Analytical and experimental investigation of bending stresses in beech plywood

ANDRZEJ MAKOWSKI

Department of Engineering Mechanics and Thermal Techniques, Poznan University of Life Sciences

Abstract: *Analytical and experimental investigation of bending stresses in beech plywood.*
The paper presents an analysis of problems connected with the determination of strength parameters in layered wood-based laminate with transverse veneer sheet configuration. Analytical calculations were performed in accordance with the classical lamination theory in thin boards within the theory of elasticity taking into consideration the Kirchhoff-Love hypothesis. Analyses were conducted on models of composite layers in a state of plane stress. Values of components of transformed stiffness matrices were estimated theoretically in the on-axis and off-axis configurations of individual laminate layers. Analytical calculations were verified experimentally. Results of the analyses are presented in the form of tables and graphs.

Keywords: laminate, wood composites, plywood.

INTRODUCTION

Plywood as a structural material exhibits properties comparable to those of composite materials. It is produced by gluing thin, intercrossing wood layers of various thickness, referred to as veneers. They are produced by off-centre cutting or peeling of round wood of various species. Physical and mechanical properties of plywood are dependent mainly on the wood species, orientation and thickness of glued veneers, as well as the type and method of gluing. Due to its properties and easy workability, e.g. steam bending of various shapes has found numerous applications in the construction industry, boat building, furniture industry, shipbuilding to name a few. As a natural and environmentally friendly material it is being used with increasing frequency, as manifested in the worldwide upward trends for the production of plywood observed in recent years. Novel applications are being searched for also for modified plywood e.g. by combining it with other materials in order to enhance its strength properties and reduce its defects. The application of plywood in engineering structures as a load-bearing element requires strength analysis, particularly in the critical nodes where stresses accumulate, e.g. at the contact points with metal or wood joints. It was attempted in this study to provide an analytical description of the distribution of stresses and strains in a layered beam element using the bending theory for thin-walled material layers. The analytical analysis was verified experimentally.

MATERIAL AND METHODS

Material properties of each plywood layer may be described using an orthotropic material model. The main directions of orthotropic material are connected with anatomical directions in wood: longitudinal L, radial R and tangential T [1,3,5,6]. Bending strength tests of beech veneer plywood in accordance with the EN 310 standard were performed on small samples cut from one sheet of 18 mm in thickness. Each sample was collected maintaining the longitudinal direction of the grain on the face surfaces (counter veneers). The method of fastening and loading of bending samples is presented in Fig. 1. Testing results for the bending samples together with geometrical dimensions are given in Table 1.
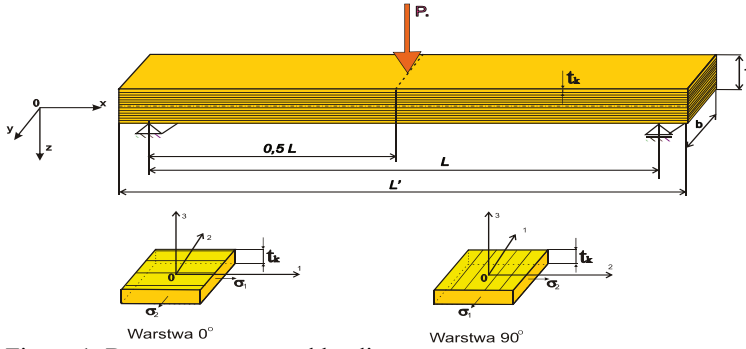
Figure 1. Beam geometry and loading

Tab. 1. Geometrical and strength parameters of bending samples

Sample no.	Length L'		Thickness t	Width b	Deflection f	Bending load P	Modulus E	Bending strength MOR
	L'	L						
	[mm]	[mm]	[mm]	[mm]	[mm]	[kN]	[N/mm²]	[N/mm²]
1	410.1		17.9	50.3	14.855	2.482	9410	82.82
2	409.9	360	17.8	50.3	15.030	2.266	8923	76.39
3	410.4		17.9	50.1	15.568	2.448	9336	81.86
4	410.1		17.9	50.3	12.891	2.164	9004	72.44
5	410.3		17.9	50.3	15.578	2.420	9504	81.30
6	410.2		17.9	50.4	16.627	2.587	9604	86.95
Val. aver.	410.167		17.883	50.283	13.425	2.394	9296	80.293

