

MODELLING THE EFFECTS OF REPEATED WHEEL LOADS ON SOIL PROFILES

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A b s t r a c t. Compaction of soil caused by increasing mechanical loads and repeated wheeling may result in reduced soil productivity. The physical response of soils to such loading is analysed with a non-linear finite element program using incremental tangential moduli with incremental loading and unloading from known initial conditions. During each load increment an iterative procedure is used to determine more accurately the stresses and the stress dependent moduli. This program is designed to model the linear elastic and plastic behaviour of soils with stress and moisture dependent properties with strain softening and irreversible load responses. The relevant material parameters are discussed and suggested methods of measuring them in standard soil mechanics test procedures are proposed. The use of such models in predicting soil behaviour must always involve a balanced integration of the measurement of the material parameters and boundary conditions for the problem, the use of appropriate analytical techniques and field verification. Practical examples are given to show how this has been done in interpreting field experiments and to confirm that the techniques can predict the physical response of soil profiles to repeated wheeling.

K e y w o r d s: compaction, wheel loads, soil strength, finite element modelling

INTRODUCTION

The physical effects of repeated wheel loads on soil profiles are an important problem in agriculture. The resulting compaction and aggregate degradation causes problems, not only with water infiltration, aeration and root penetration, but also results in the overall economic problem of a decline in crop production. Many experimental programs involving

field sampling and density or porosity measurements have defined the problem in terms of the intermediate and final response of agricultural soils to wheel loads [28]. Several research workers have measured the stress distribution under wheel loads in field trials [2,3,10-12,15]. These field measurements of bulk density and stress distribution provide an accurate quantitative picture of the physical processes and responses at a given site and at given times [8,9]. However, the results of physical measurements obtained from field experiments cannot be extrapolated readily to other wheel loads and configurations, other initial soil structural conditions and, particularly, to other soil moisture conditions, even on the same soil profile. Extrapolation to other soil profiles in different climatic and environmental conditions is even more difficult. While this is not impossible, the amount of field testing and measurements required would be enormous. An important example of this is the large variation in soil water content and soil water or matric potential profiles that can occur in the field. For example, a wet surface soil over dry subsoils may result in serious degradation whereas the reverse may not cause any problems.

Predictive or physical modelling procedures, when used in conjunction with field and laboratory measurements, can be of great

benefit, not only in extrapolating field data to other sites, environmental conditions, treatments and wheel loads, but also in understanding more fully the physical processes involved. They can also be used in planning field trials, assessing different strategies for treating problems and determining the effects of loading the soil at different times of the year. Such procedures have been used in both agricultural and geotechnical engineering applications. Those used, particularly in agricultural applications, have tended to use simple one dimensional models [1,27]. Even where two dimensional models have been used, including those using three-dimensional radial symmetry, the soil behaviour has generally been assumed to be linear elastic, non-hysteretic and in a few cases perfect rigid plasticity has been used [7]. These techniques may work to some extent with dense dry or saturated soils. However, with loose unsaturated soils, which is the condition desired in agricultural applications, they are difficult to apply successfully.

This paper describes modelling techniques which have been successfully used in predicting the physical response of soils in a wide range of geotechnical engineering applications in both saturated and unsaturated soils. These techniques utilize soil properties which are non-linear and hysteretic functions of soil water suction, soil stresses, pre-consolidation stresses and bulk density. As a result they can model problems where soil water suction not only varies in space, but also when coupled with a water flow analysis [25], where soil water suction varies with time. They can also utilize several failure criteria more appropriate to unsaturated soils and the combined effects of soil swelling in clay soils. While the work described in this paper has been restricted to two-dimensional plane strain or axisymmetric problems, the techniques are extended to true three dimensional problems, such as multi-wheel and axle configurations.

This paper also demonstrates how using simple laboratory test data, good agreement

can be obtained between computer predictions and field measurements.

MATHEMATICAL MODEL

The mathematical model used is a simple linear finite element program, which can solve the basic load-deformation differential equation. This equation can be written as:

$$\{F\}_e = [K]_e \{\delta\}_e \quad (1)$$

where $\{F\}_e$ is the force vector at the nodes of the finite elements defining the region, $[K]_e$ is the stiffness matrix of the assembled finite elements for the region analysed, $\{\delta\}_e$ is the displacement vector at the nodes of the finite elements defining the region.

