Drewno 2016, Vol. 59, No. 196
DOI: 10.12841/wood.1644-3985.128.05

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APPLICATION OF ADHESIVE JOINTS IN REINFORCEMENT AND RECONSTRUCTION OF WEAKENED WOODEN ELEMENTS LOADED AXIALLY

The paper concerns the formulation and analysis of an adhesive joint model, aimed at reinforcing or reconstructing weakened wooden elements. The joint is modeled as a plane stress problem of the theory of elasticity. It is assumed that wood is an orthotropic material. The reinforcement of an element is achieved by means of attaching a covering plate, while reconstruction is carried out by introducting an insert into the weakened (deteriorated) zone of an element. The influence of varying thickness of plates and inserts on the stress states in the adherends and adhesive is analyzed. The analyses are related to axially loaded elements.

Keywords: wood, orthotropy, adhesive joint, element reinforcement, reconstruction of weakened element, stress concentrations

Introduction

An adhesive joint is made of two adherends in a state of plane stress connected at common surfaces by an adhesive. It is assumed that the adherends and the adhesive have constant or moderately changing thickness.

The adhesive joint is modeled as a two-dimensional plane element parallel to the 0XY plane in a Cartesian set of co-ordinates 0XYZ. Projections of the adherends and adhesive onto the 0XY plane form identical figures of an arbitrary shape.

It is assumed that the bending effects in adherends are small and negligible. Thus, it is further assumed that stresses are constant across adherend thickness and form plane stress states parallel to the 0XY plane. The layout of an adhesive joint is presented in figure 1.

The thickness of the adherends is represented by functions $g_1 = g_1(x, y)$ and $g_2 = g_2(x, y)$. The mid-plane of the adhesive is given by the function $s = s(x, y)$. Adhesive thickness $t = t(x, y)$ is always larger than zero.

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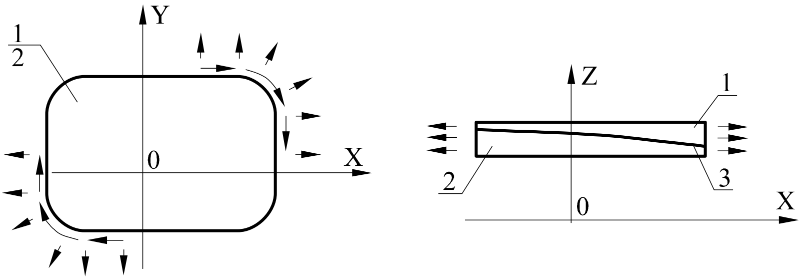


Fig. 1. Layout of adhesive joint. 1 – adherend 1, 2 – adherend 2, 3 – adhesive

The adherends are made from orthotropic materials and the principal axes of orthotropy coincide with the X and Y axes. An orthotropic material in a plane stress state is described using the moduli of longitudinal deformation E_{kx} , E_{ky} , the shear modulus G_{kxy} and Poisson's ratios ν_{kxy} , ν_{kyx}. The adhesive is modeled as a linearly elastic isotropic medium described using the following material constants: Young's modulus E_s , shear modulus G_s and Poisson's ratio ν_s , where $E_s = 2(1 + \nu_s)G_s$. The adhesive is subjected to shear stresses $\tau_x = \tau_x(x, y)$, $\tau_y = \tau_y(x, y)$ tangential to the adhesive mid-plane and stress $\sigma_N = \sigma_N(x, y)$ normal with respect to it. It is assumed that the stresses in the adhesive are constant across its thickness.

If an adherend thickness at its edge is more than zero, then we describe the edge as *unsharp*. Stresses acting at the unsharp edges of an adherend k are denoted as p_{kx} and p_{ky} (k = 1, 2). It is assumed that stresses p_{kx} and p_{ky} are parallel to the X and Y axes, respectively, and that they are constant across the thickness. They are understood as external loading to the adherend edges. The thickness of an adherend along the entire edge or its fragment can be zero. In this case, the edge is called *sharp*.

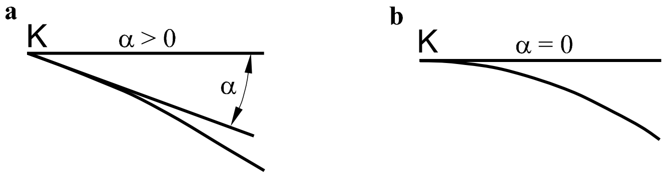


Fig. 2. Cross-section at two types of sharp edges K: a – obtuse sharp edge, b – tangential sharp edge

If a sharp edge K is defined by the external surfaces of two adherends forming an angle $\alpha > 0$, then the edge is called an *obtuse sharp edge* (fig. 2a). If a sharp edge has the external surfaces of two adherends, which are mutually tangential ($\alpha = 0$), then the edge is called a *tangential sharp edge* (fig. 2b). No boundary loading is defined at sharp edges.

Displacements of adherends 1 and 2 are given by the functions $u_1 = u_1(x, y)$ and $u_2 = u_2(x, y)$ in the direction of the X axis and by the functions $\upsilon_1 = \upsilon_1(x, y)$

and $\upsilon_2 = \upsilon_2(x, y)$ in the direction of the Y axis. The displacements u_1 , u_2 , υ_1 , υ_2 are considered as unknowns. Equations of the theory of elasticity in displacements and boundary conditions for a plane stress state were formulated in research by Rapp [2010, 2015]. Having found the functions of the displacements u_1 , u_2 , υ_1 , υ_2 , the stress and strain states for the adhesive and adherends may be expressed.