Analytical strength testing of the wood-based composite at the macroscopic scale was based on the bending theory of thin-walled layers according to the Kirchhoff-Love hypothesis [2,4,7]. For a single veneer layer we may assume an orthotropic structural model, comparable to the unidirectional fiber -reinforced composite. The reinforcement is provided by cellulose fibers, arranged along the tree stem and filled mainly with lignin. By limiting our considerations to linear relationships within the applicability of Hooke's law, the dependence between stresses and strains is expressed as follows [2,5]:

$$\{\sigma\} = [Q][T]\{\varepsilon\} = \left[Q^*\right]\{\varepsilon\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11}^* & Q_{12}^* & Q_{16}^* \\ Q_{12}^* & Q_{22}^* & Q_{26}^* \\ Q_{16}^* & Q_{26}^* & Q_{66}^* \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{1.1}$$

The transformed stiffness matrix $[Q^*]$ of an orthotropic layer „k" in the on-axis configuration expressed using engineering constants takes the form:

$$[Q^*] = [Q]_k = \begin{bmatrix} mE_{11} & mv_{12}E_{22} & 0 \\ mv_{21}E_{11} & mE_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_k \tag{1.2}$$

where: $m=(1-v_{12}v_{21})^{-1}$ assuming that $\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}$,

E_1 – the so-called longitudinal modulus of elasticity, E_2 – the so-called transverse modulus of elasticity in shear, G_{12} – modulus of shear, v_{12} – the so-called major Poisson's coefficient, v_{21} – the so-called minor Poisson's coefficient.

Theoretical calculations were performed for the wood-based laminate with material properties of European beech (*Fagus sylvatica L.*). Material constants were adopted after literature data [3,6], as mean values amounting to density of 690 kg/m³, 12% moisture content, moduli of

elasticity and Poisson's coefficient $E_L=E_{11}=14000$MPa, $E_T=E_{22}=1160$MPa, $G_{12}=S_{66}$ $=1080$MPa, (major) $\nu_{LT}=\nu_{12}=0{,}52$, (minor) $\nu_{TL}=\nu_{21}=0{,}043$. The reduced stiffness matrix for a single laminate layer in the function of engineering constants according to the equation is as follows [2,5]:

$$[Q] = \begin{bmatrix} mE_{11} & m\nu_{12}E_{22} & 0 \\ m\nu_{21}E_{11} & mE_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} = \begin{bmatrix} 14{,}32 & 0{,}62 & 0 \\ 0{,}62 & 1{,}19 & 0 \\ 0 & 0 & 1{,}08 \end{bmatrix} \text{[GPa]} \tag{1.3}$$

The analysed plywood with thickness $t = 18$mm was composed of $N =13$ intercrossing veneer layers glued at right angles. At such an arrangement of plies it is classified as symmetrical laminate with an odd number of layers. The laminate was composed of alternating glued layers arranged at an angle of grain of $0°$ and $90°$ in relation to the laminate axis x, while the core has a lamination angle of $0°$.

Each of the two elementary layers of the whole laminate has a different transformed stiffness matrix $[Q^*]^k$. The constitutive equation for the k-th ply is as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}^{(k)} = [Q^*]^{(k)} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}^{(k)} \tag{1.4}$$

The above equation defines stresses found in each laminate layer. In order to determine the volume of strains in individual layers of bending laminate, we apply geometrical relationships linking strain and curvature of the core:

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z\, k_x \\ \varepsilon_y &= \varepsilon_y^0 + z\, k_y \\ \gamma_{xy} &= \gamma_{xy}^0 + z\, k_{xy} \end{aligned} \tag{1.5}$$

Thus equation (1.4) may be written as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}^{(k)} = [Q^*]^{(k)} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + [Q^*]^{(k)} z \begin{bmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{bmatrix} \tag{1.6}$$

By replacing stresses with external forces applied on the laminate [N], [M] this equation takes the form [2,7]:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{bmatrix} \quad \text{or} \quad \begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k^0 \end{Bmatrix} \tag{1.7}$$

where: **A** – extensional stiffness matrix,
B – bending-extensional stiffness matrix,
D – bending stiffness matrix.