This mathematical model was developed over 30 years ago and is described in detail in, e.g., Zienkiewicz [29]. While 6-node triangular elements were available, the simplest elements, namely 3-node linear triangular elements have been used in this paper. These elements have the property of linearly varying forces and displacements over the triangle and have constant values of the stresses, strains and stiffness parameters over the area of each triangle. While not suited for regions of high stress/strain concentrations, 3-node elements permit a much larger number of nodes and elements to represent a given region than 6-node triangles for a given computer capacity. This is important in non-homogeneous and highly non-linear soils. They also give better numerical stability after failure when used in the computer program worked with in this paper.

As already mentioned, the finite element method used and described by Eq. (1) is a linear analysis. However, by using a multiple pass iterative-incremental method [24], it is possible to extend the single pass linear method into an effective non-linear hysteretic analysis of transient loads on soil profiles.

In this non-linear method, the analysis commences with known initial stresses, strains and soil water suctions. Increments of load and/or displacements are then applied. These loads and/or displacements represent small increments

along the total load/deformation paths being followed in the analysis. The stiffness matrices $[K]_e$ for each element are calculated using the initial stresses, strains and soil water suctions for that increment. The results of the solution of Eq. (1) then give upgraded values of stress, strain and soil water suction, which can be used to give new improved estimates of element stiffnesses $[K]_e$. The analyses are repeated iteratively, (i.e., with the same load and displacement increments) until the calculated stresses and strains do not vary more than a certain error margin. Then a new set of load and displacement increments are applied and the iterative procedure is repeated, followed by further increments until the whole load and deformation path is completed.

However, the most important part of the modelling procedures is the mathematical description of the material properties, called here the material model. This has already been described by Richards [24], but a detailed description will be given here for completeness.

MATERIAL MODEL

Basic continuum model

The hyperbolic model of Nelson [18] was modified by Richards [22] and used to describe the continuum representing the non-linear elastic behaviour of the soil up to yield. This model can be summarised by the simplified relationships:

$$K = k_1 \sigma_m^n + k_2 h^p + k_o \quad (2)$$

and

$$G = (g_1 \sigma_o^m + g_2 h^q) F_s + g_o \quad (3)$$

where $F_s = 1 - (\tau/\tau_f)^r$ for loading and = 1.0 for unloading, K is the bulk modulus, G is the shear modulus, σ_o is the previous maximum value of octahedral or mean stress, $1/3(\sigma_1 + \sigma_2 + \sigma_3)$ and is equivalent to the pre-consolidation pressure, which is input as data and updated during the analysis, h is the soil water suction, τ is the shear stress, τ_f is the yield

stress - according to the yield criteria used (usually a function of stress, σ , cohesion, C and friction angle, ϕ), and $k_1, k_2, k_o, g_1, g_2, g_o, n, m, p, q$ and r are material constants.

These equations can readily be programmed into the finite element program code where K and G are calculated for continuum elements. The equations for K and G can be expressed as functions of the components of soil water suction, namely the matrix and solute components [4,23]. The equations are also equivalent to the effective stress equations if k_2, g_2 and m are identical to k_1, g_1 and n . Some typical results of analyses using these more complete equations have already been described [21-23]. However, the simplified Eqs (2) and (3) were used in the work described below, as the suction components were not measured in the experimental program.

When a continuum element behaving elastically starts to fail, it no longer acts as a continuum but as a discontinuity or a series of discontinuities. Richards [24] modified a fixed joint element used in rock mechanics [5] to give a variable angle discontinuous element with non-linear hyperbolic shear properties and shear stress release and redistribution to simulate strain softening [16,30]. An important aspect of this element was that it was fully compatible with the continuum elements and therefore could replace them at the onset of failure and vice versa when failure ceases during unloading. The finite element formulation for the joint elements is:

$$\{F\}_e = [K_j]_e \{\delta\}_e \quad (4)$$

where $\{F\}_e$ is the nodal force vector, $[K_j]_e$ is the stiffness matrix for joint element, [24], $\{\delta\}_e$ is the nodal displacement vector.

