Mathematical support of the system of computer-aided design of screw extruders

Valeriy Dyadychev, Vyacheslav Zharikov

Volodymyr Dahl East-Ukrainian National University,
Molodizhny bl., 20a, Lugansk, 91034, Ukraine, e-mail: dvv@snu.edu.ua
Received June 18.2013: accepted July 13.2013

Summary. Exact analytical calculation of the screw pair, the most widespread working body of extruder, is difficult. The necessity of the creation of CAD system for screw extruders is explained. The functions of the provided computer-aided design system are considered. Mathematical models of motion of the material recycled in the extruder are presented. The finite element analysis and the theory of heat conduction are used for the calculation of the stressed state of the material.
Key words: extruder, CAD, finite element approach, equilibrium equation.

INTRODUCTION

Technologies of extrusion considering its high productivity are widely used in food, chemical and other industries. Exact analytical calculation of the screw pair, the most widespread working body of extruder, is difficult because of the variety of complex shapes of extruders and properties of the recycled material. Calculation of screw extruders nowadays is made by approximate, as a rule empirical formulas, with a large number of assumptions [17].

Creation of an automated instrument of the design of screw units which enables to make theoretically grounded kinematic, strength calculations, calculations of wear resistance, calculations of stress state in different points of the recycled material will improve the quality of the made design decisions in their synthesis, increase the accuracy of calculation and informativity of it for a designer. Creation of new constructional solutions using the methods of virtual engineering saves time and costs related to the design through the use of computer simulation [8].

OBJECTS AND PROBLEMS

The functions of the proposed system of automation of the screw extruders design:

1. Structural synthesis of the construction. Engineer-user of the system makes a block scheme using specified primitives of the elements of the extruder. Selection of different geometry (cylindrical or cone body shape, flat or corrugated walls and so on.) is possible at this stage. The database of primitives required for this function contains data caused by the system developer. The results of structural synthesis are displayed on the screen and may be stored in the database of constructions [10].

2. Editing of existing solutions. Extruder elements selected in the process of structural synthesis, as a rule, have adjustable parameters, such as the angularity of the screw line, thickness of the blades and so on. The system enables the engineer-user to vary these parameters within the acceptable limits from the viewpoint of feasibility of the design. The results are recorded in the database of designs.

3. Engineering calculations. Design automation system performs the necessary calculations based on specified by the engineer-user structure, design parameters, parameters of the technological mode (operating pressure in front of the matrix, rotation speed of the screw and so on),

the properties of the recycled material. The system should provide the possibility of such types of calculations as heat and strength calculations, calculations of the stress state в in different point of the recycled material and the calculations of wear resistance. The results of calculations are displayed on screen in the form of dependencies diagrams, as well as may be printed.

4. Analysis of the parameters impact. Design automation system allows to display diagrams of dependencies of the chosen parameters according to the results of engineering calculations (for example, the moment dependence on the angular velocity of rotation or on the angularity of the screw line).

5. Modeling of technological process. As a result of emergency situations in the production process, such as sudden increase or decrease of pressure, boundary temperature conditions, the use of the plant for a new material with fundamentally different properties, the indicators of the efficiency of the extruder may change abruptly. Design automation system allows the engineer-user to simulate the behavior of the designed solution in emergency situations [18].

6. Development of documentation. According to the chosen by the engineer-user standards of the compilation of design documentation the system produces design drawings necessary for the performance of the designed solution and the explanatory notes to them.

The basis of the system of the screw extruders design automation is mathematical support of engineering calculations [1].

Analysis of the behavior of the recycled material in different points of the workspace of the extruder allowed to reveal several areas with different mathematical models. The main areas are the following: the zone of fluidity, the zone of mixing and compression, the zone of sealing and filling. In close proximity to the matrix the material is in quite mixed condition, close to the continuum in classical meaning [15]. Pressure caused by the resistance of the matrix and the product pressing-out, on the one hand, and by the pressure enters this zone under the influence of the rotary screw of the material, on the other hand, it is transmitted in all directions without changes according to the law of Pascal. The movement of the material is described by Navier–Stokes equations and by the continuity equation:

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + \mu\Delta\vec{v} - \frac{1}{\rho}\nabla p + \vec{f},$$

$$\nabla\cdot\vec{v} = 0,$$

where: ∇ – Hamilton operator, Δ – Laplace operator, t – time, μ – Newtonian viscosity of the material, ρ – density, p – pressure, $\vec{v} = (v_x, v_y, v_z)$ – vector field of velocities, $\vec{f}$ – vector field of body forces [13].

