provides income at rate q and requires storage costs at rate u, it is $r - q + u$, and so on [Hull 2012].

Product options

Product options are written on the product of prices of two underlying assets. Exchange rate is often one of these assets. It is expressed in domestic currency. The other is the price of asset that is traded on foreign stock exchange markets. Such a construction is referred to as foreign domestic option [Gudaszewski and Łukojć 2004]. Furthermore, product options can be used for hedging the revenue of a company, because the revenue is the product of the commodity sales and the product price. Then the price of product is the price of the first security (S_1) and sold production is the value of the second security (S_2).

On the day of expiration a product call option gives the following payoff:

$$\max(S_1 \cdot S_2 - X, 0),$$

and a product put option similarly:

$$\max(X - S_1 \cdot S_2, 0).$$

Formulas for their pricing were given by Haug [2007], respectively for a call option:

$$c = e^{-rT}[FN(d_2) - X N(d_1)] \tag{7}$$

and a put option:

$$p = e^{-rT}[X N(d_1) - FN(d_2)] \tag{8}$$

where:

$$F = S_1 S_2 e^{(b_1 + b_2 + \rho\sigma_1\sigma_2)T} \tag{9}$$

d_1, d_2 and $\hat{\sigma}$ follow equations (4), (5) and (6).

Value of product option depends, among other things, on correlation between two underlying assets. The higher positive correlation, the more changeable the product of prices. The values of underlying assets change in an analogical way, and rise or decline is increased by multiplying two prices. The stronger positive correlation, the higher the call and put options premiums and chi always takes positive values. Figure 2 presents product call and put prices depending on the value of correlation between underlying assets.

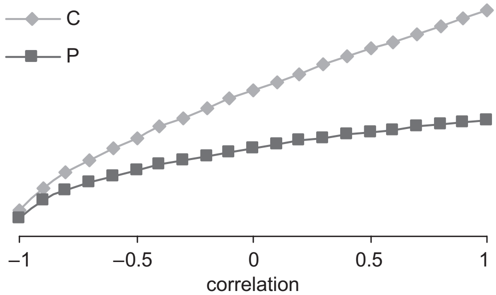

Fig. 2. Product call and put prices as a function of the correlation between underlying assets log-returns

Source: Own elaboration.

Spread options

Spread options, just as quotient options, are characterized by the fact that their payoffs depend on relative changes in the prices of two assets. These options may be used when investor wants to alter his/her exposure or construct in a synthetic way exposure to asset to which he/she has no or limited access for certain reason. Buying options on the difference between the price of product and the price of material used to manufacture it, enterprise may protect its margin of operating profit. This practice is particularly popular among companies refining crude oil. In 1994 the New York Mercantile Exchange placed on the market options on the difference between the price of lead-free petrol and the price of crude oil. Other examples of crack spread options may include: heating oil versus crude oil, gas oil versus crude oil, jet oil versus crude oil, white sugar versus raw sugar [Nelken 2000].

On the day of expiration a spread call option gives the following payoff:

$\max(S_1 - S_2 - X, 0)$,

whereas a spread put option payoffs:

$\max(X - S_1 + S_2, 0)$.

Thus, a holder of a spread call option will exercise it if the spread between the prices of the first and the second asset is higher than the exercise price. A spread put option is to be exercised when the spread between prices is lower than the exercise price.

Spread options values may be evaluated by the use of some Black-Scholes model modification proposed by Kirk [1995]. Its generalized form was provided by Haug [2007]:

– for a call:

$$c \approx (Q_2 S_2 e^{(b_2 - r)T} + X e^{-rT})[S N(d_1) - N(d_2)] \tag{10}$$

– for a put:

$$p \approx (Q_2 S_2 e^{(b_2 - r)T} + X e^{-rT})[N(-d_2) - S N(-d_1)] \tag{11}$$

where:

$$d_1 = \frac{\ln(S) + (\sigma^2 / 2)T}{\sigma \sqrt{T}} \tag{12}$$

$$d_2 = d_1 - \sigma \sqrt{T} \tag{13}$$

$$S = \frac{Q_1 S_1 e^{(b_1 - r)T}}{Q_2 S_2 e^{(b_2 - r)T} + X e^{-rT}} \tag{14}$$

$$\sigma \approx \sqrt{\sigma_1^2 + (\sigma_2 F)^2 - 2\rho \sigma_1 \sigma_2 F} \qquad 15)$$

$$F = \frac{Q_2 S_2 e^{(b_2 - r)T}}{Q_2 S_2 e^{(b_2 - r)T} + X e^{-rT}} \tag{16}$$

and Q_1, Q_2 are quantities of assets 1 and 2 respectively.

