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Hydraulically Effective Straight Drop Structure Calculation According to DIN 19661-2

Abstract

In order to stabilize the bottom of a channel, it is often necessary to reduce the slope, so that the critical tractive force is not exceeded. The excess absolute slope is then incorporated into river bottom protection structures, such as: drop structures, chutes, cascades and sills.

When a drop structure is used, it must be established that there is a double change in state of flow with a hydraulic standing or steady jump. The hydraulic effectiveness depends on the energy dissipation in the jump. For this reason, it must be ensured that a minimum drop H_w exists. The type of flow arising must be calculated hydraulically and the dimensions of the stretches to be protected must be determined.

In Germany, the Standards for these structures are listed in the Guide-lines DIN 19661-2 (1996): Hydraulic Structures – River Bottom Protection Structures, which covers both the hydraulic dimensions and the structural engineering design.

Key words: river bottom protection structures, straight drop, hydraulic effectiveness, types of flow

Introduction

For the hydraulic dimensions of a straight drop structure the following calculations are necessary.

1. The effective height of the straight drop H_w is determined.

2. The type of flow after the structure is determined.

Calculation of the depth $h_{iA} \gtrless h_{iD}$:

a) h_{iA} from the headwater,

b) h_{iD} from the tailwater.

3. The stretches to be protected and the end sill are determined.

Hydraulics at the straight drop

The hydraulics at a drop essentially depend on the type of flow upstream and downstream the structures. The aim should be to obtain a subcritical (streaming) flow ($F < 1$) in both areas; if there is a supercritical (shooting) flow ($F > 1$) in the headwater, there is no hydraulic effectiveness. If there is a streaming flow above the drop and a change of flow in the area of the structure, the critical depth h_{gr} ($F = 1$) is reached shortly before the crest of the drop. Behind the structure a second change in flow with a hydraulic jump and a surface roller must occur. The flow condition in the headwater is determined by

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the cross section at a distance of $3 \cdot h_{gr}$ up to $4 \cdot h_{gr}$ before the crest. In the tailwater, the cross section should be assumed in the undisturbed area behind the drop. Figure 1 shows a drop structure with a double change in the state of flow (local phenomenon). It is necessary to have a free stretch of water with a uniform flow between two straight drops, so that the formation of the surface roller is not influenced.

For a flow condition with the critical depth h_{gr} in the headwater, the flow types a) to d). Figure 2 shows the different surface profiles of a jump, are relevant, depending on the height H of the straight drop or on the depth of the streaming tailwater h_u. The hydraulic effectiveness depends on the energy dissipation in the jump. This requires for there to be a shooting of $F > 1.7$ at the foot of the surface roller and thus flows of $F > 0.5$ behind the