Due to the cylindrical shape of the body these equations are convenient to use in cylindrical coordinate system $\vec{v} = (v_\perp r, v_\perp \varphi, v_\perp z)$.

Because of stationarity of the flow inside the extruder the velocity derivatives are equal to zero in time. Due to the fact that the gravitational force is negligible compared to the internal forces in the layers of the material, the relevant components can be neglected [6].

As the boundary condition the condition of "adhesion" to the cylinder appears (velocity in a thin layer close to the cylinder equals to zero). For the same reason the velocity in the area of the material which borders with the screw is equal to the velocity of the screw rotation: $v_\varphi = wr$, $v_r = v_z = 0$.

The initial conditions considering the stationarity of the process are absent.

In the zone of mixing and compression the mixing is not completed and the pressure is small compared to the previous zone; the material shows the properties of pseudoplastic body. Mechanical properties of the material are presented by rheological equation, that relates the shear stress to the velocity of the shear of the material [2].

The matched condition with previous zone is the equality of pressure in the zone of the fluidity to the normal stress in the zone of mixing and compression.

The intense displacement of the added in the process of filling air and the sealing of the recycled material takes place in the zone of sealing. Because of this the shear stress on the body of the extruder changes direction in relation to the zone of mixing and compression [9].

Thus, the equality to zero of the shear and normal stress can be chosen as the condition of matching with the zone of mixing and compression. The computational model of motion of the recycled material is similar to the model for the previous zone, except for the continuity equation because of the compressibility of the material [16].

The ingoing material is received in the zone of ignition inside the extruder. This usually occurs under the influence of gravity, but the pressing of the material by the previous mixing device is also possible. There is a problem of the continuity of the feed [5].

The condition of matching with the zone of sealing is the occurrence of normal stress parallel to the axis of the extruder. We can apply for the quantitative analysis in this zone a statistical approach with the research of such values as the average size of air chambers and their number per unit of volume of the material [19].

The most appropriate for engineering calculations is the finite element approach [8] successfully applied in different industrial sectors.

Adequate from technological and computational points of view method of splitting the working space of the screw extruder into finite elements is shown in the figure 1:

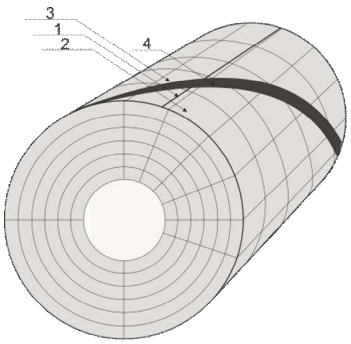

Fig. 1. Splitting the working space into finite elements: 1 - element adjacent to the blade; 2 - element not adjacent to the blade; 3 - element closing the ring; 4 - blade

The working space of the extruder is split into finite elements in three directions. The working space is split into discs by the surfaces perpendicular to the axis of the screw [3].

Discs, in their turn, are divided into rings by means of cylindrical surfaces the axis of which coincides with the axis of the screw. We obtain sectoral parts by breaking the rings with radial planes [4].

In each ring let us distinguish by the feature of neighborhood with the blade of the screw the following:

- element of the type 1 adjacent to the blade and experiencing the stress transfered to it and geometrically approximated to a triangular prism;
- elements of the type 2, not adjacent to the blade and geometrically approximated to a rectangular parallelepiped;
- element of the type 3, closing the ring and which is the mirror image of the element of the type 1 relative to the blade [20].