Analogically to previous cases, correlation between assets has an impact on the value of spread options. High positive correlation implies that S_1 and S_2 are subject to a similar change. Difference between them is characterized by lower volatility and value of call option is lower then. The same is the case with put option, and chi takes the same value for spread call and put options with analogical input variables. Figure 3 presents spread call and put prices depending on the value of correlation between underlying assets.

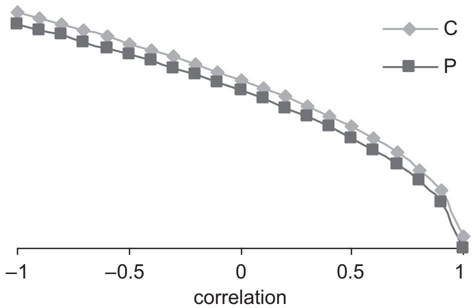

Fig. 3. Spread call and put prices as a function of the correlation between underlying assets log-returns

Source: Own elaboration.

As far as spread options are concerned, value of options is also determined by the spread between two assets. The higher the spread, the higher the call option and the lower the put option premiums.

Rainbow options

Rainbow options, likewise other multi-asset options, are written on more than one underlying asset. At the same time, every asset is analyzed individually, which makes rainbow options different from basket ones. The majority of rainbow options are stock options or stock indexes options. In this case, payoff is a difference between maximum or minimum value of underlying asset and exercise price (for call options) or between exercise price and maximum or minimum value of underlying asset (for put options) on a prespecified expiry date. If this difference is not positive, payment is not made. Depending on the number of underlying assets, rainbow options are referred to as two-colour, three-colour, four-colour etc. However, two-colour options constitute the majority. Rainbow options may be used by investors who are not sure about the market on which they want to invest their resources.

Although Kolb and Overdahl [2007] described series of different rainbow options (call on the best of two risky assets and cash, call on the maximum of two risky assets, call on the better of two risky assets, put on the maximum of two risky assets, call on the minimum of two risky assets, call on the worse of two risky assets, and put on the minimum of two risky assets), here we focus on the following four basic ones:

- call on the minimum of two assets paying off: $\max[\min(S_1, S_2) - X, 0]$,
- call on the maximum of two assets paying off: $\max[\max(S_1, S_2) - X, 0]$,
- put on the minimum of two assets paying off: $\max[X - \min(S_1, S_2), 0)$,
- put on the maximum of two assets paying off: $\max[X - \max(S_1, S_2), 0]$.

Formulas for pricing options on the minimum or the maximum of two assets, proposed for the very first time in 1982 by Stulz, are cited by Haug [2007]:

– for call on the minimum of two assets:

$$c_{\min}(S_1, S_2, X, T) = S_1 e^{(b_1 - r)T} M(y_1, -d; -\rho_1) + S_2 e^{(b_2 - r)T} M(y_2, d - \sigma\sqrt{T}; -\rho_2) - \\ - Xe^{-rT} M(y_1 - \sigma_1\sqrt{T}, y_2 - \sigma_2\sqrt{T}; \rho) \quad (17)$$

where:

$$d = \frac{\ln(S_1 / S_2) + (b_1 - b_2 + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad (18)$$

$$y_1 = \frac{\ln(S_1 / X) + (b_1 + \sigma_1^2 / 2)T}{\sigma_1\sqrt{T}} \quad (19)$$

$$y_2 = \frac{\ln(S_2 / X) + (b_2 + \sigma_2^2 / 2)T}{\sigma_2\sqrt{T}} \quad (20)$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad (21)$$

$$\rho_1 = \frac{\sigma_1 - \rho\sigma_2}{\sigma} \quad (22)$$

$$\rho_2 = \frac{\sigma_2 - \rho\sigma_1}{\sigma} \quad (23)$$

and $M(\bullet)$ – bivariate normal cumulative probability;