The subject of the paper

An adhesive joint is considered with an adherend 2 loaded axially by a force N, with an unloaded adherend 1 attached to it. If adherend 2 of a constant thickness has adherend 1 attached, then the total thickness of both adherends 1 and 2 is more than that of adherend 2 alone.

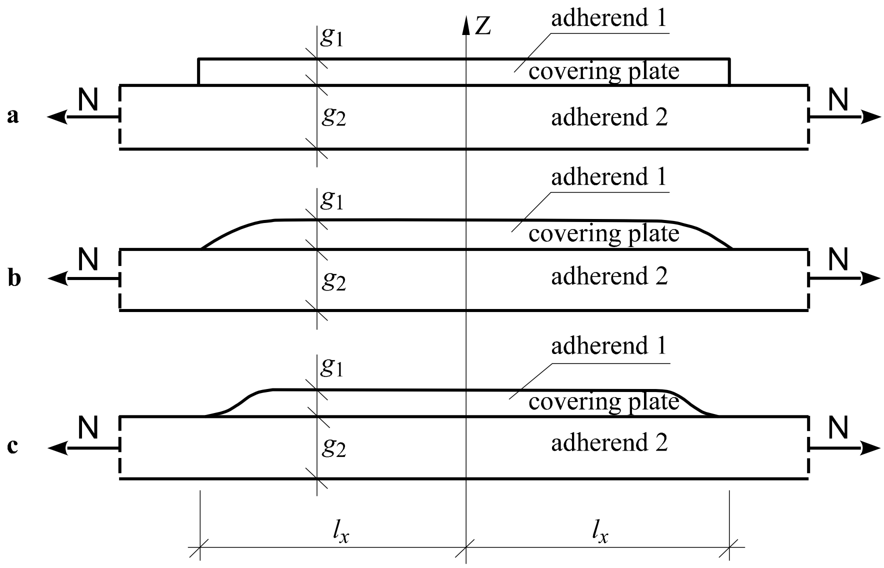


Fig. 3. Variants of reinforcement of adherend 2 using covering plates with: a – constant thickness, b – obtuse sharp edges, c) tangential sharp edges

Such an adhesive joint can be treated as a reinforcement of adherend 2 using *a covering plate* (adherend 1). The considered variants of such reinforcement using covering plates with various edge shapes are presented in figure 3.

If the material in adherend 2 is locally damaged or there are voids, then these zones can be replaced with a new adherend 1 in such a way that the total thickness of the adhesive joint is equal to the original thickness of element 2. Such an adhesive joint can be considered a reconstruction of the cross-section of element 2 by means of an insert (element 1). Some variants of reconstruction with inserts with variously shaped edges are presented in figure 4.

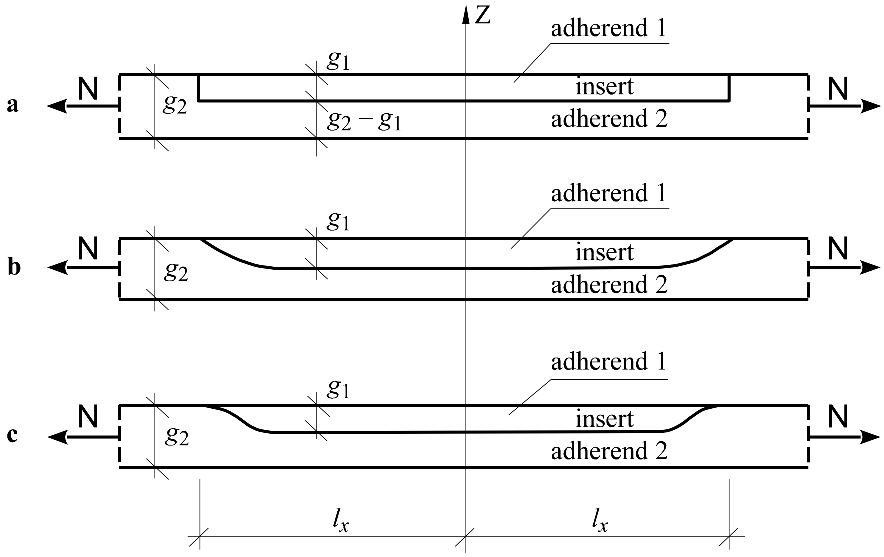


Fig. 4. Variants of reconstruction of adherend 2 using inserts with: a – constant thickness, b – obtuse sharp edges, c) with tangential sharp edges

Zones for the anchoring of covering plates or inserts should be short. Stresses in adherends between these zones should be constant and equal and stresses in the adhesive equal to zero. The adhesive in the anchoring zones should be free of stress concentrations.

Meeting these conditions greatly depends on the edge type and the varying thickness of inserts and covering plates in the anchoring zones.

In this paper, the influence of the shapes of the covering plates and inserts on the stress state in the adhesive and adherends is analyzed. In addition, formulae for the anchoring zone length for inserts and covering plates are derived.

Influence of covering plate and insert shape on stress state in joint

Adherend 2 carries all the loading at the edges determined by $x = \pm l_x$. In the range $-l_x < x < l_x$ both adherends 1 and 2 carry the load. Stresses in the adhesive at the edges $x = \pm l_x$ are relatively high. This section of the adhesive surface is considered the anchoring zone of the covering plate or insert. It is assumed that the adhesive joint in the anchoring zone carries a suitably large part of the load.