The dynamic equilibrium of the elements of type 1, 2 and 3 is determined by the total action of forces shown in figures 2, 3 and 4 respectively. To make a force calculation we direct the axis x in the circle direction, the axis y parallel to the axis of the screw, the axis z in the direction from the worm shaft to the cylindrical body. Having equated the total force and the total moment of force to zero we make an equilibrium equation for the element of the type 2 (for the elements of the types 1 and 3 the equations are made similarly):

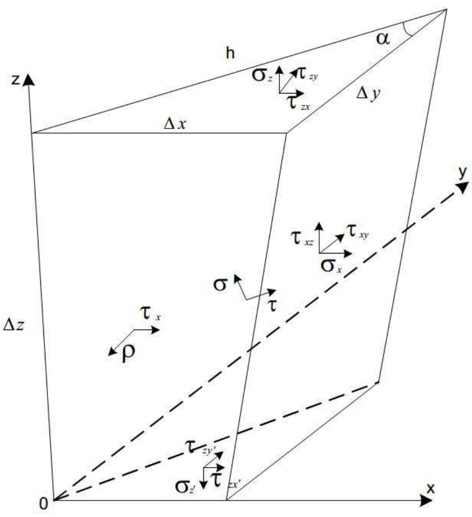

Fig. 2. Model of the element of the type 1

$$(\tau_x + \tau_{x'})\cdot \Delta x\Delta z + (\sigma_{x'} + \sigma_x)\cdot \Delta y\Delta z + (\tau_{zx} + \tau_{zx'})\cdot \Delta x\Delta y = 0,$$

$$(\rho' + \rho)\cdot \Delta x\Delta z + (\tau_{zy} + \tau_{zy'})\cdot \Delta y\Delta z + (\tau_{zy} + \tau_{zy'})\cdot \Delta x\Delta y = 0,$$

$$(\tau_{xz} + \tau_{xz'})\cdot \Delta y\Delta z + (\sigma_z - \sigma_{z'})\cdot \Delta x\Delta z = 0,$$

$$(\sigma_z - \sigma_{z'})\Delta x\Delta y\frac{\Delta y}{2} - \tau_{zy}\cdot \Delta x\Delta y\Delta z + (\rho - \rho')\Delta x\Delta z\frac{\Delta z}{2} + (\tau_{xz} - \tau_{xz'})\Delta y\Delta z\frac{\Delta y}{2} - (\tau_{xy} - \tau_{xy'})\Delta y\Delta z\frac{\Delta z}{2} = 0,$$

$$(\sigma_{x'} - \sigma_x)\Delta y\Delta z\frac{\Delta z}{2} + (\sigma_{z'} - \sigma_z)\Delta x\Delta y\frac{\Delta x}{2} + \tau_{zx'}\Delta x\Delta y\Delta z + (\tau_x + \tau_{x'})\Delta x\Delta z\frac{\Delta z}{2} - \tau_{xz'}\Delta x\Delta y\Delta z = 0,$$

$$(\rho' - \rho)\Delta x\Delta z\frac{\Delta x}{2} + (\sigma_x - \sigma_{x'})\Delta y\Delta z\frac{\Delta y}{2} - (\tau_{zx} + \tau_{zx'})\Delta x\Delta y\frac{\Delta y}{2} + (\tau_{zy} + \tau_{zy'})\Delta x\Delta y\frac{\Delta x}{2} - \tau_{x'}\Delta x\Delta z\Delta y + \tau_{xy'}\Delta x\Delta z\Delta y = 0.$$

An additional equation is necessary for unambiguous solution of this system of equations regarding the unknowns ρ', $\tau_{x'}$, $\sigma_{z'}$, $\tau_{zx'}$, $\tau_{zy'}$, $\tau_{xz'}$, $\tau_{xy'}$. For its generation let us turn to the theory of heat conduction and make the energy balance equation in the environment of the material.

Mechanical and thermodynamic properties do not change over time taking into account the stationarity of the considered process. Thus, the work of the viscous friction forces is equal to the amount of heat that is emitted from the layer of the material [11]. Let the temperature distribution in the direction from the body to the worm shaft be known and the change of the temperature of the material during the processing be negligibly small [7].