– for call on the maximum of two assets:

$$c_{\max}(S_1, S_2, X, T) = S_1 e^{(b_1 - r)T} M(y_1, d; \rho_1) + S_2 e^{(b_2 - r)T} M(y_2, -d + \sigma\sqrt{T}; \rho_2) - \\ - Xe^{-rT} [1 - M(-y_1 + \sigma_1\sqrt{T}, -y_2 + \sigma_2\sqrt{T}; \rho) \quad (24)$$

– for put on the minimum of two assets:

$$p_{\min}(S_1, S_2, X, T) = Xe^{-rT} - c_{\min}(S_1, S_2, 0, T) + c_{\min}(S_1, S_2, X, T) \quad (25)$$

where:

$$c_{\min}(S_1, S_2, 0, T) = S_1 e^{(b_1 - r)T} - S_1 e^{(b_1 - r)T} N(d) + S_2 e^{(b_2 - r)T} N(d - \sigma\sqrt{T}) \quad (26)$$

– for put on the maximum of two assets:

$$p_{\max}(S_1, S_2, X, T) = Xe^{-rT} - c_{\max}(S_1, S_2, 0, T) + c_{\max}(S_1, S_2, X, T) \tag{27}$$

where:

$$c_{\max}(S_1, S_2, 0, T) = S_2 e^{(b_2 - r)T} + S_1 e^{(b_1 - r)T} N(d) - S_2 e^{(b_2 - r)T} N(d - \sigma\sqrt{T}) \tag{28}$$

Coefficient of correlation between underlying assets is a significant parameter affecting the value of rainbow option, too. The following rule is observed for a call on the minimum of two assets and for a put on the maximum of two assets: the closer the correlation coefficient is to +1, the more expensive the options, and the closer the correlation coefficient is to –1, the lower the option premiums. Different relation is observed in the case of call on the maximum of two assets and put on the minimum of two assets: the closer the correlation is to +1, the less expensive the options, whereas the closer the correlation is to –1, the more expensive the options [Pruchnicka-Grabias 2006]. Figures 4 and 5 present rainbow call and put prices depending on the value of correlation between underlying assets.

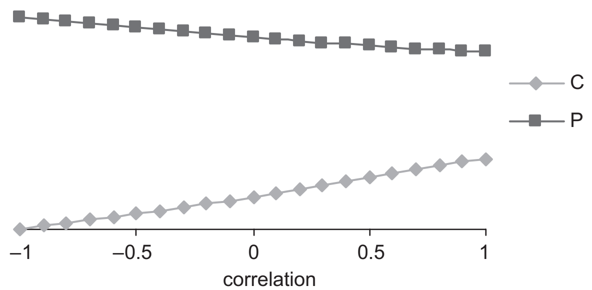

Fig. 4. Prices of rainbow call and put on the minimum of two assets as a function of the correlation between underlying assets log-returns

Source: Own elaboration.

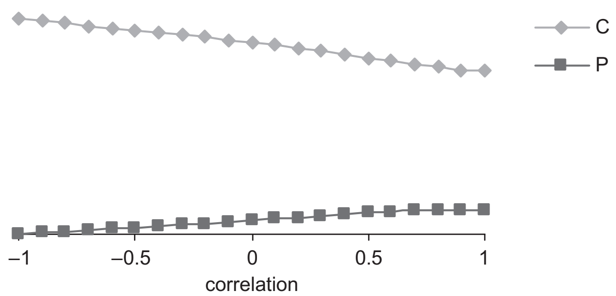

Fig. 5. Prices of rainbow call and put on the maximum of two assets as a function of the correlation between underlying assets log-returns

Source: Own elaboration.