A stress state in the adhesive in the anchoring zone depends on adherend thickness at the edges $x = \pm l_x$. For further analysis it is assumed that covering plates and inserts may have constant thickness g_1 = const, as in figures 3a and 4a, varying thickness $g_1(x, y)$ with obtuse sharp edges given by formulae (1) – as in figures 3b and 4b, or with tangential sharp edges given by formulae (2) – as in figures 3c and 4c.

$$g_1(x,y) = \begin{cases} \dfrac{5g_1}{l_x}(x+0.7l_x)+\dfrac{3}{2}g_1 & \text{for} \quad -l_x \le x \le -0.9l_x\,, \\ -\dfrac{25g_1}{2l_x^2}(x+0.7l_x)^2+g_1 & \text{for} \quad -0.9l_x \le x \le -0.7l_x\,, \\ g_1 = \text{const} & \text{for} \quad -0.7l_x \le x \le 0.7l_x\,, \\ -\dfrac{25g_1}{2l_x^2}(x-0.7l_x)^2+g_1 & \text{for} \quad 0.7l_x \le x \le 0.9l_x\,, \\ -\dfrac{5g_1}{l_x}(x-0.7l_x)+\dfrac{3}{2}g_1 & \text{for} \quad 0.9l_x \le x \le l_x\,. \end{cases} \tag{1}$$

$$g_1(x,y) = \begin{cases} \dfrac{200g_1}{9l_x^2}(x+l_x)^2 & \text{for} \quad -l_x \le x \le -0.85l_x\,, \\ -\dfrac{200g_1}{9l_x^2}(x+0.7l_x)^2+g_1 & \text{for} \quad -0.85l_x \le x \le -0.7l_x\,, \\ g_1 = \text{const} & \text{for} \quad -0.7l_x \le x \le 0.7l_x\,, \\ -\dfrac{200g_1}{9l_x^2}(x-0.7l_x)^2+g_1 & \text{for} \quad 0.7l_x \le x \le 0.85l_x\,, \\ \dfrac{200g_1}{9l_x^2}(x-l_x)^2 & \text{for} \quad 0.85l_x \le x \le l_x\,. \end{cases} \tag{2}$$

An adhesive joint made of wooden adherends with two planes measuring $2l_x \times 2l_y = 10.0$ cm × 8.0 cm, $g_1 = 0.2$ cm and $g_2 = 1$ cm is analyzed. In a timber trunk one can distinguish an element approximately characterized by plane orthotropy – for instance, a plank cut from a trunk in a radial plane (fig. 5b).

In such a plank, in a plane stress state, the principal directions of orthotropy coincide with the direction parallel to the wood grain $\mathrm{X} = L$ and the radial direction perpendicular to the wood grain $\mathrm{Y} = R$. It is assumed that in both the adherends the wood grain direction is parallel to the X axis. The material constants for spruce wood were taken from Neuhaus [1994]:

- elasticity modulus in the direction parallel to the wood grain $E_x = 1.2 \cdot 10^6$ N/cm^2,
- elasticity modulus in the direction perpendicular to the wood grain $E_y = 0.8 \cdot 10^5$ N/cm^2,
- shear modulus $G_{xy} = 0.6 \cdot 10^5$ N/cm^2,

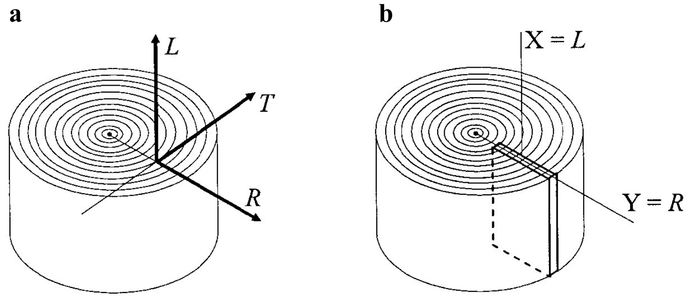


Fig. 5. Wood anisotropy: a – anatomical directions, b – a plank in a radial plane

– Poisson's ratios ν_{xy} = 0.03 and ν_{yx} = 0.45 (notation of ν_{xy}, ν_{yx} by Rapp [2015]).

The following data were assumed for the adhesive: thickness t = 0.04 cm, $G_s = 0.45 \cdot 10^5$ N/cm^2, $E_s = 1.215 \cdot 10^5$ N/cm^2. Then $\nu_s = 0.35$.

It is assumed that the adhesive joints are loaded axially by forces N = 8 N in a form of a normal stress 1 N/cm^2 uniformly distributed along the edges $x = \pm l_x$ of adherend 2.

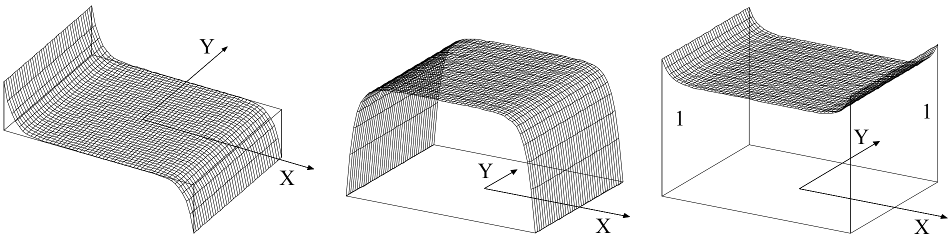
The loading N yields the stresses τ_x, τ_y, and σ_N in the adhesive and plane stress states σ_{kx}, σ_{ky} , and τ_{kxy} in the adherends (k = 1, 2). Force N is carried by the adhesive as a stress n_x as a resultant of the shear and normal stresses τ_x and σ_N, respectively, which are parallel to the X axis, and by the adherends as the normal stresses σ_{1x} , and σ_{2x}.

