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Computer-aiding of mathematical modeling of the carrot (*Daucus carota* L.) root shape

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Abstract: *Computer-aiding of mathematical modeling of the carrot* (*Daucus carota* L.) *root shape*. The developed method refers to computer aided mathematical modeling of the carrot root. The Bézier curves were used in mathematical model for description of the carrot root shape. Two smoothly connected Bézier curves described half of longitudinal contour of carrot root cross section. After rotation of these curves, the obtained equations were subjected to scaling. The proposed mathematical model can serve the generating the 3*D* solids that are similar in respect of the shape and basic dimensions of carrot roots. In mathematical model it is possible to change the values of parameters that determine basic geometric dimensions (diameters and length of root) and the root shape (coordinates of nodal and check points of the Bézier curves).

Key words: computer aiding, carrot root, shape, mathematical modeling.

INTRODUCTION

The carrot (*Daucus carota* L.) belongs to vegetables that take about 17% of their cultivation [Świetlikowska 2008]. It is used in food industry. The raw material for processing is a storage root. The carrot contains carotene [Świetlikowska et al. 2008], sugars, vitamins of groups *B*, *C* and *E* and others, as well as minerals: potassium, calcium, iron, copper, phosphorus. The raw materials of agricultural origin differ in respect of shape and are characterized by substantial variability of dimensions. The carrot root shape, as report Świetlikowska at al. [2008] after Orłowski et al. [1995], is spherical, oval, cylindrical, conical and wedge-shaped.

The shape of carrot root is taken into consideration during designing of its processing technology [Górnicki et al. 2009; Nowacka et al. 2010; Stępień 2009; Stopa et al. 2008], as well as during designing of machines and equipment for its cultivation and processing [Babik and Dudek 2000; 2003; 2006; Dudek et al. 2007]. Janaszek and Trajer [2010] proposed the method for undertaking decisions on carrot processing usability based on simplified information on the carrot roots colour. Waszkiewicz et al. [2008] investigated the effect of technical and exploitation parameters of the combine on the selected energetic indices during carrot roots harvesting.

The geometric features of raw plant materials of agricultural origin, as report Frączek and Wróbel [2009], are simulated with various accuracy by many researchers [Kęska and Feder 1997; Donev 2004; Mieszkalski 2011]. Frączek and Wróbel [2009] undertook an attempt towards application of computer graphics for 3D reconstruction of such objects. Frączek and Wróbel [2006] point out the use of image-models in evaluation of raw materials; nowadays, when dynamic development of informatics and computer techniques take place, such models will be more important and will have practical application. Mabille and Abecassis [2003] proposed the parametric equations for modeling the shape of wheat grain. With the use of image-models one will be able to describe the shape of wider and wider groups of raw materials of biological origin. In practice, there are needed the general methods for description of the raw materials' shapes with sufficient accuracy and at small labour input. This requirement is met by mathematical models. The author of this work undertook an attempt towards application of mathematical model to description of carrot root shape 3*D*.

This work aimed at elaboration of mathematical models to describe the carrot root shape with the use of Bézier curves.

METHODICS

The research material was carrot (*Daucus carota* L.) of hybrid variety Rodos *F*1, originated from 2012 cultivation. There were selected and photographed the three carrot roots of different shapes. There were measured the maximal diameters at the root upper part cross section (*D*1, *D*2), the minimal diameters (*d*1, *d*2) at the root lower cross section and the root length (*a*). The measurements were executed with the use of a slide caliper with accuracy 0.1 mm.

Detailed description of biological objects shapes can be obtained from Bézier curves, that are commonly used in the computer aided design programs and computer graphics [Foley et al. 2001]. The mathematical models were developed with the use of two connected Bezier curves, that created the half of carrot root longitudinal contour; they allowed to create the solid surfaces for given dimensions, of the shape similar to the selected carrot roots. The mentioned 3*D* solid models were visualized with the use of computer program Mathcad v. 14.

RESULTS OF MEASUREMENTS

The results of measurements on carrot roots are presented in Table 1.

Exemplary photos of carrot roots are presented in Figure 1.