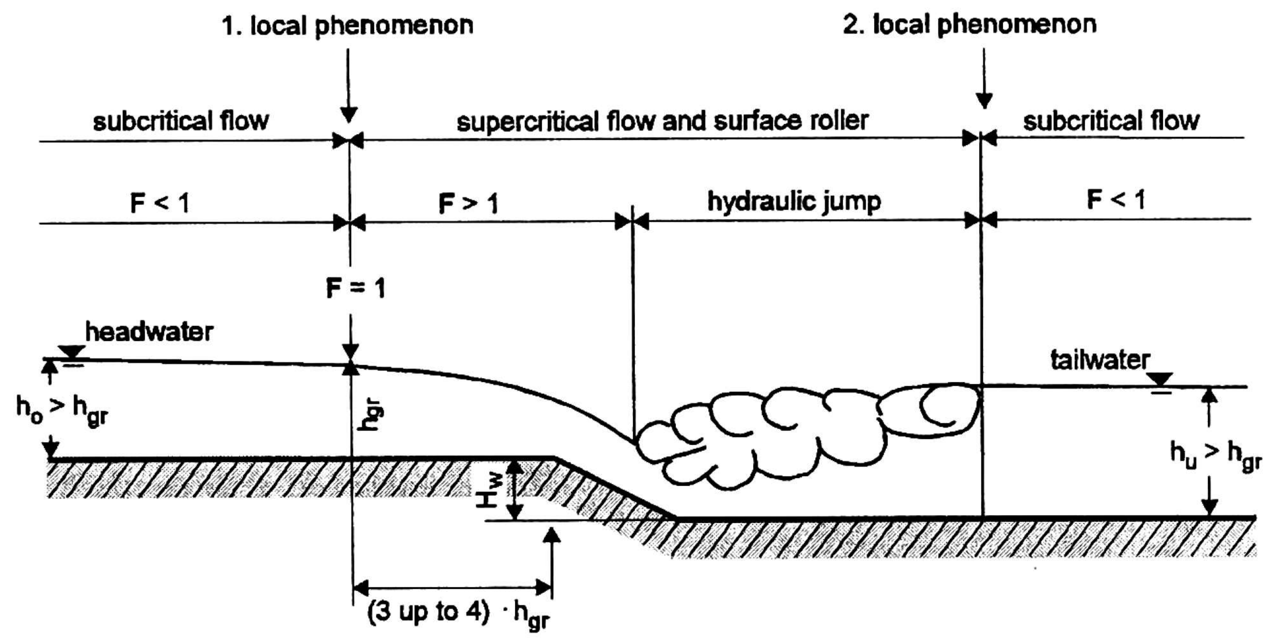

Fig. 1. Drop structure with a double local phenomenon

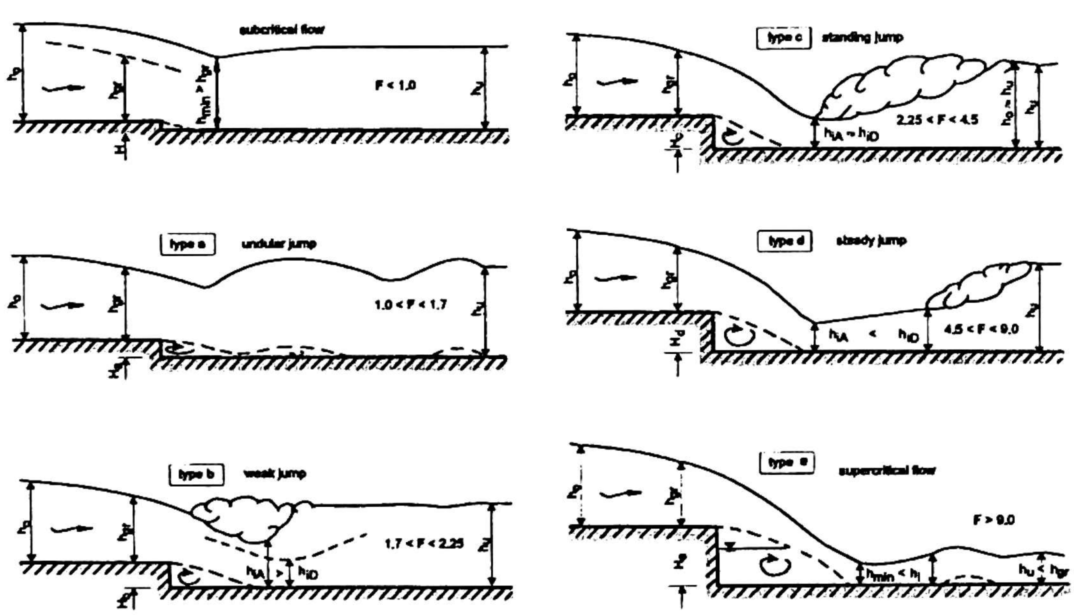

Fig. 2. Various types of flow at the straight drop

roller. A better formation of the surface roller arises according to Henderson (1966), when in the shooting area $F > 2.25$, which corresponds to $F < 0.32$ in the streaming area. The effect of the Froude number and the energy dissipation in the tailwater is shown in table.

Table. Determination of the type of flow at the straight drop

F_1	Surface roller	Type by DIN	Dissipation
1.0–1.7	none	a	none
1.7–2.25	weak	b	low
2.25–4.5	standing	c	good
4.5–9.0	steady	d	high
9.0	none	–	low

When $F > 0.5$ or when a leaving surface roller occurs, because the jump travels downstream the hydraulic effectiveness can be achieved by taking special measures such as, changing the discharge area, installing sills or stilling basins.

First of all, the type of flow, figure 3 shows the effect of tailwater depth on the formation of a hydraulic jump, must be determined by comparing the shooting water depths h_i, which arise from the conditions in the tailwater and in the headwater too. Instead of a comparison by methods of computation, the result can be taken from a nomogram (fig. 4).