According to Fourier law of heat conduction, the power of thermal losses in a steady state is proportional to the temperature gradient [14]:

$$|P_1| = kS|grad(T)|,$$

where: P_1 - total power of thermal losses, S - area through which the heat passes, $grad(T)$ - velocity of the temperature change along the normal axis, k - thermal conductivity coefficient.

The work of viscous friction forces dA in the infinitesimal area with the volume dV for the case of constant density of the material is determined by the equation [15]:

$$\frac{dA}{dVdt} = -\tau\dot{\gamma}.$$

Let us consider work of friction forces in the layers of the material in the planes parallel to z0x, z0y, y0x independently of one another:

$$x0y: |P_2| = \int_{\Delta v} \tau\dot{\gamma}dV = \Delta x\Delta y\int_0^{\Delta z} \tau\dot{\gamma}dz,$$

$$y0z: |P_3| = \int_{\Delta v} \tau\dot{\gamma}dV = \Delta y\Delta z\int_0^{\Delta x} \tau\dot{\gamma}dx,$$

$$x0z: |P_4| = \int_{\Delta v} \tau\dot{\gamma}dV = \Delta x\Delta z\int_0^{\Delta y} \tau\dot{\gamma}dy,$$

where: τ and $\dot{\gamma}$ are the vector sums of voltages or shear rates acting in the directions parallel to the given plane inside the element, which depend on the integration variable [12].

Let us write the energy balance equation:

$$|P_1| = |P_2| + |P_3| + |P_4|,$$

By means of the rheological law the shear rate $\dot{\gamma}$ can be expressed through stress τ, and thus excluded from the abovementioned integrals. The rheological law can also include temperature. In the case, when the temperature is given only on the borders of the finite elements, internal values can be approximated linearly, considering the boundary values T_1 and T_0:

$$T = \psi(T_1 - T_0) + T_0,$$

where: ψ - dimensionless coordinate located in the interval [0; 1].

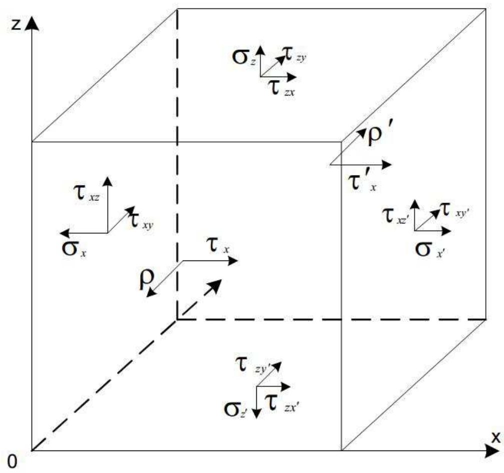

Fig. 3. Model the element of the type 2

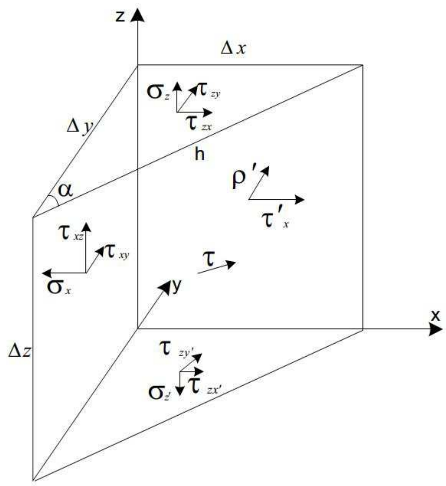

Fig. 4. Model the element of the type 3

Within a small finite element the internal stress distribution τ along the axis can also be considered approximately linear, that is determined by the boundary values τ_0 and τ_1:

$$\tau = \psi(\tau_1 - \tau_0) + \tau_0 .$$

Turning to the integration through the non-dimensional coordinate, we obtain:

$$\int_0^{\Delta x} \tau \dot{y} dx = \Delta x \int_0^1 \tau \dot{y} d\psi ,$$

$$\int_0^{\Delta y} \tau \dot{y} dy = \Delta y \int_0^1 \tau \dot{y} d\psi ,$$

$$\int_0^{\Delta z} \tau \dot{y} dz = \Delta z \int_0^1 \tau \dot{y} d\psi .$$

These integrals are possible concerning the values on the opposite faces of the element τ_0 and τ_1, using the rheological law for the recycled material.

