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Scale cutoffs and the limits of fractal soil structure

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A b s t r a c t. Fractal models of soil structure have not been explored in full. In particular, very little is known on the actual limits of prefractal behaviour of soil system. The objectives of the present work were (i) to estimate fractal scale limits from experimental bulk density – and porosity – aggregate size data sets and (ii) to search for links between fractal scaling limits and soil properties. A total of eighteen published data sets were used. They spanned a wide range of soil types and/or treatments. All the considered data sets covered more than two orders of magnitude in the independent variable (n>2), with a scaling factor ranging between b=1.66 and b=6.41. Both the maximum number of model iterations, i_{max}, and n correlated positively with soil organic carbon. Knowing the limits of fractal behaviour, it is possible to make reliable predictions of soil physical properties at the scale of the representative elementary volume (REV).

K e y w o r d s: fractals, soil structure, bulk density, porosity, soil organic matter

INTRODUCTION

The study of mechanical and hydraulic soil properties using fractal dimensions has revealed interesting details of soil behaviour (Bird and Perrier, 2003; Bird *et al*., 2000; Eghball *et al*., 1993; Filgueira *et al*., 1999; Rieu and Sposito, 1991a,b). Fractal structures appear after infinite iterations over an initiator (Mandelbrot, 1983) while real physical systems show fractal structure within upper and lower limits. These points have been addressed in seminal papers (Avnir *et al*., 1998; Malcai *et al*., 1997; Thompson *et al*., 1987; Turcotte, 1989). However, estimations of lower/upper scale cutoffs or maximum number of iterations from experimental data are still scarce. Recently, Millán *et al*. (2006) used soil water retention data for estimating the lower scale cutoff of pore radii. The objectives of the present work were (i) to estimate fractal scale limits from experimental bulk density and porosity aggregate size data sets and (ii) to search for links between fractal scaling limits and soil properties.

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MATERIALS

The starting point is the bulk density/size relation (Rieu and Sposito, 1991a) :

$$\frac{\rho_i}{\rho_0} = \left(\frac{x_i}{x_0}\right)^{D_m - 3}, \tag{1}$$

where: ρ_i is the bulk density of the aggregate at the i-th level, ρ_0 is the bulk density of the largest aggregate, x_i is the aggregate size at the i-th level, x_0 is the length of the largest aggregate and D_m is the mass fractal dimension of the system. Eq. (1) has its complement in terms of aggregate porosity:

$$\varphi_i = 1 - \left(\frac{x_{\min}}{x_i}\right)^{3-D_m}, \tag{2}$$

where φ_i is the partial porosity corresponding to the aggregate of size x_i.

Most studies only consider Eq. (1), while D_m has been the main objective of those investigations. In addition to D_m, x_0 and $x_{\min}$ are important parameters imposing the conditions $\rho_i \neq 0$ and $\varphi_i < 1$ when $i \to i_{\max}$ as expected for real prefractal structures. Note that many studies consider x_0 as the aggregate over the upper sieve, while $x_{\min}$ is the smaller aggregate over the lowest sieve within a nest. For example, in a macrostructural study from aggregate size distributions between 0.25 and 10 mm, it is usually considered that x_0 =10 mm and $x_{\min}$ = 0.25 mm. A better choice could be to use Eqs (1) and (2) for estimating x_0 and $x_{\min}$. Other questions are the number of orders of magnitude spanned by the aggregate size, n, and the maximum number of iterations for generating the aggregate size distributions. It is accepted that fractality requires at least n=2 while for experimental fractal analysis $i_{\max} << \infty$. That is, the relationship between x_0 and $x_{\min}$ is:

$$x_0 = x_{\min} (10^n) \quad (3)$$

$$n = \log\left(\frac{x_0}{x_{\min}}\right). \quad (3')$$

From Eq. (3') one can compute $i_{\max}$ using a well known relationship:

$$\frac{x_0}{x_{\min}} = b^{i_{\max}}, \quad (4)$$

$$i_{\max} = \frac{n}{\log b}. \quad (4')$$

In both cases $b > 1$ is a scaling factor.

Data sets and methods

In the present study a total of 18 published data sets were used, from Filgueira *et al*. (1999) (T1 to T7 treatments), Eghball *et al.* (1993) (six data sets), Chepil (1950) (three data sets), Wittmus and Mazurak (1958) (one data set), and Millán and Orellana (2001) (one data set). All of these data sets reported ρ_i values as a function of x_i. Partial porosities, φ_i, were computed using the equation:

$$\varphi_i = 1 - \frac{\rho_i}{\rho_p}, \quad (5)$$

assuming $\rho_p = 2.65$ Mg m^{-3} for elementary particle density.