The main difficulty in changing from a continuum element to the joint element is the determination of the joint angle, α . This is the angle between the coordinate system for the joint (i.e., normal and parallel to the joint or the n-s system) and the normal x-y system for the whole region. Using the data of Morgenstern and Tchalenko [17] and Skempton [26], the best estimate for the angle α was the

maximum or principal stress direction for associated flow material and the maximum or principal strain direction for non-associated flow material with no dilatancy.

The stiffness matrix for the joint element in the general x-y coordinate system is:

$$[K]_{xy} = [T]^T [K]_{ns} [T] \quad (5)$$

where $[T]$ is the transformation matrix containing the direction cosines of the joint angle, α .

Strain softening

Strain softening is defined as the process where soils and particularly clays undergo a reduction of shear strength and modulus during shearing. Realignment of soil particles with a reduction of soil water suction have been suggested as reasons for this [17]. In this paper, it is assumed that strain softening only occurs in the joint elements. At yield, the actual shear stresses are equal to or exceed the yield stresses for peak strength and the shear stiffness from Eq. (4) has been reduced to zero values. The yield stresses for residual strength are then calculated and the difference is redistributed until the shear stresses in the failed elements are at the residual values.

The excess shear stress along the joint at angle, α , is given by:

$$\Delta\tau = \tau - \tau_R \quad (6)$$

where τ is the actual shear stress along the joint and τ_R is the residual shear strength.

Using the „initial stress“ method [30], the excess stresses are redistributed by generating a new set of nodal forces:

$$\{F\}_e = [B]_e^T \{-\Delta\tau\} \text{vol} \quad (7)$$

where $[B]_e$ is the strain matrix, $\{-\Delta\tau\}$ is the negative value of the excess stress and vol is the volume of the element.

As the shear modulus of the failed elements is near zero, the iterative method used by Lo and Lee [16] was not required for the reduction of the stresses in the failed elements, but was required to ensure the redistribution of

these stresses to the adjacent elements not yet failed.

Tensile failure

Tensile failure is defined here as the condition when the maximum tensile stress exceeds the tensile strength of the material and failure occurs in the direction of the maximum tensile stress. The model used for the tensile failure of any element is simply to reduce any tensile stresses exceeding the tensile strength of the material to zero and redistribute them using the „initial stress“ method discussed in the previous section.

APPLICATION OF MODELLING PROCEDURES

Details of field experiments

The field trials and experimental work previously undertaken over a three-year period at 37 typical Bavarian soils gave excellent data for testing the modelling procedures [14]. This investigation was carried out to determine and predict the mechanical compressibility and trafficability of differently textured soils. The soil physical properties, measured before and after loading are also given in Lebert [14].

The soil results described here were from an agricultural soil, characterised by dense, compacted layers, especially in the subsoils, after wheeling and tillage. In the plow-pan layers, very low values for the air capacity ($<0.05 \text{ m}^3 \text{ m}^{-3}$) and air permeability ($<10 \mu\text{m}^2$) as well as high values for the penetration resistance ($>2 \text{ MPa}$) were found. The details and descriptions of the soil profile used in the work described in this paper are given in Table 1.

Soil mechanical tests

As shown in earlier work with these modelling procedures [19], the best choice of material parameters for the material model is obtained by fitting the results of the analyses of tests themselves to the test data. Conventional soil mechanics procedures do not always give good material parameters. The analyses of the test include not only the soil itself, but also the

Table 1. General description of the gleyic cambic Colluvisol derived from Loess (after Lebert, [14])

Horizon	Depth (cm)	Structure	Texture	Bulk/particle density (gcm^{-3})	Air capacity (vol. %)	Available water (vol. %)	k_f (cm s^{-1})	P_v (kPa)		Angle of internal friction ($^\circ$)		Cohesion (kPa)	
								a	b	a	b	a	b
Ap	0-25	crumbly	clayey silt	1.54	2.4	19.1	$7.8 \cdot 10^{-4}$	105	110	43.2	37.8	13.3	31.3
BvAh	25-40	subangular blocky	silty clay	1.51	2.3	17.6	$6.6 \cdot 10^{-6}$	115	105	42.4	36.7	20.4	40.4
SAh	40-50	blocky	silty loam	1.53	3.4	12.6	$7.9 \cdot 10^{-4}$	140	130	35.7	41.8	27.7	38.1
SBv	>50	blocky	sandy loam silt	1.62	9.9	13.9	$1.3 \cdot 10^{-3}$	60	60	35.2	31.8	18.6	45.1

a = -6 kPa pore water pressure; b = -30 kPa pore water pressure; k_f - mean saturated hydraulic conductivity, P_v - pre-compression stress.