A two-dimensional boundary value problem for each adhesive joint presented in figures 3 and 4 was solved using the finite difference method. The presented results are restricted to the stresses n_x, σ_{1x}, and σ_{2x} related to axial force N. They are given in figures 6-10 (no results for the joint in fig. 4a were given as they only differ from those in fig. 6 in magnitudes, see fig. 12e).

The extreme adhesive stress values n_x are found at the covering plate or insert edges in the case of constant thickness (fig. 6a). At the obtuse sharp edges stress n_x decreases by ca 50-60% (figs. 7a and 9a), while the extreme values are still located at the edges. However, stress n_x at the adhesive surface is more flattened. Stress n_x at the tangential sharp edges is equal to zero (figs. 8a, 10a) [Rapp 2015].

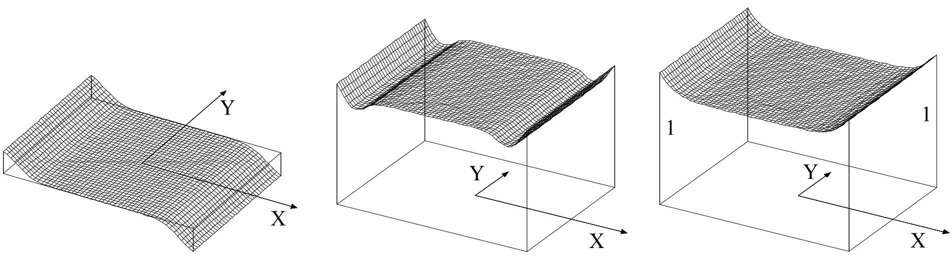
Covering plates and inserts with tangential sharp edges take the stresses from adherend 2 in a moderate way and the extreme stress n_x is found in the anchoring zone. Thus, the risk of adhesive debonding at the edge is reduced. Extreme values of stress n_x are lower than for obtuse sharp edges.

In the case of adherends of constant thickness the stress distributions σ_{1x} and σ_{2x} take the known shape. Stress σ_{1x} increases from zero at the edge $x = \pm l_x$ and quickly reaches an approximately constant level between the anchoring zones (fig. 6b). In loaded adherend 2 stress σ_{2x} at the edges $x = \pm l_x$ assumes the boundary values: 1 N/cm^2 (in the case of the covering plate) or 1.25 N/cm^2 (in the case of the insert), then decreases and quickly levels up to a constant value as in adherend 1 (fig. 6c).



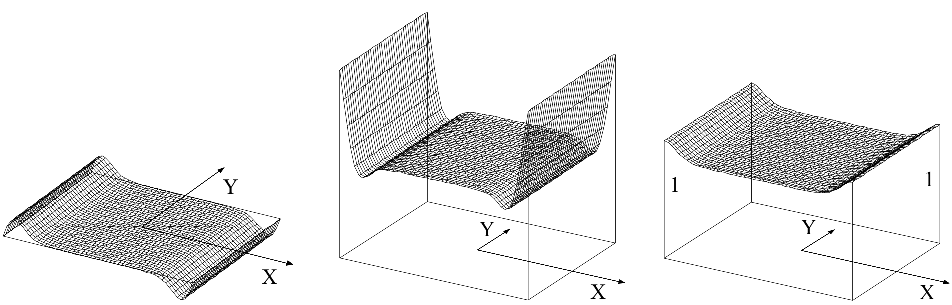
a – stress n_x b – stress σ_{1x} c – stress σ_{2x}

Fig. 6. Stresses due to axial force N = 8 N in a joint with a covering plate of constant thickness as in figure 3a. $n_x(\pm l_x, 0) = \pm 0.38215$ N/cm^2, $\sigma_{1x}(0, 0) = 0.83332$ N/cm^2, $\sigma_{2x}(\pm l_x, 0) = 1$ N/cm^2, $\sigma_{2x}(0, 0) = 0.83334$ N/cm^2



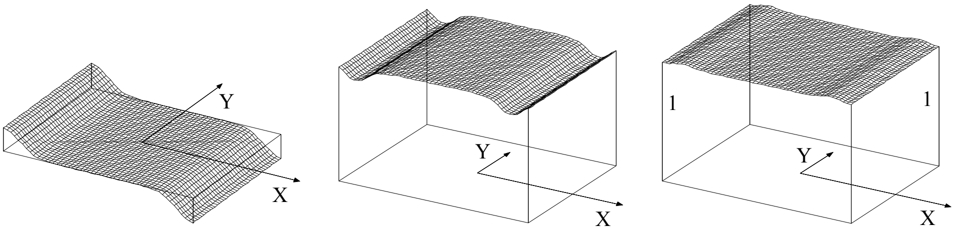
a – stress n_x b – stress σ_{1x} c – stress σ_{2x}

Fig. 7. Stresses due to axial force N = 8 N in a joint with a covering plate with obtuse sharp edges as in figure 3b. $n_x(\pm l_x, 0) = \pm 0.18173$ N/cm^2, $\sigma_{1x}(\pm l_x, 0) = 0.90865$ N/cm^2, $\sigma_{1x}(0, 0) = 0.83244$ N/cm^2, min $\sigma_{1x}(x, 0) = 0.74923$ N/m^2, $\sigma_{2x}(\pm l_x, 0) = 1$ N/cm^2, $\sigma_{2x}(0, 0) = 0.83248$ N/cm^2