Hydraulically Effective Straight Drop Structures

In the critical area, the acceleration is determined by the height of the straight drop. If it is small in relation to the water depth, it is not able to convert the subcritical flow at the drop into a supercritical flow. The streaming flow continues, at the most, a undular flow (type a) occurs, whereby the nappe, which initially dives down, rises to the water surface and more often than not forms distinct, standing waves. The straight drop is then hydraulically ineffective.

Based on the forces arising, the hydrostatic pressure (1) and force of momentum (2) for a rectangular discharge area with the formulation for the momentum principle (fig. 5) results the minimum required height H_W of the drop:

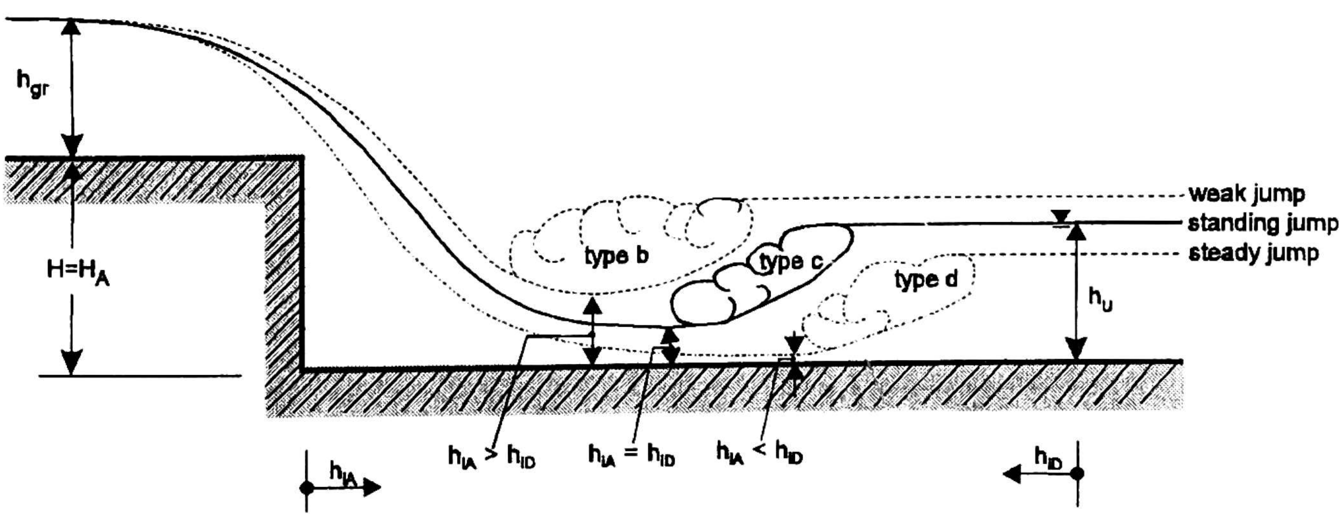

Fig. 3. Types of jump (b, c, d) at the straight drop

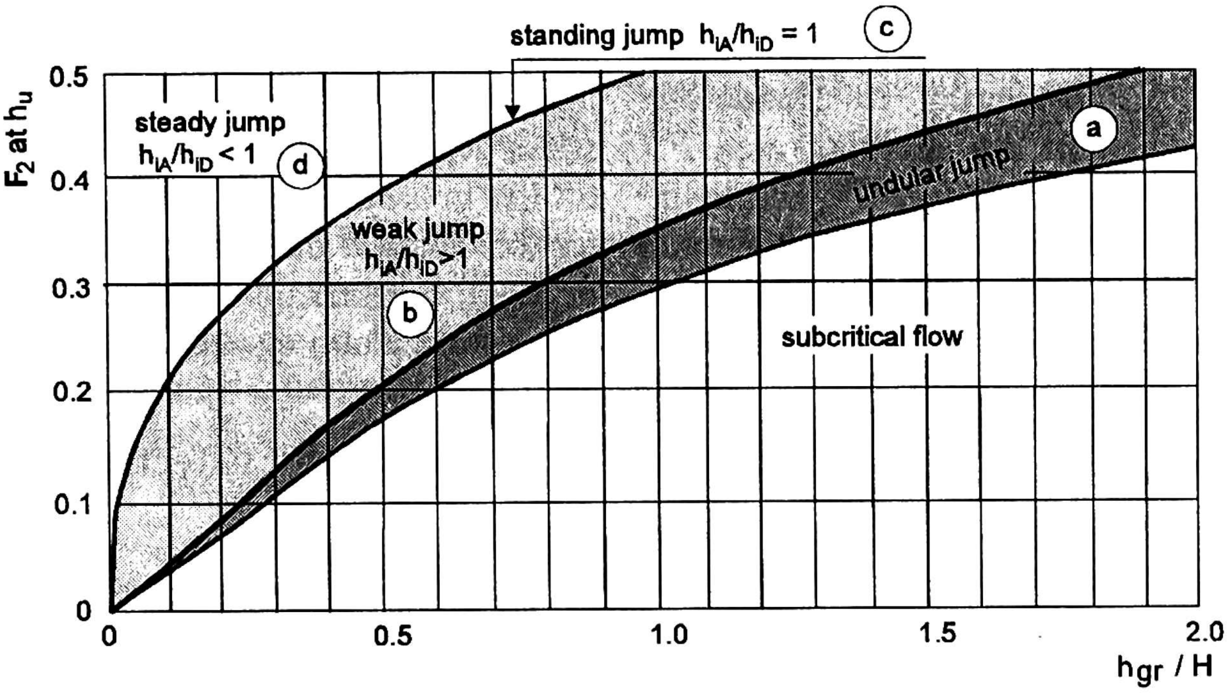

Fig. 4. Nomogram to determine the type of jump