In the case of bending of symmetrical laminates their important characteristic is connected with the fact that the bending-extensional matrix is reduced to zero [B]=0. In the case of bending of a composite at the absence of external longitudinal tensile forces acting on the composite, the stiffness matrix of the laminate is reduced to the form:

$$\{M\} = [D]\{k^0\} \tag{1.8}$$

After the transformation of equation (1.8) we may determine the bending curvature of laminate in the neutral surface:

$$\{k^0\} = [D]^{-1}\{M\} \quad \text{or} \quad \begin{Bmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}^{-1} \begin{Bmatrix} M_x \\ 0 \\ 0 \end{Bmatrix} \tag{1.9}$$

Where: bending moment along the edge is expressed as: $\{M_x\} = \begin{Bmatrix} M'/b \\ 0 \\ 0 \end{Bmatrix}$, and $[D]=[Q^*]\, t^3/12$

Transformed stiffness matrices of layer $0°$ and layer $90°$, according to (1.3) are as follows:

$$[Q]_{0°} = [Q^*]_0 = \begin{bmatrix} 14.32 & 0.62 & 0 \\ 0.62 & 1.19 & 0 \\ 0 & 0 & 1,08 \end{bmatrix} GPa \qquad [Q]_{90°} = [Q^*]_{90} = \begin{bmatrix} 1,19 & 0.62 & 0 \\ 0.62 & 14,32 & 0 \\ 0 & 0 & 1,08 \end{bmatrix} GPa \qquad (2.0)$$

$$\{M_x\} = \begin{Bmatrix} \dfrac{0,5 \cdot 2394 \cdot 180 [Nmm]}{50 [mm]} \\ 0 \\ 0 \end{Bmatrix} = 4,309 kN$$

Taking into consideration the above results we may determine elements of the inverse bending stiffness matrix $[D]^{-1}$ for the core $0°$, which are as follows:

$$[D]_0^{-1} = \begin{bmatrix} 17,691 & -0,759 & 0 \\ -0,759 & 1,469 & 0 \\ 0 & 0 & 19,05 \end{bmatrix} 10^{-4} (kNmm)^{-1} \qquad (2.1)$$

Curvatures of the neutral bending surface may be calculated from dependence (1.8) and they amount to:

$$\begin{Bmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}^{-1} \begin{Bmatrix} M_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 17,691 & -0,759 & 0 \\ -0,759 & 1,469 & 0 \\ 0 & 0 & 19,05 \end{bmatrix} 10^{-4} kNmm^{-1} \begin{Bmatrix} 4,309 \\ 0 \\ 0 \end{Bmatrix} kN =$$

$$\begin{Bmatrix} -6,329 \\ 3,272 \\ 0 \end{Bmatrix} 10^{-4} mm^{-1}$$

(2.2)

RESULTS

Values of normal deformations of individual laminate layers may be estimated from the formula (1.5). Assuming they are linear, the values of individual strains are dependent on the distance of the surface of a given layer from the laminate core. In these analyses the distribution of shear stresses, being small due to the t/L ratio, was not taken into consideration. Results of analytical calculations of normal stresses are given in Table 2 and presented in Fig. 2.

Tab. 2. Analytical values of stresses in individual laminate layers.

Layer no.		σ_x (MPa)		σ_y (MPa)	
Bottom to top		bottom	top	bottom	top
1	0°	79.330	67.363	-0.953	-0.025
2	90°	4.207	3.427	-32.676	-26.656
3	0°	54.954	42.545	-0.020	-0.016
4	90°	2.657	1.882	-20.637	-14.618
5	0°	30.136	17.727	-0.011	-0.007
6	90°	1.107	0.387	-8.299	-3.609
7	0°	6.204	-6.204	-0.002	0.0
8	90°	-0.387	-1.107	8.299	3.609
9	0°	-17.727	-30.136	0.011	0.007
10	90°	-1.882	-2.657	20.637	14.618
11	0°	-42.545	-54.954	0.020	0.016
12	90°	-3.427	-4.207	32.676	26.656
13	0°	-67.363	-79.330	0.953	0.025

An example calculation was performed for layer 1 with the orientation angle $0°$ counting from the backing layer of the laminate.

Data: $z = -9mm$, $\varepsilon_x^0 = \varepsilon_y^0 = 0$

$\varepsilon_x = \varepsilon_x^0 + z\, k_x = -9mm\ (-6.329)10^{-4}mm^{-1} = 5.699\ 10^{-3}$
$\varepsilon_y = \varepsilon_y^0 + z\, k_y = -9mm\ (3.272)10^{-4}mm^{-1} = -3.350\ 10^{-3}$

$$[\sigma]_z = [Q]\{\varepsilon\}$$

Bottom

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_9 = 10^3\,(MPa) \begin{bmatrix} 14,32 & 0,61 & 0 \\ 0,61 & 1,19 & 0 \\ 0 & 0 & 1,08 \end{bmatrix} 10^{-3} \begin{Bmatrix} 5,699 \\ -3,722 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 79,339 \\ -0,953 \\ 0 \end{Bmatrix} MPa$$