Equations (1) and (2) were fitted to experimental data using nonlinear regression for estimating D_m, x_0 and $x_{\min}$. A quasi-Newtonian method was used with a convergence criterion of 10^{-4}. Eq. (3') was applied for calculating the number of orders of magnitude spanned by the prefractal system. After estimating x_0 from experimental data, the ratio $b=x_0/x_1$ was used for calculating b, where x_1 is the aggregate size corresponding to the first fragmentation step. After that, Eq. (4') was used for estimating $i_{\max}$. Correlations were also searched between any estimated parameter and reported soil properties. Both linear and nonlinear analyses were performed using the STATISTICA^{TM} Software Package (Stat. Soft. Inc., 1998).

RESULTS

As expected, Eqs (1) and (2) yielded the same values of mass fractal dimensions. Table 1 summarizes x_0, $x_{\min}$, $i_{\max}$ and b values computed from experimental bulk density/size and partial porosity/size data sets. The upper cutoff ranged between 2.654 and 21.10 mm, the lower scale cutoff varied within the range from 1.09 10^{-4} mm (0.109 µm) to 0.182 mm (182 µm), while $2.06 \le n \le 5.20$ and b values covered the range from 1.66 to 6.41. In particular, b values were within the same range as reported by Perfect *et al*. (2002). A positive linear relationship was found between $i_{\max}$ and reported percentage of organic carbon for each soil or soil treatment as shown in Fig. 1:

$$i_{\max} = 0.558\ (2.691) + 4.772\ (1.338)\ OC,$$

$$R = 0.758\ (p < 0.10,\ N = 12). \quad (6)$$

T a b l e 1. Parameters associated with the fractal limits of the studied soils

Soil/soil treatment	x_0 (mm)	$x_{\min}$ (mm)	$i_{\max}$	n	b
T1	12.51	1.48 10^{-4}	15	4.93	2.085
T2	21.10	1.57 10^{-4}	9	5.13	3.510
T3	21.10	2.04 10^{-4}	9	5.00	3.510
T4	17.23	2.01 10^{-4}	11	4.93	2.835
T5	11.86	1.09 10^{-4}	17	5.03	1.980
T6	19.74	1.27 10^{-4}	10	5.20	3.290
T7	18.28	3.47 10^{-4}	10	4.72	3.050
Chisel	11.44	5.13 10^{-3}	8	3.35	2.610
Disk	11.69	4.88 10^{-3}	8	3.38	2.670
No-till	9.96	3.94 10^{-3}	10	3.40	2.271
Plow	15.30	2.55 10^{-3}	7	3.77	3.490
Fine sandy loam	4.00	2.90 10^{-2}	5	2.14	2.510
Silt loam	3.75	6.78 10^{-3}	7	2.74	2.350
Clay	10.23	1.85 10^{-4}	6	4.75	6.410
Sharpburg soil	2.65	4.36 10^{-3}	13	2.78	1.660
Vertisol	20.87	1.82 10^{-1}	4	2.06	3.480
C – S – C*	11.31	4.54 10^{-3}	8	3.39	2.580
S – C – S*	13.36	3.21 10^{-3}	7	3.62	3.050

*See Eghball *et al.* (1993) for details.

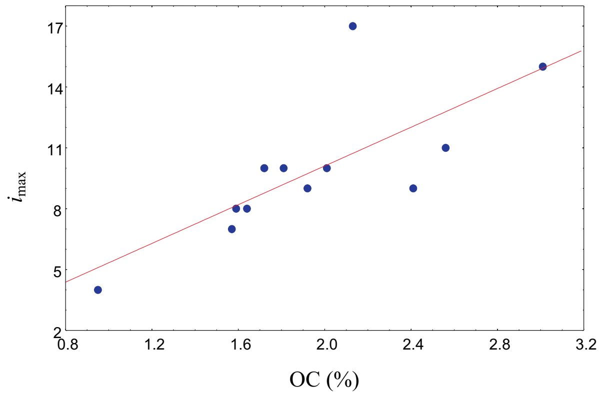

Fig. 1. Relationship between maximum number of iterations (i_{max}) and soil organic carbon (OC).