Thus, for example, for the rheological equation $\tau = \tau^* + \mu_H \dot{y} \exp(a(T - T_H))$, where τ^*- yield point, μ_H - viscosity at normal temperature T_H, a - coefficient of the temperature sensitivity of the viscosity, the integral is solved like

$$\int_0^1 \tau y' d\psi = \int_0^1 \tau \frac{\tau - \tau^*}{\mu_H \exp(a(\psi(T_1 - T_0) + T_0 - T_H))} d\psi =$$

$$= \int_0^1 \tau \frac{(\psi(\tau_1 - \tau_0) + \tau_0)(\psi(\tau_1 - \tau_0) + \tau_0 - \tau^*)}{\mu_H \exp(a(\psi(T_1 - T_0) + T_0 - T_H))} d\psi =$$

$$= \frac{1}{\mu_H} \left(\int_0^1 \left(\psi^2 (\tau_1 - \tau_0)^2 + \psi(\tau_1 - \tau_0)\tau_0\right) e(\psi) d\psi \right) +$$

$$+ \left(\int_0^1 \left(\psi(\tau_1 - \tau_0)(\tau_0 - \tau^*) + \tau_0(\tau_0 - \tau^*)\right) e(\psi) d\psi \right) =$$

$$= \frac{(\tau_1 - \tau_0)^2}{\mu_H} \int_0^1 \psi^2 e(\psi) d\psi + \frac{(\tau_1 - \tau_0)(2\tau_0 - \tau^*)}{\mu_H} \times$$

$$\times \int_0^1 \psi e(\psi) d\psi + \frac{\tau_0(\tau_0 - \tau^*)}{\mu_H} \int_0^1 e(\psi) d\psi =$$

$$= \frac{(\tau_1 - \tau_0)^2}{\mu_H} \frac{(b^2 - 2b + 2)\exp(b + c) - 2\exp(c)}{b^3} +$$

$$+ \frac{(\tau_1 - \tau_0)(2\tau_0 - \tau^*)}{\mu_H} \frac{(b - 1)\exp(b + c) + \exp(c)}{b^2} +$$

$$+ \frac{\tau_0(\tau_0 - \tau^*)}{\mu_H} \frac{\exp(b + c) - \exp(c)}{b} ,$$

where: for short it is indicated: $e(\psi) = \exp(b\psi + c)$, $b = a(T_0 - T_1)$, $c = T_H - T_0$.

Substituting the possible, concerning τ_0 and τ_1, integrals into the equation of energy balance, we can obtain the closing given system of equations and calculate the given element.

CONCLUSIONS

Thus, the finite-element approach is put into the basis of the calculation of screw extruders. Its advantage is the versatility of the usage in a wide range of the properties of the recycled material and designs of screw extruders. The further research should be focused on the further detection of typical finite elements and computational schemes for other designs, calculating the shear rate in the recycled material and the development of the appropriate algorithmic support.