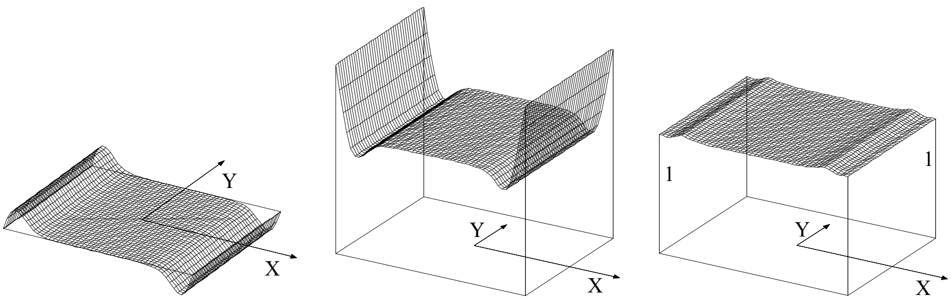
a – stress n_x b – stress σ_{1x} c – stress σ_{2x}

Fig. 8. Stresses due to axial force N = 8 N in a joint with a covering plate with tangential sharp edges as in figure 3c. max$|n_x(x, 0)| = 0.14749$ N/cm^2, $\sigma_{1x}(\pm l_x, 0) = 1.5943$ N/cm^2, $\sigma_{1x}(0, 0) = 0.83339$ N/cm^2, min $\sigma_{1x}(x, 0) = 0.71944$ N/m^2, $\sigma_{2x}(\pm l_x, 0) = 1$ N/cm^2, $\sigma_{2x}(0, 0) = 0.83345$ N/cm^2



a – stress n_x b – stress σ_{1x} c – stress σ_{2x}

Fig. 9. Stresses due to axial force N = 8 N in a joint with an insert with obtuse sharp edges insert as in figure 4b. $n_x(\pm l_x, 0) = \pm 0.18958$ N/cm², $\sigma_{1x}(\pm l_x, 0) = 0.96667$ N/cm², $\sigma_{1x}(0, 0) = 0.99971$ N/cm², min $\sigma_{1x}(x, 0) = 0.88205$ N/m², $\sigma_{2x}(\pm l_x, 0) = 1$ N/cm², max $\sigma_{2x}(x, 0) = 1.0246$ N/cm²



a – stress n_x b – stress σ_{1x} c – stress σ_{2x}

Fig. 10. Stresses due to axial force N = 8 N in a joint with an insert with tangential sharp edges as in figure 4c. max$|n_x(x, 0)| = 0.17134$ N/cm², $\sigma_{1x}(\pm l_x, 0) = 1.5942$ N/cm², $\sigma_{1x}(0, 0) = 1.0001$ N/cm², min $\sigma_{1x}(x, 0) = 0.85296$ N/m², $\sigma_{2x}(\pm l_x,$v$0) = 1$ N/cm², max $\sigma_{2x}(x, 0) = 1.0330$ N/cm²

Except for small fluctuations at the anchoring zones, stress level σ_{1x} is flat and only very slightly exceeds the stress in the adhesive joint in both cases of obtuse sharp edges – with a covering plate or with an insert. Such a model is a most convenient way to reinforce or reconstruct a cross-section of adherend 2 (figs. 7b, 9b).

In the case of tangential sharp edges for covering plates or inserts, a convenient distribution of stress n_x is accompanied by a large local increase in stress σ_{1x} at the edges of adherend 1. In the case of the covering plate, it is ca 100% (fig. 8b), and for the insert – ca 75% (fig. 10b) of the mean stress value. This is due to the fact that adherend 1 is less thick at the sharp edges.

Anchoring length for covering plate and insert

For a joint between adherends of constant thickness (figs. 3a and 4a), the anchoring length of a covering plate or an insert can be assessed analytically using a one-dimensional model, where adherend 2 is under axial tension due to edge stresses $p_{2x}^{p} = \sigma$ and $p_{2x}^{l} = -\sigma$, (σ > 0), and adherend 1 is not loaded. The function of the shear stress in adhesive τ_x is given by a known relation

$$\tau_x(x) = -\frac{G_s \sigma}{t k_0 E_{2x} \cosh k_0 l_x} \sinh k_0 x \ , \tag{3}$$

where

$$k_0^2 = \frac{G_s}{t}\left(\frac{1}{g_1 E_{1x}} + \frac{1}{g_2 E_{2x}}\right).$$

The distribution of function (3) is given in figure 11. There are regions limited by curve τ_x and the X axis along sections $l_{anch} = l_x - l_k$. An area of each region is a measure of the force carried by the adhesive joint between covering plate 1 and adherend 2 on the left and right ends of the joint. The sections l_{anch} determine the anchoring zones on the adhesive surface and the length l_{anch} is the anchoring length for a covering plate attached to the loaded element.

The length of an anchoring zone can be determined in various ways, for instance as a ratio $\tau_x(l_x) : \tau_x(l_k)$ between shear stresses at the ends of the section l_{anch} or as a ratio between the area bounded by curve τ_x and the X axis along l_{anch} and the entire area along the section [0, l_x].