After estimating x_0, x_{min}, and D_m, Rieu and Sposito (1991a) equation was used for predicting the total porosity, φ, of the initiator by setting $x_{min} = d_m$ and $x_i = d_0$. This φ value ranged from 0.407 for the Vertisol in Millán and Orellana (2001) to 0.641 for the T4 treatment in Filgueira *et al*. (1999). They were within the range reported by the authors.

A breakpoint regression detected a split in the linear relationship between i_{max} and predicted φ values:

$$i_{max} = [-42\ (11.7) + 110\ (25.5)\ \varphi]\ (\varphi \leq 0.511) + 9.33\ (0.67)\ (\varphi > 0.511),$$

$$R = 0.761\ \ (p < 0.10,\ N = 18). \qquad (7)$$

Figure 2 presents the positive linear relationship between n and the reported soil organic carbon:

$$n = 1.347\ (0.733) + 1.489\ (0.364)\ OC,$$

$$R = 0.791\ (p = 0.0022,\ N = 12). \qquad (8)$$

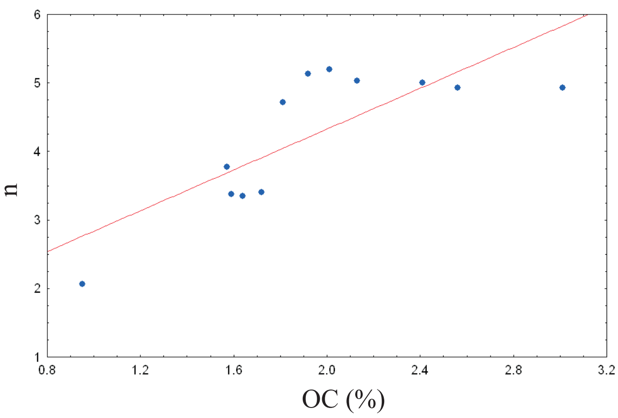

Fig. 2. Positive linear relation between the number of orders of magnitude (n) of the prefractal model and soil organic carbon (OC).

DISCUSSION

Both Eqs (1) and (2) might reveal information out of the scope of fractal dimensions. It is noted that quite different soils and/or treatments rendered a narrow range of mass fractal dimensions. For instance, D_m=2.88 for the fine sandy loam soil reported by Chepil (1950) (minimum value) while D_m= 2.95 for the clay soil reported by that author (maximum value), that is ΔD_m=0.07. Taking the same two soils as a basis for discussions, important differences were found between their fractal domains. Table 1 shows that fractal domain for the fine sandy loam soil was much smaller as compared to the clay soil (x_0=4 mm, x_{min}=0.029 mm, n = 2.14, b = 2.51 and x_0=10.23 mm, x_{min} = 1.85 10^{-4} mm, n = 4.75, b = 6.41, respectively). An open question is to search for experimental ways for estimating a true representative elementary volume (REV) for fractal analysis. A recent effort was based on the mean porosity of the soil sample (Bartoli *et al*., 2005). Equation (6) shows that increasing soil organic complex (organic carbon in this case) also increases the soil sample capacity for allowing a large number of iteration steps (i_{max}). Thus, organic matter is very important for soil fertility, but it also contributes to the generation of a more porous and stable soil structure. This means that a more porous structure contains a larger number of pore size classes which could explain the increase of i_{max}. For instance, i_{max}=15 and OC=3.01% for T1 treatment in Filgueira *et al*. (1999) while i_{max}=4 and OC=0.95 % for the Vertisol in Millán and Orellana (2001).The piecewise relationship between predicted soil sample porosity, φ, and i_{max} is very difficult for interpretation, as shown by Eq. (7). The first part of Eq.(7) complements Eq. (6). That is, i_{max} increased as the initiator porosity also increased for $\varphi \leq 0.511$. The second part of Eq. (7) indicates that $\varphi > 0.511$ does not influence the i_{max} value (i_{max}=constant). Obviously, a larger data set is needed for a sound discussion, but there is evidence that the traditional assumption that $i_{max} \to \infty$ does not hold for real prefractal structures. Equation (8) and Fig. 2 could mean that high quality soils *eg* rich in organic carbon content are more tractable within a fractal framework than a low quality one. Since organic matter generates porous, stable structures, it is possible to gain a wider range of scales of self-similar aggregates for fractal studies.

CONCLUSIONS

1. Fractal models of soil structure can be used with experimental data for estimating the limits of fractal scaling behaviour.

2. In addition to fractal dimensions or lacunarity values, other parameters such as b, i_{max}, x_0 or x_{min} could be dependent on soil physical and/or chemical properties.

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