TEKA Kom. Mot. i Energ. Roln. – OL PAN, 2011, 11B, 177-184

MATHEMATICAL MODEL TO MINIMIZE OPERATING COSTS FOR INTERCITY PASSENGER TRAFFIC

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Summary. The given paper presents an elaborated mathematical model of making optimal scheduling of passenger transport in long-distance communication. The model is based on the analysis of the distribution laws of demand, taking into account the possible presence of heterogeneous passenger flows.

Keywords: passenger transport, passenger flow, the distribution law, forecasting, schedules.

INTRODUCTION

Current systems of traffic management are not optimal, rolling stock is used with low efficiency. The reasons are the lack of validity of the routing and timetables, lack of or inadequate traffic control operations, the discrepancy of passenger fleet to the passenger flows etc. As a consequence a low level of passenger service remains.

In this paper we consider the task of organizing and planning for long-distance passenger transportation - and the use of rational combination of possible resources to meet the maximum transport work for better public services.

For any production of services the loss of two types may be possible:

- of unmet demand, in this case there is shortfall in profits;
- the excess of supply over demand, in this case there are losses from the production of services not found in demand.

The presence of excess vehicles is not profitable, because it leads to "empty carriage," and lack of vehicles also leads to losses due to shortfalls in revenue and the rise of competitors in the market of services.

Governance is associated with adjusting the distribution of departure for the distribution of transport demand. In this paper we study primarily the distribution of demand and data processing is aimed at finding and forecasting parameters of the distribution law according to which an optimal schedule of traffic is sought.

MATHEMATICAL MODEL OF THE PROCESS

We introduce the following notations:

- let it be on the route, which is denoted by W , buses go forward with X^{st} in X^{fn} , and in the opposite direction from X^{fn} to X^{st} ;
- t_{st}^{fwd} (t_{st}^{rev}) – start time of buses on the route маршруте W in the forward (backward) direction;
- t_{fn}^{fwd} (t_{fn}^{rev}) – end time of buses on the route of W in the forward (backward) direction. Where ($t_{fn}^{fwd} - t_{st}^{fwd}$), ($t_{fn}^{rev} - t_{st}^{rev}$) $\in N$;
- $T^{fwd} = \left([t_{st}^{fwd}, t_{st}^{fwd}+1], [t_{st}^{fwd}+1, t_{st}^{fwd}+2], \ldots, [t_{st}^{fwd}+n_{fwd}-1, t_{fn}^{fwd}]\right)$ - an ordered sequence of time intervals per 1 hour, where $n_{fwd} = t_{fn}^{fwd} - t_{st}^{fwd}$;
- $T^{rev} = \left([t_{st}^{rev}, t_{st}^{rev}+1], [t_{st}^{rev}+1, t_{st}^{rev}+2], \ldots, [t_{st}^{rev}+n_{rev}-1, t_{fn}^{rev}]\right)$ - an ordered sequence of time intervals per 1 hour, where $n_{rev} = t_{fn}^{rev} - t_{st}^{rev}$;
- $m_{t_i^{fwd}}^{fwd}$ ($t_i^{fwd} \in T^{fwd}$, where i - the serial number corresponding to t in T^{fwd}) – number of passengers who have tickets to each of the periods of time in the forward direction;
- $m_{t_i^{rev}}^{rev}$ ($t_i^{rev} \in T^{rev}$, where i - the serial number corresponding to t in T^{rev}) – number of passengers who have tickets to each of the periods of time in the opposite direction.

Statistical data to be processed - data on the sale of tickets for several years on the route W. Data on the ticket was originally processed for each day separately, and the algorithm processing is identical, so further in paragraphs 1-7, the algorithm is regarded as an example of day.