TABLE 1. Basic dimensions of selected carrot roots of variety Rodos F1

Cross section	Dimension	Designation of carrot root		
		A	*AB*	*ABC*
		mm	mm	mm
	Length of root (*a*)	229.1	148.5	124.4
Upper part of root	Diameter (*D*1)	55.2	45.3	33.2
	Diameter (*D*2)	61.0	48.1	34.2
Lower part of root	Diameter (*d*1)	34.1	35.9	24.2
	Diameter (*d*2)	37.3	36.7	25.7

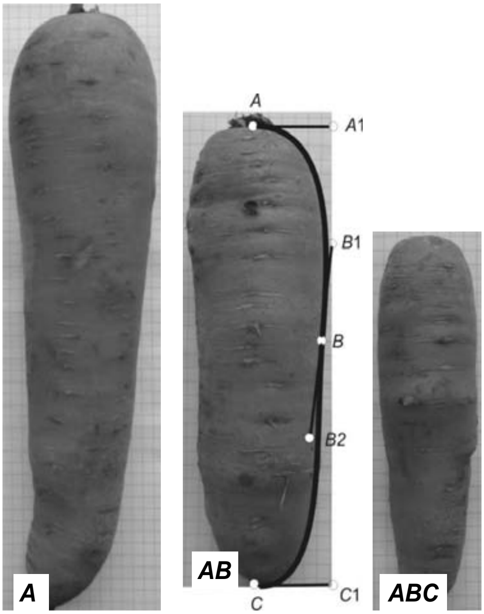

FIGURE 1. Exemplary carrot roots of variety Rodos *F*1 of successive designations *A*, *AB*, *ABC* (own elaboration, dimensions in Table 1), middle view presents two connected Bézier curves on carrot root contour

MODEL FOR CARROT ROOT SOLID SHAPE REPRESENTED WITH BÉZIER CURVES

Matrix equations of coordinates *xA*1, *zA*1 of Bézier first curve points for carrot *A*:

$$xA1_{t1} = \mathrm{xA11}\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right]^3 + \mathrm{xA12}\cdot 3\frac{\mathrm{t1}}{\mathrm{N}}\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right]^2 + \mathrm{xA13}\cdot 3\cdot\left[\frac{\mathrm{t1}}{\mathrm{N}}\right]^2\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right] + \mathrm{xA14}\cdot\left[\frac{\mathrm{t1}}{\mathrm{N}}\right]^3 \quad (1)$$

$$zA1_{t1} = \mathrm{zA11}\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right]^3 + \mathrm{zA12}\cdot 3\frac{\mathrm{t1}}{\mathrm{N}}\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right]^2 + \mathrm{zA13}\cdot 3\cdot\left[\frac{\mathrm{t1}}{\mathrm{N}}\right]^2\cdot\left[1-\frac{\mathrm{t1}}{\mathrm{N}}\right] + \mathrm{zA14}\cdot\left[\frac{\mathrm{t1}}{\mathrm{N}}\right]^3 \quad (2)$$

Matrix equations of coordinates *xA*2, *zA*2 of Bézier second curve points for carrot *A* have the following form:

$$xA2_{t} = \mathrm{xA14}\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right]^3 + \mathrm{xA22}\cdot 3\frac{\mathrm{t}}{\mathrm{N}}\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right]^2 + \mathrm{xA23}\cdot 3\cdot\left[\frac{\mathrm{t}}{\mathrm{N}}\right]^2\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right] + \mathrm{xA24}\cdot\left[\frac{\mathrm{t}}{\mathrm{N}}\right]^3 \quad (3)$$

$$zA2_{t} = \mathrm{zA14}\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right]^3 + \mathrm{zA22}\cdot 3\frac{\mathrm{t}}{\mathrm{N}}\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right]^2 + \mathrm{zA23}\cdot 3\cdot\left[\frac{\mathrm{t}}{\mathrm{N}}\right]^2\cdot\left[1-\frac{\mathrm{t}}{\mathrm{N}}\right] + \mathrm{zA24}\cdot\left[\frac{\mathrm{t}}{\mathrm{N}}\right]^3 \quad (4)$$

The matrix equations of Bézier curve coordinates for carrot *AB* and *ABC* are written down analogically.