EXAMPLES OF USING SELECTED TWO-ASSET OPTIONS ON THE GRAIN MARKET IN POLAND

In order to develop input variables for two-asset options, empirical data was used in the form of average weekly prices of cereal in Poland. It is collected as a part of Integrated System of Agricultural Market Information and available at the website of the Ministry of Agriculture and Rural Development (www.minrol.gov.pl). The aforementioned data were used for determining historical volatilities of particular underlying assets (σ), their prices on the day contracts were written (S_0) and on the days of their expiration (S_T), and correlation between the logarithmic rates of return (ρ). Two variants of option life were taken into account, namely $T_1 = 6$ and $T_2 = 12$ months. It was assumed that the options were issued on 20th July 2010. Consequently, expiry dates were 20th January 2011 and 20th July 2011 respectively[4]. Table 1 and Table 2 present variables essential for pricing. Both historical volatilities and correlation coefficients were determined on the basis of data for six months before the issue of options. It might be noticed that throughout the period under analysis the highest volatility was the case with feed barley, whereas the lowest one – with feed corn. Nonetheless, this outcome is similar to the results for milling wheat (Table 1). The highest positive correlation was observed for the pair: milling wheat – feed wheat, and the lowest (not significant at 0.05) for the pair: milling wheat – feed barley (Table 2).

Table 1. Basic parameters of underlying assets for considered two-asset options

Parameter	Commodity			
	milling wheat	feed wheat	feed barley	feed corn
σ (%)	11.8	13.4	19.9	11.6
S_0 (PLN·t^{-1})	555	555	413	602
S_{T1} (PLN·t^{-1})	930	878	794	854
S_{T2} (PLN·t^{-1})	920	882	718	963

Source: Own elaboration.

Table 2. Matrix of coefficients of correlation between log-returns of analyzed commodities

Commodity	milling wheat	feed wheat	feed barley	feed corn
Milling wheat	1	×	×	×
Feed wheat	0.78	1	×	×
Feed barley	0.03	0.40	1	×
Feed corn	0.52	0.68	0.42	1

Source: Own calculations.

[4] The dates were selected after the detailed analysis of cereal prices from 2007 through 2011. It was aimed at revealing 6- and 12-month periods reflecting different market conditions in order to show alternative consequences of adopting separate options.

In the next step of the research separate two-asset options were created end priced. As all of them were European options that could be exercised only on the day of expiration, for the purpose of their pricing one could use models, presented in the first part of the paper, being modifications of the Black-Scholes method. While pricing the options under analysis, the following rule was applied: contracts on commodities that are investment assets are evaluated analogically to contracts on an investment asset that provide no income, for example a non-dividend paying stock. 6- and 12-month WIBOR rates on 20th July 2010 were taken as risk-free rates for options with 6- and 12-month time to maturity respectively. Separate types of options were designed for pairs of commodities with different correlations.

Quotient options

Quotient options were created for the following pairs of underlying assets: milling wheat – feed corn, feed wheat – feed barley, milling wheat – feed barley. Table 3 presents input parameters for separate quotient options and results of pricing.

Table 3. Results of pricing quotient options

Parameter	Pair		
	milling wheat – feed corn	feed wheat – feed barley	milling wheat – feed barley
S_1 (PLN·t^{-1})	555	555	555
S_2 (PLN·t^{-1})	602	413	413
X (PLN·t^{-1})	0.9	1.3	1.3
σ_1 (%)	11.8	13.4	11.8
σ_2 (%)	11.6	19.9	19.9
r_1 (%)	3.99	3.99	3.99
r_2 (%)	4.25	4.25	4.25
T_1 (years)	0.5	0.5	0.5
T_2 (years)	1	1	1
Correlation	0.52	0.40	0.03
$C(T_1)$ (PLN·t^{-1})	0.04	0.13	0.12
$C(T_2)$ (PLN·t^{-1})	0.05	0.17	0.17
$P(T_1)$ (PLN·t^{-1})	0.02	0.06	0.05
$P(T_2)$ (PLN·t^{-1})	0.03	0.08	0.08

Source: Own calculations.

Payoff for the quotient call option on the pair milling wheat – feed corn, expiring six months after the date of issue, amounts to $\frac{930}{854} - 0.9 = 0.19$ minus premium (0.04). Hence, net profit obtained by the owner of this option amounts to 0.15 PLN·ton^{-1}.