1. Initial data processing

At the initial processing of data for each t_i^{fwd} (t_i^{rev}) is found $m_{t_i^{fwd}}^{fwd}$ ($m_{t_i^{rev}}^{rev}$). Data processing algorithm in the forward and backward directions is similar, so the algorithm of paragraphs 2-7 as an example of direct destinations. For simplicity, it is written:

- instead T^{fwd} - T ;
- instead of $m_{t_i^{fwd}}^{fwd}$ - m_i (where i – the serial number corresponding to t in T);
- instead of t_i^{fwd} - t_i (where i – the serial number corresponding to t in T)

2. Reflection of the real length of time on the interval [0, 1] as a necessary condition for the beta distribution

In the future, to obtain the theoretical distribution function of the data a beta distribution will be used, for this it will need to use an ordered sequence of time intervals T , turned into a:

- x - an ordered sequence of number of segments belonging to the interval [0,1];

- $\bar{x}$ - an ordered sequence of numbers belonging to the interval [0, 1]

An ordered sequence of numerical segments x belonging to the interval [0, 1], is computed as follows. Every $t_i \in T$ – the length of time, that is $t_i = [t_1^i, t_2^i]$, is transformed into a sequence of x according to the formulas:

$$x_1^i = \frac{t_1^i - t_{st}^{fwd}}{t_{fn}^{fwd} - t_{st}^{fwd}} \text{ и } x_2^i = \frac{t_2^i - t_{st}^{fwd}}{t_{fn}^{fwd} - t_{st}^{fwd}}, \quad (1)$$

and $x_i = [x_1^i, x_2^i]$ that is computed $t_i \in T$, while any $x_i \in [0,1]$.

An ordered sequence of numbers $\bar{x}$, which belongs to the interval [0,1] is computed as follows. From an ordered sequence of segments x an ordered sequence of numbers is received $\bar{x}$, where $\bar{x}$ is the middle of the segment x_i:

$$\bar{x}_i = \frac{x_1^i + x_2^i}{2}. \quad (2)$$

3. Partitioning data on flows

When a general law of distribution is not unimodal, which means that it is heterogeneous, the data are broken down into multiple threads, each of which is unimodal.

Due to the fact that in most cases within days there are two maxima - so-called clock "peak" - morning and evening, we consider the distribution of transport demand during the day as a bimodal distribution law. In all other cases, the data must be split in a similar way. Due to the fact that in most cases within days there are two maxima - so-called clock "peak" - morning and evening, we consider the distribution of transport demand during the day as a bimodal distribution law. In all other cases, the data must be split in a similar way. All the calculations in the future will be for the two streams of the distribution law. The result of decomposition is presented in table 1.

Table 1. **Result of splitting into two streams of data**

THE FIRST STREAM			
t	$t_1 = [t_{st}^{fwd}, t_{st}^{fwd} + 1]$	…	$t_l = [t_{st}^{fwd} + l - 1, t_{st}^{fwd} + l]$
x	$x_1 = [x_1^1, x_2^1]$	…	$x_l = [x_1^l, x_2^l]$
$\bar{x}$	$\bar{x}_1 = \frac{x_1^1 + x_2^1}{2}$	…	$\bar{x}_l = \frac{x_1^l + x_2^l}{2}$
m	m_1	…	m_l
THE SECOND STREAM			
t	$t_l = [t_{st}^{fwd} + l - 1, t_{st}^{fwd} + l]$	…	$t_{n_{fwd}} = [t_{st}^{fwd} + n_{fwd} - 1, t_{fn}^{fwd}]$
x	$x_l = [x_1^l, x_2^l]$	…	$x_{n_{fwd}} = [x_1^{n_{fwd}}, x_2^{n_{fwd}}]$
$\bar{x}$	$\bar{x}_l = \frac{x_1^l + x_2^l}{2}$	…	$\bar{x}_{n_{fwd}} = \frac{x_1^{n_{fwd}} + x_2^{n_{fwd}}}{2}$
m	m_l	…	$m_{n_{fwd}}$

Each thread represents a unimodal distribution and data processing algorithm is similar in different threads, so in paragraphs 4-7, an algorithm for data flow on the example of the first stream is given.

4. Obtaining the empirical distribution function flow

To obtain the empirical distribution function we will use the data obtained after the initial processing of statistical data. The algorithm to obtain the empirical distribution function $\widetilde{F}$:

4.1 You must obtain the empirical distribution of the flow that is to calculate for each $t_i \in T$ the probability $\widetilde{p}_i$:

$$\widetilde{p}_i = \frac{m_i}{\sum_i m_i}. \tag{3}$$

4.2 Empirical function flow distribution $\widetilde{F}$ is as follows:

$$\widetilde{F}_1 = \widetilde{p}_1, \widetilde{F}_2 = \widetilde{F}_1 + \widetilde{p}_2, \ldots, \widetilde{F}_l = \widetilde{F}_{l-1} + \widetilde{p}_l \tag{4}$$