According to the criterion defined by the ratio $\tau_x(l_x) : \tau_x(l_k)$ one gets

$$\tau_x(l_x) = -\frac{G_s \cdot \sinh k_0 l_x}{t k_0 E_{2x} \cosh k_0 l_x} \cdot \sigma \ , \qquad \tau_x(l_k) = -\frac{G_s \cdot \sinh k_0 l_k}{t k_0 E_{2x} \cosh k_0 l_x} \cdot \sigma$$

and

$$\frac{\tau_x(l_x)}{\tau_x(l_k)} = \frac{\sinh k_0 l_x}{\sinh k_0 l_k} = \frac{e^{k_0 l_x} - e^{-k_0 l_x}}{e^{k_0 l_k} - e^{-k_0 l_k}} \ . \tag{4}$$

Usually $e^{-k_0 l_x}$ and $e^{-k_0 l_k}$ are small when compared to $e^{k_0 l_x}$ and $e^{k_0 l_k}$. If one neglects $e^{-k_0 l_x}$ and $e^{-k_0 l_k}$ in the expression (4) the following relation yields

$$\frac{\tau_x(l_x)}{\tau_x(l_k)} = \frac{e^{k_0 l_x}}{e^{k_0 l_k}} = e^{k_0 (l_x - l_k)} \ . \tag{5}$$

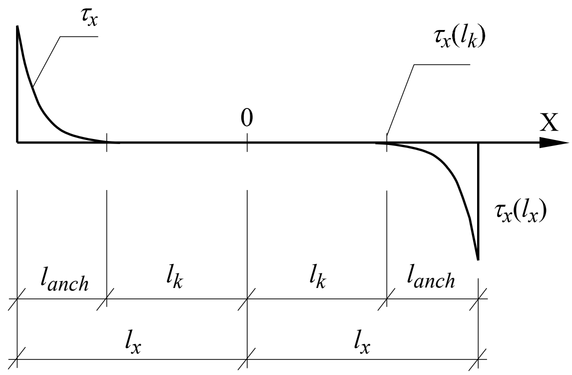


Fig. 11. Stress distribution τ_x in adhesive for a joint with a covering plate loaded axially as in figure 3a

Introducing the anchoring length $l_{anch} = l_x - l_k$, one gets from (5)

$$l_{anch} = \frac{1}{k_0} \ln \frac{\tau_x(l_x)}{\tau_x(l_k)} \,. \tag{6}$$

If N_1 denotes an axial force in a covering plate at the point $x = 0$, then in the one-dimensional model

$$N_1 = \int_0^{l_x} |\tau_x(x)| dx \,. \tag{7}$$

According to the second criterion the anchoring force, denoted by N_{anch}, is defined as part of the force N_1

$$N_{anch} = p\, N_1, \tag{8}$$

where $0 < p < 1$. It means that the anchoring zone carries $p \cdot 100\%$ of force N_1. The anchoring force N_{anch} as a result of stress τ_x in the adhesive in the anchoring zone l_{anch} can be given by the following formulae

$$N_{anch} = \int_{l_k}^{l_x} |\tau_x(x)| dx \,. \tag{9}$$

Substitution of function τ_x from (3) to the relations (7) and (9) yields

$$N_1 = \int_0^{l_x} |\tau_x(x)| dx = \frac{G_s \sigma}{t k_0^2 E_{2x} \cosh k_0 l_x} (\cosh k_0 l_x - 1) =$$

$$= \frac{G_s \sigma}{t k_0^2 E_{2x} \cosh k_0 l_x} \left(\frac{e^{k_0 l_x} + e^{-k_0 l_x}}{2} - 1 \right), \tag{10}$$

$$\mathrm{N}_{anch} = \int_{l_k}^{l_x} \left|\tau_x(x)\right| dx = \frac{G_s\sigma}{tk_0^2 E_{2x}\cosh k_0 l_x}(\cosh k_0 l_x - \cosh k_0 l_k) =$$

$$= \frac{G_s\sigma}{tk_0^2 E_{2x}\cosh k_0 l_x}\left(\frac{e^{k_0 l_x} + e^{-k_0 l_x}}{2} - \frac{e^{k_0 l_k} + e^{-k_0 l_k}}{2}\right). \tag{11}$$

Neglecting the terms $e^{-k_0 l_x}$ and 1 in (10), and $e^{-k_0 l_k}$ in (11) one gets approximate formulae

$$\mathrm{N}_1 = \frac{G_s\sigma}{tk_0^2 E_{2x}\cosh k_0 l_x}\cdot\frac{e^{k_0 l_x}}{2} \quad \text{and} \quad \mathrm{N}_{anch} = \frac{G_s\sigma}{tk_0^2 E_{2x}\cosh k_0 l_x}\left(\frac{e^{k_0 l_x}}{2} - \frac{e^{k_0 l_k}}{2}\right)$$

Substitution of these relations to condition (8) leads to

$$e^{k_0 l_k} = (1-p)e^{k_0 l_x} \quad \text{and} \quad k_0 l_k = \ln(1-p) + k_0 l_x .$$

From the equation $l_{anch} = l_x - l_k$ one gets the following relation

$$l_{anch} = \frac{1}{k_0}\ln\frac{1}{1-p} . \tag{12}$$

In this way two formulae, (6) and (12), defining the anchoring length for a covering plate in an axially loaded adhesive joint, depending on the definition of the anchoring zone, have been formulated. If the following condition is met

$$\frac{\tau_x(l_x)}{\tau_x(l_k)} = \frac{1}{1-p} . \tag{13}$$

then they yield the same value of anchoring length.