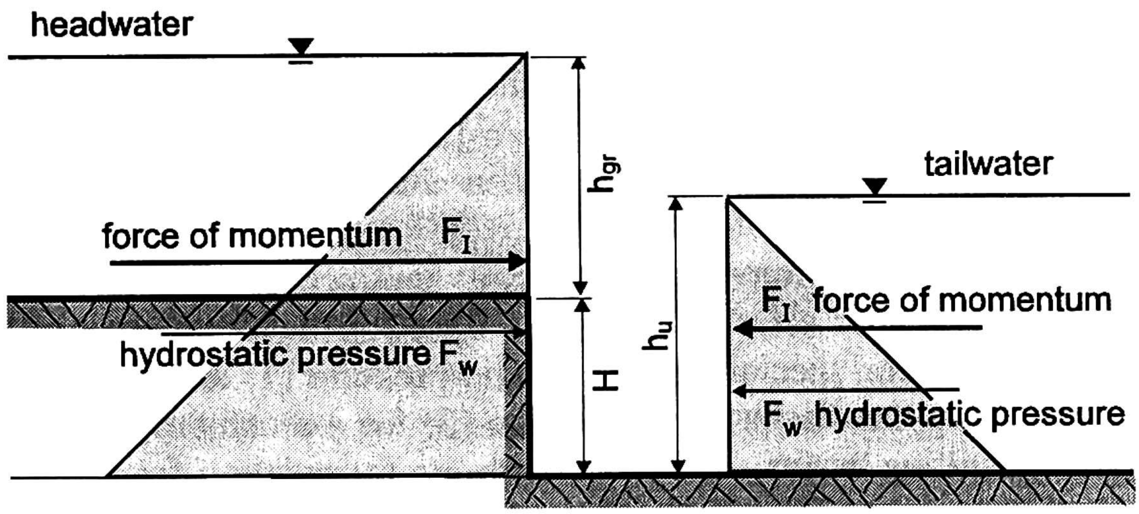

Fig. 5. Formulation for the momentum principle at the drop

$$F_W = 1/2 \cdot \rho \cdot g \cdot h^2 \cdot s \qquad (1)$$

$$F_I = \rho \cdot Q \cdot v = \rho \cdot A \cdot v^2 \qquad (2)$$

In addition to Musterle (1930) the following method of computation by *Eq.* (3) results:

$$H_W \geq \left(-1 + \sqrt{n_u^2 + \frac{2}{n_u} - 2}\right) \cdot h_{gr} \qquad (3)$$

Substituting $n_u = \dfrac{h_u}{h_{gr}} = \boldsymbol{F}^{-2/3}$

becomes:

$$H_W \geq [(\boldsymbol{F}^{-4/3} + 2 \cdot \boldsymbol{F}^{2/3} - 2)^{1/2} - 1] \times$$

$$\times\ h_{gr} = \varphi \cdot h_{gr} \qquad (4)$$

The comparison of this equation by Bleines (1951) with model tests at the straight drop with both a perpendicular and a steeply inclined wall has shown, that the theoretically derived *Eq.* (4) results in heights, which are too low for the drop. It can, however, be used for chutes with a bottom gradient of 1:4 to 1:12.