5. The primary estimation of the parameters of beta distribution

The primary parameters of the beta distribution are calculated by the method of moments:

$$\alpha_0 = \frac{MX^2(1-MX)}{DX} - MX \quad \text{and} \quad \beta_0 = \alpha_0(\frac{1}{MX} - 1). \tag{5}$$

To calculate α_0 and β_0 to find mathematical mean Mx and variance Dx :

$$Mx = \sum_i \bar{x}_i p_i \text{ and } Dx = \sum_i (\bar{x}_i)^2 p_i - (Mx)^2. \tag{6}$$

Once Mx and Dx would be found, α_0 and β_0 computed.

6. The theoretical distribution function of the flow

Algorithm to obtain the theoretical distribution function:

6.1 You must obtain the theoretical flow distribution that is to count theoretical probability ($\hat{p}_i$) for each $t_i \in T$:

$$\begin{aligned} &\hat{p}_1 = \hat{p}_{t_1} = \hat{p}_{x_1} \\ &\hat{p}_2 = \hat{p}_{t_2} = \hat{p}_{x_2} \\ &\ldots\ldots\ldots\ldots\ldots \\ &\hat{p}_{l-1} = \hat{p}_{t_{l-1}} = \hat{p}_{x_{l-1}} \\ &\hat{p}_l = \hat{p}_{t_l} = \hat{p}_{x_l} \end{aligned} \tag{7}$$

The theoretical probability calculated by the formula:

$$\hat{p}_{a;b}(\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_a^b x^{\alpha-1}(1-x)^{\beta-1} dx, \text{ where } [a,b] \in [0,1] \tag{8}$$

$$\hat{p}_1 + \hat{p}_2 + \ldots + \hat{p}_{l-1} + \hat{p}_l = 1. \tag{9}$$

6.2 The theoretical distribution function is calculated as follows:

$$\hat{F}_1 = \hat{p}_1, \hat{F}_2 = \hat{F}_1 + \hat{p}_2, \ldots, \hat{F}_l = \hat{F}_{l-1} + \hat{p}_l. \tag{10}$$

7. The optimization of parameters and_ of theoretical distribution function of the flow.

To optimize the parameters α and β the theoretical distribution function the method of global search is applied. For this we introduce a new feature:

$$d(\alpha,\beta) = \|\hat{F} - \tilde{F}\| = \sqrt{\sum_i (\hat{F}_i - \tilde{F}_i)^2} \ . \tag{11}$$

Such α and β must be found for a function $d(\alpha,\beta)$ to be minimal, that is to rate the difference between the theoretical distribution function $\hat{F}$ of the flux and flow empiric distribution function $\tilde{F}$ to be the lowest.

The function $d(\alpha,\beta)$ is optimized by using the combined global search for the conservation of information.

Prediction of beta distribution parameters

Using the algorithm described above in paragraphs 1-7, the parameters α and β are calculated for all days (for which we have data on the ticket) for all flows in the forward and reverse direction. After that you can predict the parameters of beta distribution for the next period of time.

To align our series we apply a polynomial of the third degree [Kolyakina A., Pozhidaev V. 2009]:

$$\hat{y} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ . \tag{12}$$

The parameter estimates will be obtained using the method of least squares. As a result, for each day of the next year we will get the theoretical values α and β for all flows in the forward and reverse direction. Using these α and β, for each day separately constructed the demand curve for passenger car traffic is constructed and the timetable is made up.

The construction of passenger demand curve

In the forward and backward the way to build passenger demand curves is the same, so we will consider a method for constructing the curve in the forward direction.

According to the formula of density β - distribution:

$$f(x) = \frac{1}{\mathrm{B}(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \tag{13}$$

where: $0 \le x \le 1$, $\alpha > 0$, $\beta > 0$,

using the parameters obtained after the prediction of beta distribution for each flow - α_i, β_i (i – number of flow) the demand curve is constructed. An example of the demand curve is shown in Figure 1.

Upon receipt of the demand curves (the distribution parameters of each stream), passenger traffic in the forward and reverse directions, the timetable of passenger vehicles is made up.