In the case of call option expiring after one year, payoff is calculated in the following way: $\frac{920}{963} - 0.9 = 0.05$. Once the premium is taken into consideration, the profit is zero. In the case of both put options on the aforementioned pair, i.e. milling wheat – feed corn, one should allow them to expire worthless and thus lose the premium he/she paid (0.02 and 0.03 PLN·ton^{-1} respectively). Analogical analysis for the pair feed wheat – feed barley enables one to draw the following conclusions: the holder of both types of call options cannot exercise them profitably and the options expire worthless, so he/she incur losses corresponding to premiums paid (0.13 and 0.17 PLN·ton^{-1}). Put option expiring after six months enables the owner to earn net profit amounting to 0.13 PLN·ton^{-1}. By contrast, put option expiring after one year generates net loss of 0.01 PLN·ton^{-1}. Nevertheless, it ought to be exercised (to acquire a long position in the option the trader paid 0.08 PLN·ton^{-1}, so it would be foolish not to exercise). Both call options on the pair milling wheat – feed barley expire worthless. Put option expiring after six months generates net profit amounting to 0.08 PLN·ton^{-1}. Option expiring after one year should be exercised in order to minimize losses.

Product options

Product options were created for the following pairs of underlying assets: feed wheat – feed barley, milling wheat – feed corn, milling wheat – feed barley. Pricing results are reported in Table 4.

Table 4. Results of pricing product options

Parameter	Pair		
	feed wheat – feed barley	milling wheat – feed corn	milling wheat – feed barley
S_1 (PLN·t^{-1})	555	555	555
S_2 (PLN·t^{-1})	413	602	413
X (PLN·t^{-1})	230,000	330,000	230,000
σ_1 (%)	13.4	11.8	11.8
σ_2 (%)	19.9	11.6	19.9
r_1 (%)	3.99	3.99	3.99
r_2 (%)	4.25	4.25	4.25
T_1 (years)	0.5	0.5	0.5
T_2 (years)	1	1	1
Correlation	0.40	0.52	0.03
$C(T_1)$ (PLN·t^{-1})	20,335	29,834	19,771
$C(T_2)$ (PLN·t^{-1})	32,973	47,993	32,200
$P(T_1)$ (PLN·t^{-1})	11,865	11,258	11,311
$P(T_2)$ (PLN·t^{-1})	14,045	13,156	13,294

Source: Own calculations.

On the base of results for the pair feed wheat – feed barley, given in Table 4, one can state that both call options generated net profits: the option with 6-month time to maturity: 446,797 PLN·ton^{-1} (payoff: 697,132 – 230,000 = 467,132 offset by the premium of 20,335 PLN·ton^{-1}) and the option with 12-month time to maturity: 370,303 PLN·ton^{-1} (payoff: 633,276 – 230,000 = 403,276 offset by the premium of 32,973 PLN·ton^{-1}). In the case of the pair both put options should not be exercised producing losses equal to premiums paid. Analogous situation occurs for the pair milling wheat – feed corn (net profit on the 6-month call option equals 434,386 PLN·ton^{-1}, and on the 12-month call option 507,967 PLN·ton^{-1}, while put options expire worthless) and for the pair milling wheat – feed barley as well. Net profits on 6- and 12-month call options equal respectively 488,649 and 398,360 PLN·ton^{-1}. Put options should not be exercised.

Spread options

Spread options were created for the following pairs of underlying assets: feed corn – feed barley, feed corn – feed wheat, milling wheat – feed barley. Results of their pricing are displayed in Table 5.

Table 5. Results of pricing spread options

Parameter	Pair		
	feed corn – feed barley	feed corn – feed wheat	milling wheat – feed barley
S_1 (PLN·t^{-1})	602	602	555
S_2 (PLN·t^{-1})	413	555	413
X (PLN·t^{-1})	190	50	140
σ_1 (%)	11.6	11.6	11.8
σ_2 (%)	19.9	13.4	19.9
r_1 (%)	3.99	3.99	3.99
r_2 (%)	4.25	4.25	4.25
T_1 (years)	0.5	0.5	0.5
T_2 (years)	1	1	1
Correlation	0.42	0.68	0.03
$C(T_1)$ (PLN·t^{-1})	24.69	15.32	31.66
$C(T_2)$ (PLN·t^{-1})	36.52	22.60	45.35
$P(T_1)$ (PLN·t^{-1})	21.93	17.33	26.90
$P(T_2)$ (PLN·t^{-1})	29.61	23.52	37.52

Source: Own calculations.