important elements of the apparatus, such as loading patterns, sample holder and membranes. All soil boundaries are modelled by interfacial or joint elements discussed above. As a result of using this technique, experimental factors such as end restraint, side friction and restraint and uneven stress distribution can be taken into account. As it is not possible to calculate the material parameters directly, the simple conventional soil mechanics parameters are usually taken as estimates of the initial values. The parameters can then be improved by a trial and error procedure, with good agreement usually being achieved within three to four trials. A computer program using a multi-point curve fitting procedure is now being developed to carry this out automatically and statistically.

As shown by Richards [23,24], no conventional soil mechanics test can provide all the material parameters uniquely. However, the use of two quite different tests can give very useful parameters. The use of triaxial, plane strain, unconfined or confined compression tests as one test and the direct or simple shear as the other test have been found to be adequate. As an example of how the material parameters were obtained, the data from both confined compression and direct shear tests for the four soil horizons and the plow-pan is used in the following section.

Material parameters

As the first step, simple calculations of the initial value of K , (K_0) versus σ_m , assuming $\sigma_2 = \sigma_3 = 0.5 \sigma_1$ were made from the confined compression tests and the initial value of G (G_0) versus σ_m from the direct shear tests. Plotting K_0 and G_0 versus σ_m on a log-log plot produces near linear plots [22], from which straight line fitting gives estimates of k_1 , g_1 , n and m in Eqs (2) and (3). First estimates of the cohesion, C , and angle of friction, ϕ , were made from the direct shear box tests using the simple Mohr-Coulomb theory. Finite element analyses were then carried out on both the confined compression and direct shear tests using the first estimates for the material

parameters representing each soil horizon and a range of stresses. These analyses included the whole test apparatus including the metal end caps and shear box with shear elements between soil and metal and the rubber membranes [24].

Invariably these first estimates are not satisfactory due to the over-simplified conventional procedures used to obtain the parameters. Trial and error procedures using experience with the material model were used to improve the fitting of the predicted results with the experimental data. Examples of typical results of the fitting process are shown in Fig. 1 for the confined compression test and in

Fig. 2 for the direct shear test. The final accepted fit for the shear test on soil from horizon 2 is shown in Fig. 3. The final material parameters for all four soil horizons plus the plow-pan used in the field analyses of the gley cambic Colluvisol at Erding (profile 14), were determined from the values given in Table 2.

The horizon 2a was added during the analyses of profile 14, as it eliminated stress concentrations at the bottom of horizon 2 and gave better agreement with field data. This is a technique used in road pavement engineering to allow for tensile failure in stiff layers overlying weaker layers.

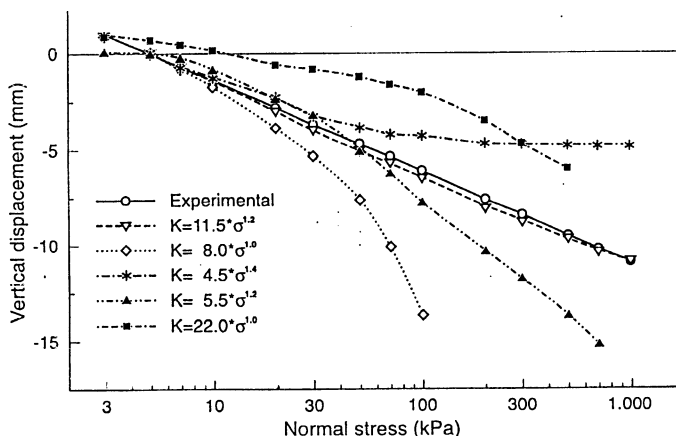


Fig. 1. Vertical displacement versus normal stress from a confined compression test: a comparison of experimental results and predictions using different parameters for bulk modulus (K).