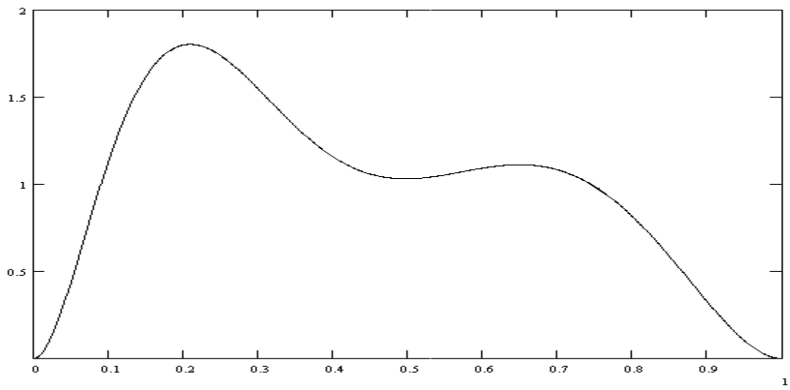

Fig. 1. The demand curve of passenger traffic on the route
"Lugansk - Sverdlovsk" forward

Scheduling of traffic

The algorithm to schedule the motion is identical for all days, so further d an algorithm to schedule the movement of vehicles for one day is described. This algorithm is implemented by the programmed way.

The passenger demand curve in the forward direction and OX the axis form a figure, let's note it as F. The passenger demand curve in the opposite direction and the axis OX form a figure, let's note it as F'.

The algorithm to schedule the motion:

1. In the forward direction:
   - the first bus always leaves at $x_1 = 0$, the bus number will be av_1, the bus, which travels to x_1, is chosen randomly from those buses that can travel at this time from X^{st};
   - in order to determine the departure time of the next bus - x_2, you need to find a value of x_2' that area of the figure F, bounded by straight lines $x = x_1$ and $x = x_2'$, multiplied by the number of passengers in the stream, in which received figure is equal to the capacity of the bus av_1. The following cases are possible:
     - if the area of the figure F, bounded by straight lines $x = x_1$ and $x = x_2^{n_{fwd}}$, multiplied by the corresponding number of passengers is less than or equal to the capacity of the bus av_1, it means that the search for the schedule in the forward direction is over, and the difference between the capacity av_1 and the area – these are the losses of underutilization of the bus;

- if you find the desired value x'_2, but there is no bus that can travel at this time, we are looking for $x''_2 > x'_2$, such that for any $x \in [x'_2, x''_2)$ there is no bus, and to have at least one bus that would go. Then $x_2 = x''_2$ and av_2 is randomly selected of those buses that can go to x_2;
- the area of the figure F, bounded by straight lines $x = x'_2$ and $x = x''_2$, multiplied by the appropriate number of passengers –these are the losses (the number of people who at that time were not provided with the transportation services);
- if you find the desired value of x'_2 and there are buses that can travel at this time, then $x_2 = x'_2$ and av_2 is chosen randomly.

- the search of the subsequent departure time of buses and determination of their numbers is similar to the previous search - x_d, av_d, is calculated using x_{d-1}, av_{d-1}.

2. In the opposite direction: the search for timetables of buses in the opposite direction is similar to scheduling in the forward direction in the opposite direction up the schedule using the figure F'.
3. Once time and number of the bus in either direction are found, one should randomly determine to continue the search schedule in a given direction, or go to the search schedule in the other direction (this is necessary for the rotation search).

When scheduling the random selection is used, therefore, when you use an algorithm with the same demand curves different schedules are obtained. After selecting the optimal schedule, the time of departure of transport is back transferred from $x \in [0,1]$ into real time.

CONCLUSIONS

The application of this mathematical model allows minimizing costs in the organization of passenger traffic. With the accumulation of statistical information, the predicted passenger demand curves can be changed that allows to take trends in demand for road transport into account , when scheduling the movement of vehicles.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ МИНИМИЗАЦИИ ПРОИЗВОДСТВЕННЫХ ЗАТРАТ ДЛЯ МЕЖДУГОРОДНИХ ПАССАЖИРСКИХ ПЕРЕВОЗОК

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Аннотация. Приведена математическая модель составления оптимальных графиков движения пассажирского автотранспорта в междугороднем сообщении. Модель построена на анализе законов распределения спроса, с учетом возможности наличия неоднородных пассажиропотоков.

Ключевые слова: пассажирские перевозки, пассажиропотоки, закон распределения, прогнозирование, расписание движения.