Analysis of results obtained for the pair feed corn – feed barley suggests an investor not to exercise the 6-month call, whereas an owner of the spread call with 12-month time to maturity captures net profit equal to 18.48 PLN·t^{-1} (payoff: 963 – 718 – 190 = 55 minus premium amounting 36.52). Opposite situation occurs for put options: one should exercise the option with 6-month time to maturity achieving net profit of 108.07 PLN·ton^{-1} (payoff 190 – 854 + 794 = 130 PLN·t^{-1} reduced by 21.93 PLN·t^{-1} premium) and should not exercise the option with 12-month time to maturity (the loss equals the value of premium). Analogous results were obtained for the pairs feed corn – feed wheat and milling wheat – feed barley. Call options with 6-month time to maturity expire worthless. Call options with 12-month time to maturity produce net profits: for the pair feed corn – feed wheat 8.40 PLN·t^{-1}, and for the pair milling wheat – feed barley 16.65 PLN·ton^{-1}. Exercise of the 6-month spread put on the pair feed corn – feed wheat generates net profit equal to 56.67 PLN·t^{-1}, whereas exercise of the option on the pair milling wheat – feed barley produces net loss of 22.90 PLN·t^{-1}, though lower than the loss an investor would suffer from not exercising the option. Put options with 12-month time to maturity should not be exercised.

Rainbow options

Rainbow options were created for the following pairs of underlying assets: milling wheat – feed wheat, milling wheat – feed corn, milling wheat – feed barley. Tables 6 and 7 display results of pricing options on the minimum of two risky assets (Table 6) and options on the maximum of two risky assets (Table 7).

All rainbow call options on the minimum and on the maximum of two risky assets bring their owners net profits if they decide to exercise them. In the case of the pair

Table 6. Results of pricing options on the minimum of two risky assets

Parameter	Pair		
	milling wheat – feed wheat	milling wheat – feed corn	milling wheat – feed barley
S_1 (PLN·t^{-1})	555	555	555
S_2 (PLN·t^{-1})	555	602	413
X (PLN·t^{-1})	555	580	500
σ_1 (%)	11.8	11.8	11.8
σ_2 (%)	13.4	11.6	19.9
r_1 (%)	3.99	3.99	3.99
r_2 (%)	4.25	4.25	4.25
T_1 (years)	0.5	0.5	0.5
T_2 (years)	1	1	1
Correlation	0.78	0.52	0.03
$C(T_1)$ (PLN·t^{-1})	17.45	10.32	2.58
$C(T_2)$ (PLN·t^{-1})	28.09	19.47	7.43
$P(T_1)$ (PLN·t^{-1})	19.79	27.75	78.63
$P(T_2)$ (PLN·t^{-1})	23.80	29.65	80.72

Source: Own calculations.

Table 7. Results of pricing options on the maximum of two risky assets

Parameter	Pair		
	milling wheat – feed wheat	milling wheat – feed corn	milling wheat – feed barley
S_1 ($PLN \cdot t^{-1}$)	555	555	555
S_2 ($PLN \cdot t^{-1}$)	555	602	413
X ($PLN \cdot t^{-1}$)	555	580	500
σ_1 (%)	11.8	11.8	11.8
σ_2 (%)	13.4	11.6	19.9
r_1 (%)	3.99	3.99	3.99
r_2 (%)	4.25	4.25	4.25
T_1 (years)	0.5	0.5	0.5
T_2 (years)	1	1	1
Correlation	0.78	0.52	0.03
$C(T_1)$ ($PLN \cdot t^{-1}$)	33.54	42.72	67.02
$C(T_2)$ ($PLN \cdot t^{-1}$)	52.68	62.15	83.13
$P(T_1)$ ($PLN \cdot t^{-1}$)	9.28	5.38	1.13
$P(T_2)$ ($PLN \cdot t^{-1}$)	10.79	6.70	2.31

Source: Own calculations.