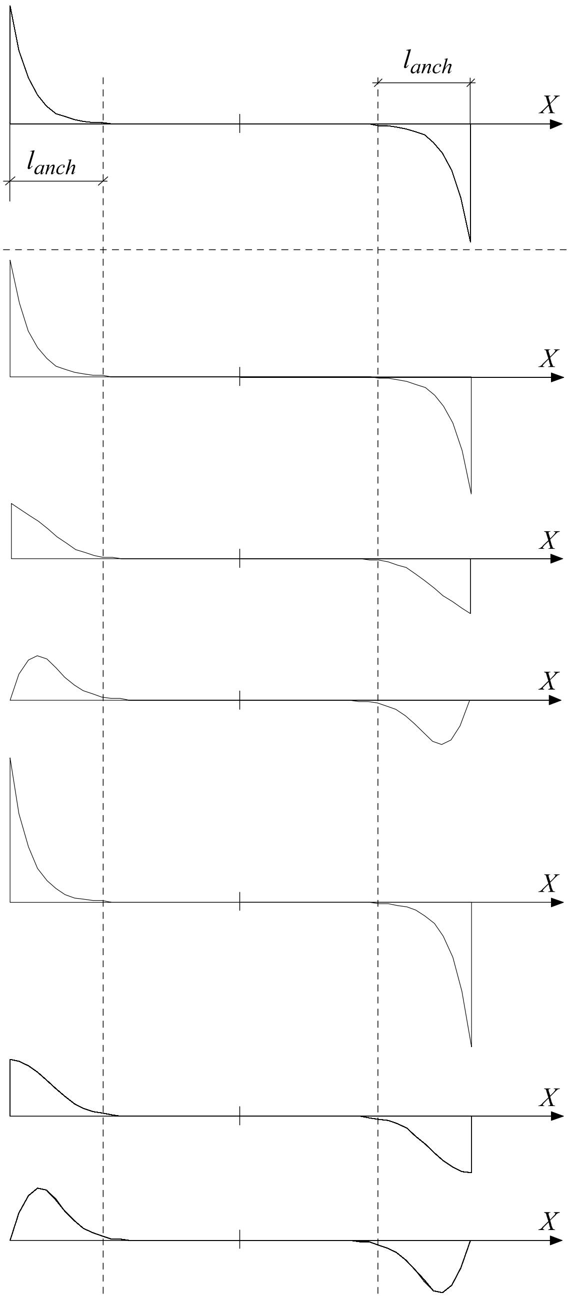
For instance, if $\tau_x(l_x) : \tau_x(l_k) = 100$, then relation (13) leads to $p = 0.99$. This means that the anchoring zone carries 99% of the entire load acting on the adhesive joint. In this case, the anchoring length is

$$l_{anch} = \frac{\ln 100}{k_0} \approx \frac{4.6}{k_0} = 4.6\cdot\sqrt{\frac{tg_1 g_2 E_{1x} E_{2x}}{G_s(g_1 E_{1x} + g_2 E_{2x})}} . \tag{14}$$

If the width $2l_y$ of the adhesive joint is large enough to have a plane stress strip in its central zone, then in the one-dimensional model with an orthotropic material one has to substitute

$E'_{1x} = E_{1x}/(1-\nu_{1xy}\nu_{1yx})$ and $E'_{2x} = E_{2x}/(1-\nu_{2xy}\nu_{2yx})$ for E_{1x} and E_{2x}.

Formulae (6), (12) and (14) give good approximations of the anchoring length of a covering plate l_{anch} for the one-dimensional model loaded axially,



a) One-dimensional plane stress model of a joint with a covering plate of constant thickness as in fig. 3a

$n_x(\pm l_x, 0) = \pm 0.39260$ N/cm^2
$l_{anch} = 1.95495$ cm ≈ 1.955 cm

Two-dimensional models

b) A joint with a covering plate of constant thickness as in figure 3a
$n_x(\pm l_x, 0) = \pm 0.38215$ N/cm^2

c) A joint with a covering plate with obtuse sharp edges as in figure 3b
$n_x(\pm l_x, 0) = \pm 0.18173$ N/cm^2

d) A joint with a covering plate with tangential sharp edges as in figure 3c
$\max|n_x(x, 0)| = \pm 0.14749$ N/cm^2

e) A joint with an insert of constant thickness as in figure 4a
$n_x(\pm l_x, 0) = \pm 0.46752$ N/cm^2

f) A joint with an insert with obtuse sharp edges as in figure 4b
$n_x(\pm l_x, 0) = \pm 0.18958$ N/cm^2

g) A joint with an insert with tangential sharp edges as in fig. 4c
$\max|n_x(x, 0)| = \pm 0.17134$ N/cm^2

Fig. 12. Stress profiles n_x in adhesive of joints with covering plates and inserts loaded axially by force $\mathbf{N}$ = 8 N. Graphical comparison of anchoring lengths for covering plates and inserts

if the adhesive joint has medium or small deformability. Then the values $k_0 l_x$ and $k_0 l_k$ are sufficiently high and the anchoring length is relatively short, which is important from a practical point of view. The accuracy of these remarks can be checked in the adhesive joint presented in figure 3a. For the plane stress state, one gets $k_0 = 2.35564$ 1/cm. Then

$$e^{k_0 l_x} = e^{11.7782} \approx 130380 \text{ and } e^{-k_0 l_x} = e^{-11.7782} \approx 0.00000767.$$

For $p = 0.99$, one gets $l_{anch} = 1.95495$ cm from (11). The anchoring length calculated for the adhesive joint with an insert, presented in fig. 4a, is $l_{anch} = 1.91545$ cm.