The coordinates of nodal and check points that occur in equations (1, 2, 3, 4) for the first and second Bézier curves for carrot roots *A*, *AB*, *ABC* are written down in matrixes 5, 6, 7:

$$\begin{bmatrix} xA11 & zA11 \\ xA12 & zA12 \\ xA13 & zA13 \\ xA14 & zA14 \\ xA22 & zA22 \\ xA23 & zA23 \\ xA24 & zA24 \end{bmatrix} = \begin{bmatrix} 0 & 229.1 \\ 52 & 229.1 \\ 28 & 132 \\ 27 & 119 \\ 26 & 98 \\ 31 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} xAB11 & zAB11 \\ xAB12 & zAB12 \\ xAB13 & zAB13 \\ xAB14 & zAB14 \\ xAB22 & zAB22 \\ xAB23 & zAB23 \\ xAB24 & zAB24 \end{bmatrix} = \begin{bmatrix} 0 & 148.5 \\ 28 & 148.5 \\ 27 & 112 \\ 24 & 80 \\ 21 & 48 \\ 32 & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} xABC11 & zABC11 \\ xABC12 & zABC12 \\ xABC13 & zABC13 \\ xABC14 & zABC14 \\ xABC22 & zABC22 \\ xABC23 & zABC23 \\ xABC24 & zABC24 \end{bmatrix} = \begin{bmatrix} 0 & 124.4 \\ 24 & 124.4 \\ 18 & 71 \\ 17 & 55 \\ 16 & 39 \\ 21 & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

In vector 8 there is given number of meridians and parallels in the model that describes the carrot root shape in relations to a single Bézier curve, while the range variables are written down in vector 9:

$$\begin{bmatrix} N \\ n \\ n1 \end{bmatrix} = \begin{bmatrix} 30 \\ 50 \\ 10 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} t \\ j \\ t1 \end{bmatrix} = \begin{bmatrix} 0...N \\ 0...N \\ 0...N-1 \end{bmatrix} \tag{9}$$

In order to obtain the solid of revolution that represent carrot root *A*, one should rotate the first Bézier curve (equations 10, 11, 12), then the second Bézier curves (equations 13, 14, 15):

$$\begin{aligned} XA1_{t1,\,j} &= xA1_{t1} \cdot \sin(\phi_j) \\ YA1_{t1,\,j} &= xA1_{t1} \cdot \cos(\phi_j) \\ ZA1_{t1,\,j} &= zA1_{t1} \end{aligned} \tag{10}$$

$$\begin{aligned} XA2_{t,\,j} &= xA2_{t} \cdot \sin(\phi_j) \\ YA2_{t,\,j} &= xA2_{t} \cdot \cos(\phi_j) \\ ZA2_{t,\,j} &= zA2_{t} \end{aligned} \tag{11}$$

where: $$\phi_j = \frac{2 \cdot \pi \cdot j}{N} \tag{12}$$

Matrixes obtained from rotation of Bézier curve should be connected with columns with the use of Mathcad *stack* procedure:

$$\begin{aligned} Xa &:= \mathrm{stack}(XA1, XA2) \\ Ya &:= \mathrm{stack}(YA1, YA2) \\ Za &:= \mathrm{stack}(ZA1, ZA2) \end{aligned} \tag{13}$$

Dimensions (*a*, *D1*, *D2*, *d1*, *d2*) from the real carrot root measurements *A*, *AB*, *ABC* are placed in matrix 14.

$$\begin{bmatrix} aA & aAB & aABC \\ D1A & D1AB & D1ABC \\ D2A & D2AB & D2ABC \\ d1A & d1AB & d1ABC \\ d2A & d2AB & d2ABC \end{bmatrix} = \begin{bmatrix} 229.1 & 148.5 & 124.4 \\ 55.2 & 45.3 & 33.2 \\ 61 & 48.1 & 34.2 \\ 34.1 & 35.9 & 24.2 \\ 37.3 & 36.7 & 25.7 \end{bmatrix} \quad (14)$$

In order to obtain the set dimensions (*a*, *D1*, *D2*, *d1*, *d2*) of carrot root model one should execute the scaling of equations 13. The scaled matrix equations of coordinates *XA*, *YA*, *ZA* of net nodal points for surface describing *A* carrot root shape have the following form:

$$XA = \frac{D1A}{\max(Xa) - \min(Xa)} \cdot Xa \quad (15)$$

$$YA = \frac{D2A}{\max(Ya) - \min(Ya)} \cdot Ya \quad (16)$$

$$ZA = \frac{aA}{\max(Za) - \min(Za)} \cdot Za \quad (17)$$

Similar calculations should be executed for carrot roots designated as *AB* and *ABC*.

The 3*D* models for carrot roots *A*, *AB* and *ABC* are presented in Figure 2.