milling wheat – feed wheat exercise of the 6-month call on the minimum of two risky assets generates net profit amounting 305.55 $PLN \cdot t^{-1}$ (payoff: min (930, 878) – 555 = = 323 PLN·t–1 minus premium equal to 17.45 $PLN \cdot t^{-1}$), whereas exercise of the 6-month call on the maximum of two risky assets produces net profit amounting 341.46 $PLN \cdot t^{-1}$ (payoff: max (930, 878) – 555 = 375 $PLN \cdot t^{-1}$ minus premium of 33.54 $PLN \cdot t^{-1}$). Net profits for 12-month contracts are the following: for call on the minimum of two risky assets 298.91 $PLN \cdot t^{-1}$ and for call on the maximum on two risky assets 312.32 $PLN \cdot t^{-1}$. On the day of expiration rainbow call options on the pair milling wheat – feed corn generate the following profits: the 6-month call on the minimum of two risky assets: 263.68 $PLN \cdot t^{-1}$, the 12-month call on the minimum of two risky assets: 320.53 $PLN \cdot t^{-1}$, the 6-month call on the maximum of two risky assets: 307.28 $PLN \cdot t^{-1}$, the 12-month call on the maximum of two risky assets: 320.85 $PLN \cdot t^{-1}$.

Then, exercise of rainbow call options on the pair milling wheat – feed barley also let their owners gain profits: 291.42 $PLN \cdot t^{-1}$ (the 6-month call on the minimum of two risky assets), 210.57 $PLN \cdot t^{-1}$ (the 12-month call on the minimum of two risky assets), 362.98 $PLN \cdot t^{-1}$ (the 6-month call on the maximum of two risky assets) and 336.87 $PLN \cdot t^{-1}$ (the 12-month call on the maximum of the maximum of two risky assets). All put options on the minimum and on the maximum of two risky assets regardless the length of the time to maturity should not be exercised. Hence, their owners incur losses corresponding to the premiums already paid.

Generally, taken into consideration call and put two-asset options expiring after one year, regardless of the type (i.e. quotient, product, spread and rainbow), are more expensive than options expiring after six months. The relation is analogical to the relation for standard options: the longer time to maturity, the higher values of both call and put options.

CONCLUSIONS

As a result of progressing globalization, investment and business activity often entails taking actions on many markets simultaneously. Such an activity and its aspects are exposed to significant risk comprising many factors. Therefore, it is essential to provide effective complex protection in the form of multi-asset options that are more adequate than hedging strategies involving a great number of assets each of which protects against a single risk factor. Furthermore, using multi-asset options may reduce the cost of adopted strategy. The purchase of one multi-asset option instead of several single options minimizes transaction costs since the price of correlation options is always lower than the cost of buying a series of single options. However, due to the fact there are numerous variables determining the value of multi-asset options, many risk factors are to be monitored both by option writers and buyers. Nevertheless, correlation options are effective at reducing the risk faced by enterprises.

The aim of the paper was presenting the description and methods for pricing selected correlation options, with special reference to two-asset options. Examples of using such options on Polish grain market were presented as well. Commodity correlation options may be effective tool for minimizing the risk faced by companies operating also in agri-food sector. Producers and food processing enterprises can make use of multi-asset options in order to protect themselves against the risk from change in the price of raw materials and final products and to protect the margin of operating profit. Such entities are, for instance, mills, sugar plants, producers of spirit products, producers of meat products etc. Exporters, selling their products on a number of foreign markets, can also make use of multi-asset options, e.g. in order to eliminate exchange rate risk. Nonetheless, before adopting a particular tool, one should familiarize with its construction and attributes, which will enable him/her to assess potential gains and losses (depending on the state of the market).

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CHARAKTERYSTYKA I WYCENA WYBRANYCH OPCJI DWUCZYNNIKOWYCH ORAZ PROPOZYCJE ICH WYKORZYSTANIA NA RYNKU ZBÓŻ W POLSCE

Streszczenie. Opcje wieloczynnikowe są kontraktami, których wypłata zależy od co najmniej dwóch instrumentów bazowych. Mogą to być akcje, indeksy, waluty lub towary. Istotnym czynnikiem, wpływającym na wartość opcji wieloczynnikowych, jest korelacja stóp zwrotu instrumentów bazowych. Dlatego często są one określane mianem opcji korelacyjnych. W pracy przedstawiono charakterystykę i modele wyceny wybranych opcji korelacyjnych: opcji ilorazowych, iloczynowych, rozpiętościowych i dwukolorowych opcji tęczowych. Podano również przykłady ich zastosowania na rynku zbóż w Polsce.

Słowa kluczowe: opcje wieloczynnikowe, korelacja, modyfikacje modelu Blacka-Scholesa, rynek zbóż

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