The anchoring length l_{anch} from the one-dimensional model represents a good approximation of the anchoring lengths for covering plates and inserts in two-dimensional models of adhesive joints loaded axially. For verification purposes, the distributions of stress n_x in the adhesive for the one-dimensional model and two-dimensional adhesive joints shown in figures 3 and 4 are presented in figure 12 on the same scale.

In figures 12a-e, the adhesive is parallel to the 0XY plane, therefore $n_x = \tau_x$. In the cases shown in figures 12f and 12g, the adhesive surfaces in the anchoring zones are curved. Thus, the distributions of stress n_x parallel to the X axis (stress n_x is a result of stresses τ_x and σ_N) are shown to enable a comparison of the results.

It can be seen in figures 6-10 that stresses τ_x and σ_N (as well as n_x) in the adhesive are almost constant along the Y axis. Thus, the graphs of the function of n_x along the X axis, presented in fig. 12, are representative. The anchoring length $l_{anch} \approx 1.955$ cm is measured to scale in figure 12a as calculated from a plane stress strip in the one-dimensional model, and it is depicted by dashed lines in the remaining figures 12b-g.

The length l_{anch} calculated from the one-dimensional model is a good approximation of the anchoring length in two-dimensional models.

Reinforcement and reconstruction zones for an element

For the assumed value $p = 0.99$ in the anchoring zone, the joint carries 99% of force N_1, i.e. the total force carried by the joint along $0 \le x \le l_x$. It can be concluded from the equality $\mathsf{N}_1(x) + \mathsf{N}_2(x) = \mathsf{N}$ that in section $0 \le x \le l_x - l_{anch}$ the following inequalities hold:

$$\mathrm{N}_1 \ge \mathrm{N}_1(x) \ge 0.99\ \mathrm{N}_1 \quad \text{and} \quad \mathsf{N} - \mathsf{N}_1 \le \mathsf{N}_2(x) \le \mathsf{N} - 0.99\ \mathsf{N}_1 .$$

Similarly, in section $-l_x + l_{anch} \le x \le 0$ the inequalities

$$0.99\ \mathsf{N}_1 \le \mathsf{N}_1(x) \le \mathsf{N}_1 \quad \text{oraz} \quad \mathsf{N} - 0.99\ \mathsf{N}_1 \ge \mathsf{N}_2(x) \ge \mathsf{N} - \mathsf{N}_1$$

are correct. Thus, in region $-l_x + l_{anch} \leq x \leq l_x - l_{anch}$ stresses σ_{1x}, σ_{2x} in adherends 1 and 2 are approximately constant. In this zone of the joint, stress $n_x = \tau_x$ is negligible or equal to zero (fig. 12), so displacements and strains in both adherends are approximately: $u_1 \approx u_2$ and $\varepsilon_{1x} \approx \varepsilon_{2x}$. Hence, for normal stress in the adherends the relation $\sigma_{1x} : \sigma_{2x} \approx E_{1x} : E_{2x}$ is true.

If the adherends are made from identical materials, then in zone $-l_x + l_{anch} \leq x \leq l_x - l_{anch}$, the normal stresses σ_{1x} and σ_{2x} in adherends 1 and 2 are approximately identical and constant, as illustrated in figs. 6-10.

The internal zone of adhesive joint $-l_x + l_{anch} \leq x \leq l_x - l_{anch}$ can be considered a reinforcing zone for element 2 in the case of the covering plate or a reconstruction zone for the cross-section of element 2 in the case of the insert.

Conclusions

In the case of axial loading, extreme values of stress n_x in the adhesive occur at the edges of the covering plates and inserts. In the case of obtuse sharp edges, stress n_x is reduced by ca 50-75%. For tangential sharp edges, adhesive stress n_x is zero. A covering plate or an insert with tangential sharp edges takes stresses from adherend 2 in a moderate way and extreme stress n_x in the adhesive is located in the anchoring zone. Thus, the risk of debonding at the edges is reduced. The maximum values of stress n_x are then lower than the stress in the case of the obtuse sharp edges.

In the cases of adherends with constant thickness, stress σ_{1x} in the covering plates or inserts increases from zero at the edges $x = \pm l_x$ and quickly stabilizes at an approximately constant level between the anchoring zones. Stress σ_{2x} at the edges $x = \pm l_x$ of the loaded adherend 2 assumes boundary values and then decreases to a constant level as in adherend 1.

The level of stress σ_{1x} in the cases of covering plates and inserts having obtuse sharp edges is flattened except for insignificant fluctuations and it only very slightly exceeds the values of the stresses acting on the adhesive joint. Such a case is most efficient for reinforcing and reconstructing element 2.

In the cases of covering plates or inserts with tangential sharp edges, stress distribution n_x in the adhesive is advantageous. However, a local increase in stress at the edges of the covering plates and inserts is not. In the case of the covering plate, it amounts to ca 100%, and in the case of the insert – to ca 60% of the mean stress. The reason for this increase is the fact that adherend 1 is less thick at the sharp edges.

The anchoring length for the covering plates and inserts calculated from the one-dimensional model is a good approximation of the anchoring length in two-dimensional models.

An overview of problems related to reinforcing and reconstructing weakened elements in various technical fields can be found in Ahn and Springer [2000];

Bahei-El-Din and Dvorak [2001]; Kaye and Heller [2002]; Boss et al. [2003]; Kumar et al. [2006]; and Wang and Gunnion [2008].

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