VERIFICATION OF CARROT ROOT MODEL SOLIDS

The worked out mathematical models that describe the shape of carrot roots by rotation of the two connected Bézier curves were subjected to dimensional verification. The characteristic verifying dimensions for carrot root were: maximal diameters (*D*1, *D*2) minimal diameters (*d*1, *d*2), length of root (*a*). The verifica-

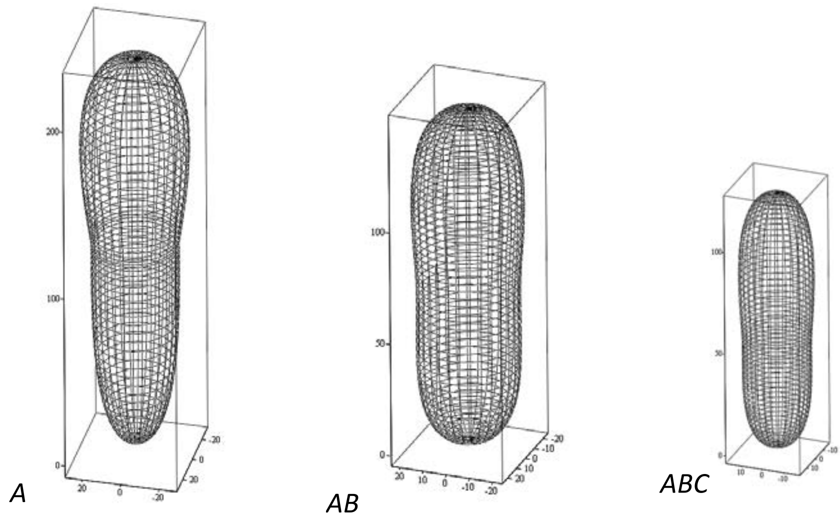

FIGURE 2. 3*D* models for carrot roots *A*, *AB*, *ABC*

TABLE 2. Verification results for *A*, *AB*, *ABC* carrot root model

Designation of root	Result (mm) determined on basis of: max(X) – min(X), max(Y) – min(Y), max(Z) – min(Z)				
	a	*D*1	*D*2	*d*1	*d*2
A	229.1	55.2	61	34	37.6
AB	148.5	45.3	48.1	35.2	37.4
ABC	124.4	33.2	34.2	24.4	25.1

tion results for *A*, *AB*, *ABC* carrot root models are presented in Table 2.

Comparing the dimension results obtained on the basis of models (Tab. 2) and the real carrot root dimensions (Tab. 1) one can find, that the shapes of solids represented by carrot root models (Fig. 2) are similar to the real carrot roots presented in Figure 1. The proposed mathematical models can serve generating the 3*D* solids similar in respect of shape and basic dimensions of carrot roots and can be used in agricultural engineering and food engineering.

CONCLUSIONS

1. The proposed mathematical model represented by Bézier curve can serve generating the 3*D* solids similar in respect of shape and basic dimensions of carrot roots.

2. In the proposed mathematical model one can change the value of parameters that determine the basic geometric parameters and the root shape (coordinates of nodal and check points of Bézier curves); it enables to generate any (within the species) solids similar to carrot roots in respect of the shape and basic dimensions.

REFERENCES

BABIK J., DUDEK J. 2006: Nowa maszyna do zwalczania chwastów w uprawach warzyw na redlinach. Journal of Research and Applications in Agricultural Engineering, Vol. 51 (2), p. 7–12.

BABIK J., DUDEK J. 2000: New, complex machine for ridge forming, and simultaneous sowing vegetable crops. Vegetable Crops Research Bulletin 53: 103–110.

BABIK J., DUDEK J. 2003: New design of machine for weed control on the ridges. Abstracts. EWRS Working Group "Weed Management Systems in Vegetable Crops" Workshop Report of Skierniewice meeting: 25. http://www.agr.unipg.it/ewrsveg/

DUDEK J., BABIAK J., DYŚKO J. 2007: Technologia kroplowego nawadniania warzyw korzeniowych uprawianych na redlinach. Journal of Research and Applications in Agricultural Engineering, Vol. 52 (3), p. 32–36.

DUDEK J., BABIK J. 2006: Urządzenie do mechanicznego umieszczania węży nawadniających w glebie, w uprawie warzyw korzeniowych na redlinach. Problemy Inżynierii Rolniczej. Nr 1 (51), p. 127–134.

DONEV A. et al. 2004: Improving the density of jammed disordered packings using ellipsoids. Science, Vol. 303, p. 990–993.