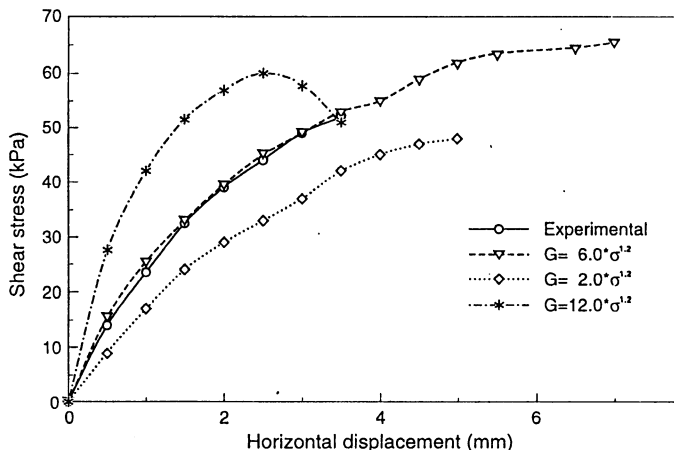


Fig. 2. Shear stress versus horizontal displacement from a direct shear test: a comparison of experimental results and predictions using different parameters for shear modulus (G).

Analyses of wheel loads on Profile 14

The geometry of the wheel load and the region representing the soil profile at profile 14 is shown in Fig. 4. The dimensions of the region were chosen from previous experience with similar problems [20,22]. The wheel load and the region were defined as a circular axisymmetric problem, which is a good approximation for a single wheel on a semi-infinite layered soil profile. Initially a uniform wheel load of 120 kPa tire pressure over a radius of 12 cm was used in the preliminary analyses. Using the material model parameters for the soil profile given above, this particular prob-

lem gave the vertical stress distribution under the full wheel load as shown in Fig. 5. The experimental data shown were measured by stress transducers installed in profile 14 under the wheel loading modelled.

As pointed out above, the large difference in stiffness between horizon 2 and the plow-pan cause some stress oscillation. Previous experience with road pavements has shown that stiff layers tend to increase the non-linear stiffness of a layer above, which exhibits frictional properties. The reverse also tends to be true. A new layer with higher stiffness properties was therefore inserted immediately above the plow-pan and this gave smoother results.

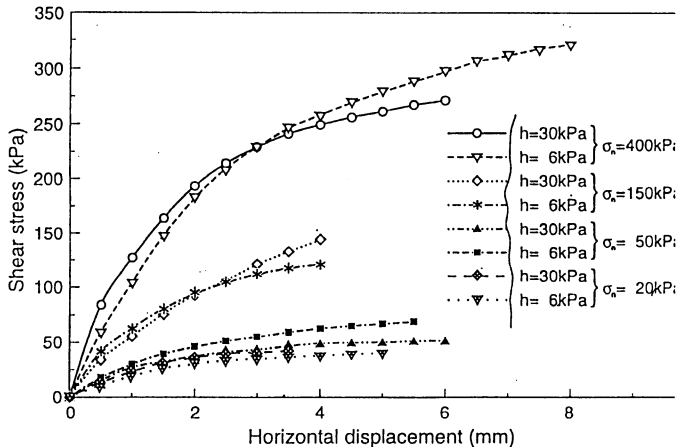


Fig. 3. Examples of direct shear test results at different soil water suctions, (h) and vertical stresses, (σ_v) for horizon 2.

Table 2. Material model parameters for the study site

	k1	ko	g1	go	n	m	p	c	ϕ
Horizon 1 (0-10 cm)	0.3	50.0	0.2	30.0	2.00	2.00	1.00	10.00	37.0
Horizon 2 (10-20 cm)	11.5	50.0	6.0	30.0	1.20	1.20	1.50	31.3	37.8
Horizon 2a (20-25 cm)	11.5	50.0	6.0	30.0	1.20	1.20	1.50	40.4	36.7
Plow-pan (25-30 cm)	16.0	50.0	10.0	30.0	1.20	1.20	1.50	50.1	41.8
Horizon 3 (30-40 cm)	16.0	50.0	10.0	30.0	1.20	1.20	1.50	38.1	41.8
Horizon 4 (40-100 cm)	7.2	50.0	3.5	30.0	1.20	1.20	1.50	45.1	31.8

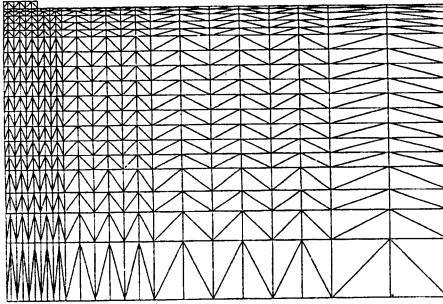


Fig. 4. Geometry of the soil profile region and loading system used in the analyses (initial mesh only).