By improving *Eq.* (4) with the measuring results at the straight drop, for the integration of the measuring values a hyperload was selected, the following *Eq.* (5) for the hydraulically effective height H_w of the drop was obtained:

$$H_W \geq [-3.97 + \\ + \sqrt{(n + 5.47)^2 - 14.15}] \cdot h_{gr} \quad (5)$$

with

$$n = \varphi = -1 + \sqrt{n_u^2 + \frac{2}{n_u} - 2}$$

and

$$n_u = \frac{h_u}{h_{gr}} = \boldsymbol{F}^{-2/3}$$

Instead of the hyperload borderline, in the German Standard a simplified representation was selected, whereby the hyperload was replaced by a straight line with a distance of 0.15 on the axis of the abscisse. The agreement between the both equations (fig. 6) is perfect. Therefore in DIN 19661-2: Guide-lines for Hydraulic Structures – River Bottom Protection Structures, the effective height H_W of the straight drop can be computed by *Eq.* (6):

$$H_W \geq [0.15 + 1.1\,((\boldsymbol{F}^{-4/3} + \\ + 2\,\boldsymbol{F}^{2/3} - 2)^{1/2} - 1)] \cdot h_{gr} = \\ = \psi \cdot h_{gr} \quad (6)$$

with

$$\psi = (0.15 + 1.1 \cdot \varphi).$$

To receive a steady surface roller, the Standard asks $\boldsymbol{F} \leq 0.5$, better $\boldsymbol{F} \leq 0.32$ for the tailwater flow. With $\boldsymbol{F} = 0.50$ can be computed: $H_W \geq 0.52 \cdot h_{gr}$ and for $\boldsymbol{F} = 0.32$: $H_W \geq 1.20 \cdot h_{gr}$. With the requirement $\boldsymbol{F} \leq 0.5$ an effective height H_W of the straight drop of at least $1/2 \cdot h_{gr}$ and for $\boldsymbol{F} = 0.32$ at $1.2 \cdot h_{gr}$ results.