Top

$z = -7.6mm,\ \ \varepsilon_x^0 = \varepsilon_y^0 = 0$

$\varepsilon_x = \varepsilon_x^0 + z\, k_x = -7.6mm\ (-6,329)10^{-4}mm^{-1} = 4.810\ 10^{-3}$
$\varepsilon_y = \varepsilon_y^0 + z\, k_y = -9mm\ (3.272)10^{-4}mm^{-1} = -2.487\ 10^{-3}$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_{7,6} = 10^3\,(MPa) \begin{bmatrix} 14,32 & 0,61 & 0 \\ 0,61 & 1,19 & 0 \\ 0 & 0 & 1,08 \end{bmatrix} 10^{-3} \begin{Bmatrix} 4,810 \\ -2,487 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 67,363 \\ -0,025 \\ 0 \end{Bmatrix} MPa$$
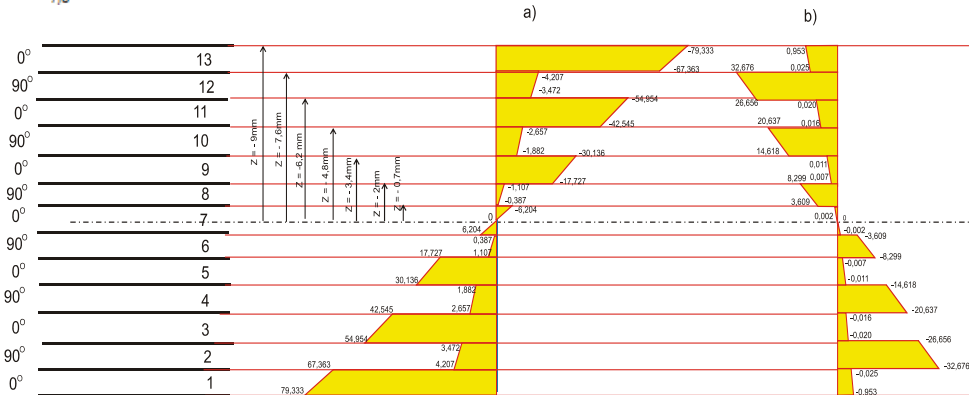

Figure 2. Stress distribution along the laminate thickness: a) σ_x, [MPa] b) σ_y, [MPa]

Figure 3 presents damage to one of the bending samples in the three-point bending test. The type of damage indicates that admissible stresses were exceeded in the bottom plywood layers. Effort in the bottom layers with the longitudinal arrangement of grain (0°) caused their transverse rupture, while in layers with the lamination angle (90°) we observe damage to the veneer lignin.

Fig.3. A sample following bending strength test

CONCLUSIONS

Results of analytical calculations were verified empirically. It shows that the adopted assumptions of the bending theory according to Kirchhoff's hypothesis in the analytical investigations confirm its applicability. It refers to the specific analysis of the distribution of strains and stresses in layers of wood-based materials. Analytical calculations of the bending laminate, clearly show variation in the distribution of stresses in individual laminate layers, and thus also different mechanical properties. Distribution of stresses in layers with the same

direction of grain is linear with variable values to the thickness of the laminate. Stresses in the elastic range are dependent on moduli of elasticity of veneers in the parallel and perpendicular direction to grain. Calculated values of normal stresses in the face layers of the laminate do not diverge greatly from the mean value determined experimentally. The presented method to analyses strength parameters in multi-ply wood based materials taking into consideration orthotropic, at slight differences from actual values confirms the advisability of the application of the theory of layer systems.

Streszczenie: *Doświatczalna i analityczna analiza naprężeń w sklejce bukowej w statycznej próbie zginania.* W pracy przedstawiono szczegółową analizę zagadnień związanych z określaniem parametrów wytrzymałościowych wielowarstwowego laminatu drewnopochodnego o krzyżowej konfiguracji warstw forniru. Obliczenia analityczne wykonano zgodnie z klasyczną teorią laminacji płyt cienkich w zakresie teorii sprężystości z uwzględnieniem hipotezy Kirchhoffa-Love'a. Przedmiotem analiz były modele warstw kompozytu znajdujące się w płaskim stanie naprężenia. Oszacowano teoretycznie wartości składowych transformowanych macierzy sztywności w konfiguracji osiowej i nieosiowej indywidualnych warstw laminatu. Analityczne obliczenia zweryfikowano badaniami doświadczalnymi. Wyniki badań przedstawiono w formie tabel i wykresów.

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Corresponding author:

Andrzej Makowski,
Department of Engineering Mechanics and Thermal Techniques,
Poznan University of Life Sciences,
60-627 Poznan,
ul. Wojska Polskiego38/42,
Poland
e-mail: makowski@au.poznan.pl