The distribution of the contact tire pressure was measured at the test site and shown to be non-uniform, and it depends on the position of the tire tread or lugs [13]. This also could account for the inconsistencies in some stress measurements due to the position of the tire lugs. Different distributions of the contact tire pressure, which give the same total wheel load and are consistent with the contact pressure measurements were analysed. These gave increased soil stresses at the surface and slightly less at depth, giving better agreement with the field measurements.

The wheel displacement versus loading for loading, unloading and reloading paths is shown in Fig. 6. This gives an irrecoverable displacement (i.e., rut depth) after one pass of 31 mm, and after two passes, 40 mm, compared with a measured depth of 55 mm. The surface displacement versus wheel load for a treaded tire is also shown in Fig. 6 and the rut depth after one pass, namely 54 mm, was in the range of those measured in the field.

As examples of the graphical outputs from the computer program, Fig. 7 shows the displacement trajectories at peak load conditions, Fig. 8 the vertical and radial (horizontal) stresses and Fig. 9 the shear stresses, strains and failure zones under the first peak wheel load.

Finally it is interesting to note that this analysis and particularly the use of Eqs (2) and (3) include the effects of the pre-consolidation pressure which has been shown to be a significant factor [14]. Figure 6 has already shown the effects of previous loading, with

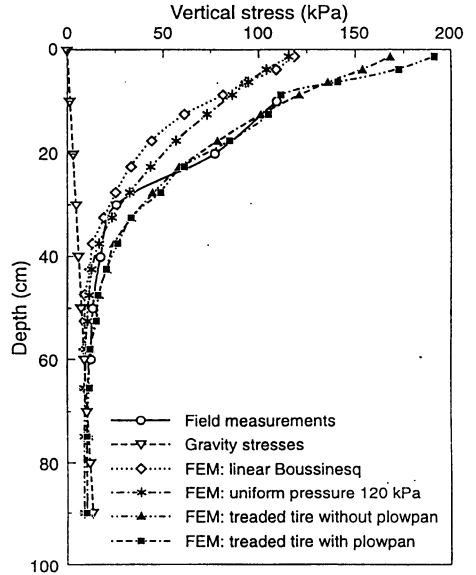


Fig. 5. The vertical stress distribution with depth under the wheel load using the average contact pressure and total wheel radius (uniform pressure) as well as the effect of the position of tyre tread with and without ploughpan. The stress distribution according to Boussinesque and due to gravity is also shown.

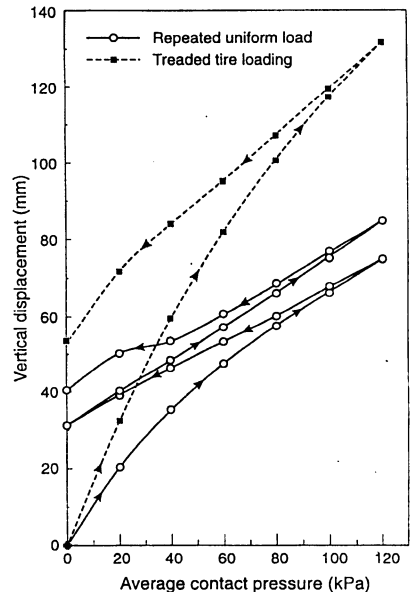


Fig. 6. The vertical displacement of the surface under the uniform wheel load as a function of the wheel load over two loading cycles. The effect of treaded tire loading on vertical displacement is also shown for several loading cycles.

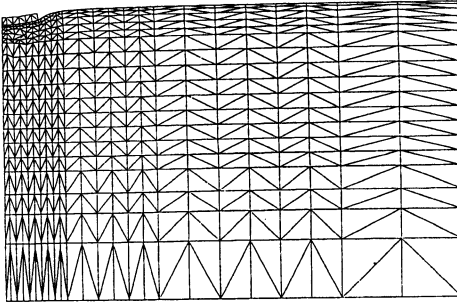


Fig. 7. The loaded displacement trajectories predicted under a treaded wheel load (distorted mesh only).