REFERENCES

1. **Abramov O.V., 2009.:** Nauchnoe obespechenie processa jekstruzii model'nyh sred na osnove krahmalosoderzhashhego syr'ja i razrabotka vysokojeffektivnogo oborudovanija dlja ego realizacii [Tekst]: Avtoreferat diss. … d.t.n. – Voronezh. 45.
2. **Bejgel'zimer Ja.E., Varjuhin V.N., Orlov D.V., Synkov S.G., 2003.:** Vintovaja jekstruzija– process nakoplenija deformacii.– Doneck: Firma TEAN. 87.
3. **Djadichev V.V., Kolesnikov A.V., 2010.:** Industrial enterprises study of automatic control systems. TEKA Kom. Mot. I Energ. Roln. – OL PAN, 10A, 126-132.
4. **Djadichev V.V., Tereshhenko T.M., Djadichev A.V., 2010.:** Problems of specified quality polymer mixture preparation when utilizing waste in coextrusion equipment. TEKA Kom. Mot. I Energ. Roln. – OL PAN, 10A, 113-118.
5. **Kandyrin L.B., SurikovP.V., 2002.:** Modelirovanie processov pererabotki plastmass: Uchebno – metodicheskoe posobie. M.: MITHT. 86.
6. **Klinkov A.S., Beljaev P.S., Sokolov M.V., 2005.:** Utilizacija i vtorichnaja pererabotka polimernyh materialov: Ucheb. posobie. Tambov: Izd-vo Tamb. gos. tehn. un-ta. 80.
7. **Kochin N.E., 1965.:** Vektornoe ischislenie i nachala tenzornogo ischislenija. M.: Nauka.
8. **Li K., 2004.:** Osnovy SAPR (CAD/CAM/CAE) [Tekst] / K. Li. – SPb.: Piter. 560.
9. **Nikolis G., 1990.:** Prigozhin I. Samoorganizacija v neravnovesnyh sistemah. Per. s angl. M.: Mir. 542.
10. **Polishhuk, V.Ju., 2003.:** Proektirovanie jekstruderov dlja otraslej APK [Tekst] / V.Ju. Polishhuk. – Ekaterinburg: UrO RAN. 201.
11. **Revjako M.M., Kasperovich O.M., 2005.:** Oborudovanie i osnovy proektirovanija predprijatij po pererabotke plastmass: ucheb. Posobie / – Mn.: BGTU. 344.
12. **Romanov A.S., Semikolenov A.V., Taranenko S.N., Shahorin A.P., 2008.:** Ideal'naja i vjazkaja zhidkosti: Ucheb. Posobie/A.S. – M.: Izd-vo MGTU im. N.Je. Baumana. 64.

13. **Sagalaeva G.V., 2000.:** Spravochnik po tehnologii izdelij iz plastmass. M.: Himija. 424.
14. **Sedov L.I., 1970.:** Mehanika sploshnoj sredy. T. 1, 2. M.: Nauka.
15. **Sedov, L.I., 1970.:** Mehanika sploshnoj sredy [Tekst]. V 2 t. T. 1 / L.I. Sedov. – M.: Nauka. 492.
16. **Shul'man Z.P. 1975.:** Konvektivnyj teplomassoperenos reologicheski slozhnyh zhidkostej. M.: Jenergija.
17. **Sokolov, M.V., 2004.:** Avtomatizirovannoe proektirovanie i raschet shnekovyh mashin [Tekst] / M.V. Sokolov, A.S. Klinkov, O.V. Efremov, P.S. Beljaev, V.G. Odnol'ko. – M.: «Izdatel'stvo Mashinostroenie1». 248.
18. **Vlasov S.V., Kandyrin L.B., Kuleznev V.N., 2004.:** Osnovy tehnologii pererabotki plastmass: Uchebnik dlja vuzov. – M.: Himija. 600; il.
19. **Zubkova T.M., 2004.:** Metodicheskie materialy po modelirovaniju i optimizacii odnoshnekovyh jekstruderov/ Pod. red. L.P. Kartashova. — M.: RASHN. 34.
20. **Zubkova T.M., 2002.:** Razrabotka metodologii matematicheskogo modelirovanija tehnologicheskih obiektov // Vestnik OGU. - №2. 209-213.

МАТЕМАТИЧЕСКОЕ ОБЕСПЕЧЕНИЕ СИСТЕМЫ АВТОМАТИЗИРОВАННОГО ПРОЕКТИРОВАНИЯ ШНЕКОВЫХ ЭКСТРУДЕРОВ

Валерий Дядичев, Вячеслав Жариков

А н н о т а ц и я. В работе разъяснена необходимость создания системы автоматизации проектирования шнековых экструдеров. Рассмотрены функции предполагаемой САПР. Представлены математические модели движения перерабатываемого в экструдере. Для расчета напряженного состояния материала применены конечно-элементный анализ и теория теплопроводности.

К л ю ч е в ы е с л о в а. Экструдер, САПР, конечно-элементный подход, уравнения равновесия.