FOLEY J.D., van DAM A., FEINER S.K., HUGHES J.F., PHILLIPS R.L. 2001: Wprowadzenie do grafiki komputerowej. WNT, Warszawa.

FRĄCZEK J., WRÓBEL M. 2006: Metodyczne aspekty oceny kształtu nasion. Inżynieria Rolnicza. Nr 12, p. 155–163.

FRĄCZEK J., WRÓBEL M. 2009: Zastosowanie grafiki komputerowej w rekonstrukcji 3D nasion. Inżynieria Rolnicza. Nr 6 (115), p. 87–94.

GÓRAL D., STADNIK Ł. 2007: Zależność wybranych cech mechanicznych marchwi (daucus carota l.) i pietruszki zwyczajnej (petroselinum sativum hoffm.) od kinetyki obróbki chłodniczej metodą odwróconej fluidyzacji. Acta Sci. Pol., Technica Agraria. Nr 6 (1), p. 17–24.

JANASZEK M., TRAJER J. 2010: Zastosowanie metody wektorów nośnych oraz komputerowej analizy obrazu w klasyfikacji korzeni marchwi. Inżynieria Rolnicza. Nr 7 (125), p. 75–80.

KĘSKA W., FEDER S. 1997: Trójwymiarowa rekonstrukcja kształtu elementów roślinnych z dwuwymiarowych obrazów rastrowych. Prace Przemysłowego Instytutu Maszyn Rolniczych, p. 15–17.

MABILLE F., ABECASSIS J. 2003: Parametric modelling of wheat grain morphology: a new perspective. Journal of Cereal Science, 37, 43–53.

MIESZKALSKI L. 2011: Metoda matematycznego modelowania kształtu bryły ziarna pszenicy za pomocą parametrycznej krzywej przestrzennej i czterowęzłowej siatki. Postępy Techniki Przetwórstwa Spożywczego, Nr 1, Tom 21/38, p. 41–45.

NOWACKA M., WITROWA-RAJCHERT D., STRACHOTA W., SOBCZAK E. 2010: Wybrane właściwości promiennikowo-konwekcyjnych suszy marchwi i ziemniaka. Acta Agrophysica. Nr 16 (1), p. 127–136.

ORŁOWSKI M., REKOWSKA E., DOBROMILSKA R., SADOWSKI P., ABRAMOWICZ M. 1995: Warzywnictwo. Wydawnictwo Akademii Rolniczej w Szczecinie.

ŚWIETLIKOWSKA K. (red.) 2008: Surowce spożywcze pochodzenia roślinnego. Wyd. II uzupełnione. Wydawnictwo SGGW, Warszawa.

STĘPIEŃ B. 2009: Wpływ metody suszenia na wybrane cechy mechaniczne marchwi po ponownym uwodnieniu. Inżynieria Rolnicza. Nr 5(114), p. 251–258.

STOPA R., ROMAŃSKI L., JANKOWSKI L.J. 2008: Obrazy izochrom w modelach przekroju poprzecznego korzenia marchwi dla różnych wartości sił spójności warstw kory i rdzenia. Inżynieria Rolnicza. Nr 5 (103), p. 283–290.

WASZKIEWICZ C., LISOWSKI A., STRUŻYK A., KLONOWSKI J. 2008: Wpływ parametrów technicznych i eksploatacyjnych kombajnu na wybrane wskaźniki energetyczne podczas zbioru korzeni marchwi. Inżynieria Rolnicza. Nr 1(99), p. 389–394.

Streszczenie: *Komputerowe wspomaganie matematycznego modelowania kształtu korzenia marchwi* (*Daucus carota* L.). Proponowana metoda dotyczy matematycznego modelowania korzeni marchwi wspomaganego komputerowo. W modelu matematycznym do opisu kształtu korzenia marchwi zastosowano krzywe Béziera. Dwie gładko połączone krzywe Béziera opisywały połowę konturu wzdłużnego przekroju korzenia marchwi. Po dokonaniu obrotu tych krzywych otrzymane równania poddawano skalowaniu. Proponowany model matematyczny może służyć do generowania brył 3*D*, podobnych pod względem kształtu i podstawowych wymiarów korzeni marchwi. W modelu matematycznym istnieje możliwość zmiany wartości parametrów decydujących o podstawowych wymiarach geometrycznych (średnice i długość korzenia) oraz o kształcie korzenia (współrzędne punktów węzłowych i kontrolnych krzywych Béziera).

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