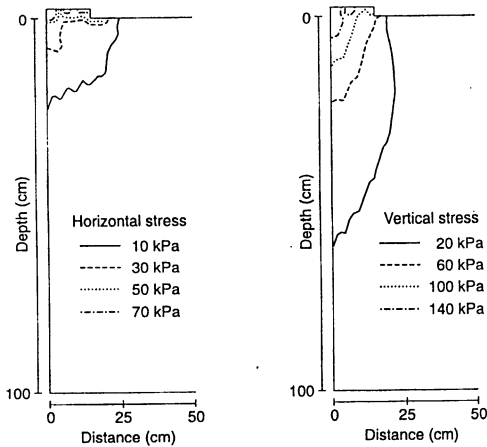


Fig. 8. The vertical (right) and horizontal (left) stresses predicted under a treaded wheel load.

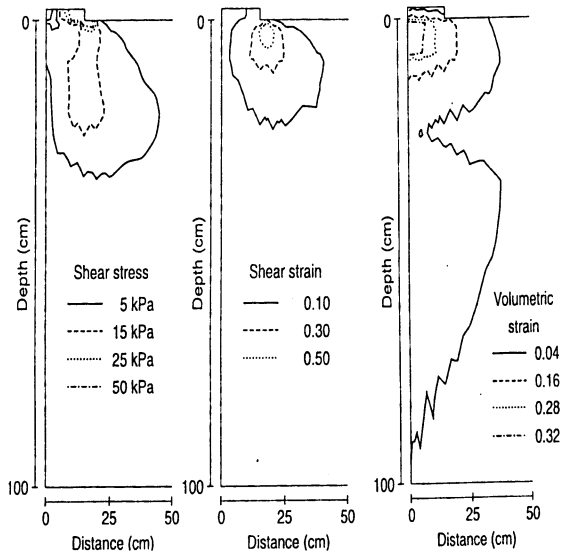


Fig. 9. The shear stresses, strains and failure zones predicted under a treaded wheel load.

subsequent loadings causing less incremental soil deformation due to the increased soil stiffnesses. Figure 10 shows the predicted results for a confined compression test compared with the experimental data and shows the hysteretic effect of the continually updated pre-consolidation pressure during unloading and reloading cycles.

CONCLUSIONS

A complete modelling procedure for the prediction of stresses, strains and displacements in agricultural soil profiles under the loading, unloading and reloading of wheel loads has been described. The deformation (elastic) and failure (plastic) material parameters are expressed as non-linear hysteretic functions of the stresses, soil water suction and the initial pre-consolidation pressure. Generally the simultaneous use of two different standard soil mechanics tests, such as a compression test and a shear test have been found suitable for the determination of these parameters. Any layered soil profile, which can be described and whose material parameters measured can be modelled in this way and its response to various wheel loadings of different radius, contact pressure distributions and total wheel loads can be analysed.

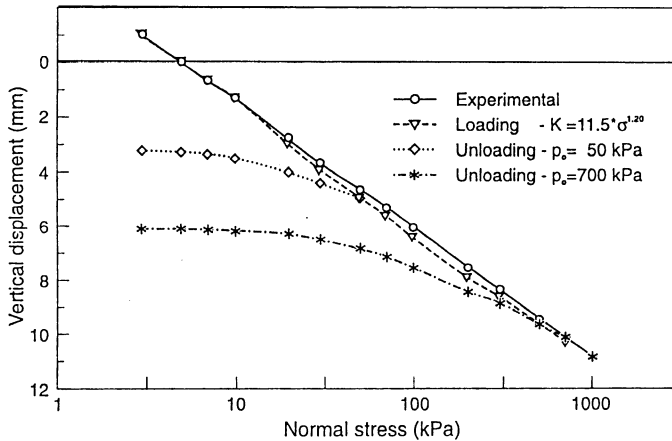


Fig. 10. The effects of pre-consolidation pressure on the prediction and measurement of the response of confined compression tests on horizon 2.

Experience with these modelling procedures using data from practical field experiments has been most encouraging. A practical case study of one field trial has been included in this paper to demonstrate the capabilities of the procedures.

With experience, it provides a quick and accurate method of predicting the response of a soil profile to various types of wheel loads at different times of the year. It is also a valuable tool for assessing the various practical treatments or remedies that are possible to prevent or remedy problems with soil compaction and structural degradation.

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