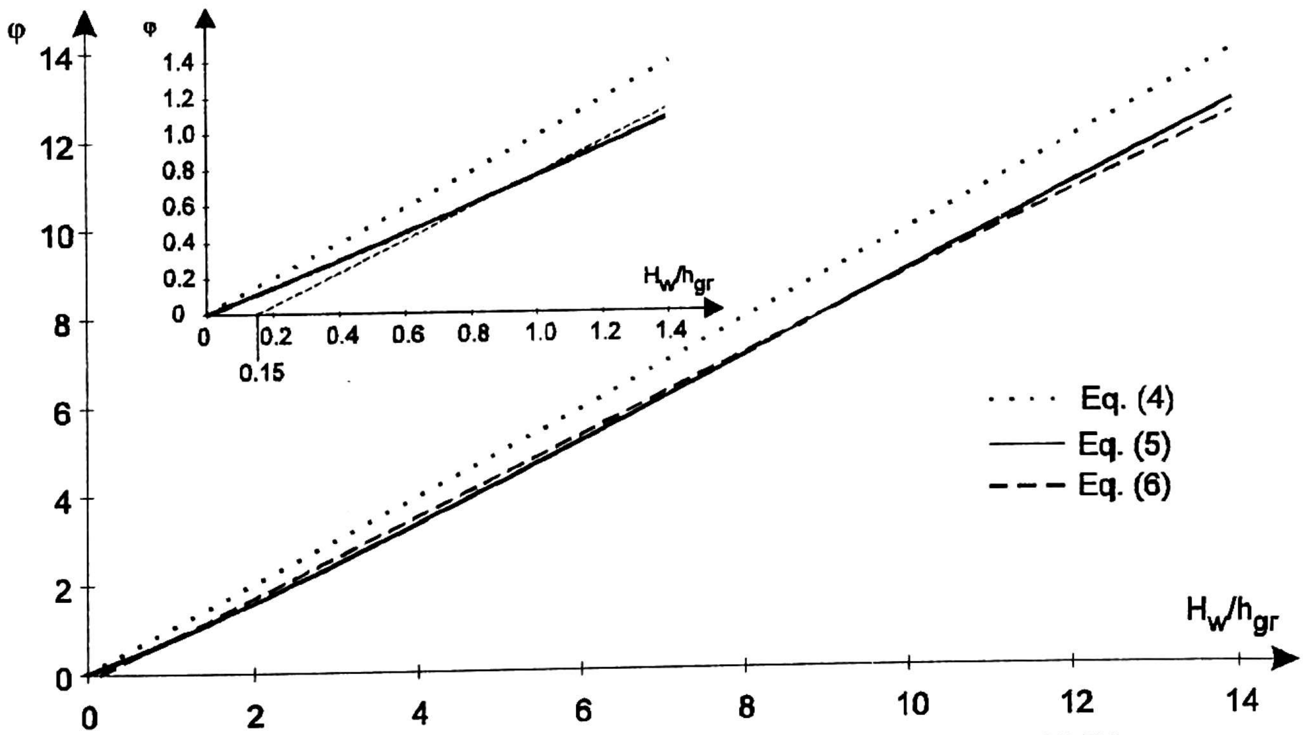

Fig. 6. Comparasion of the equations by Musterle, Bleines and DIN

Proof of The Actually Existing Type of Flow

For the borderline case, flow type c (standing jump) two formulations have the same flow patterns on the apron. The first formulation is based on the critical depth in the headwater and on the shooting flow on the apron before the surface roller (fig. 7). The second formulation considers the condition immediately before and after the surface roller (fig. 8). Thus both formulations have the same water depth $h_{iA} = h_{iD}$ immediately before the surface roller in the borderline case of flow type c. A comparison of both water depths $h_{iA} \geq h_{iD}$ ($h_{iA} \leq h_{iD}$) thus makes it possible to draw a conclusion, as to which flow type can actually be found in each investigation carried out. Thus, the results of the calculations can be checked for correctness.

Determination of $h_i = h_{iA}$ from the Headwater Conditions

According to the continuity equation of the streaming water ($Q_1 = Q_2 = v_1 \cdot A_1 = v_2 \cdot A_2$) with

$$v_1 = \sqrt{2g \cdot [(H+z) + \frac{h_{gr}}{2}]},$$

$$v_{gr} = \sqrt{g \cdot h_{gr}}$$

and

$$h_i = h_{iA}$$

the following *Eq.* (7) results:

$$h_{iA} = h_{gr} \cdot \left[\frac{2 \cdot (H+z)}{h_{gr}} + 1 \right]^{-\frac{1}{2}} \qquad (7)$$

If $\frac{1}{\sqrt{2}} = 0.7$ is set for simplification purposes, the result in the commonly used way may be writen:

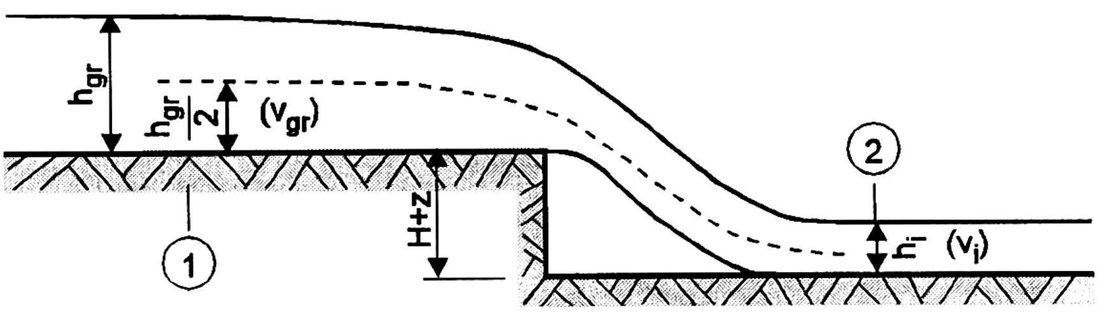

Fig. 7. Derivation of the tailwater depth h_{iA}

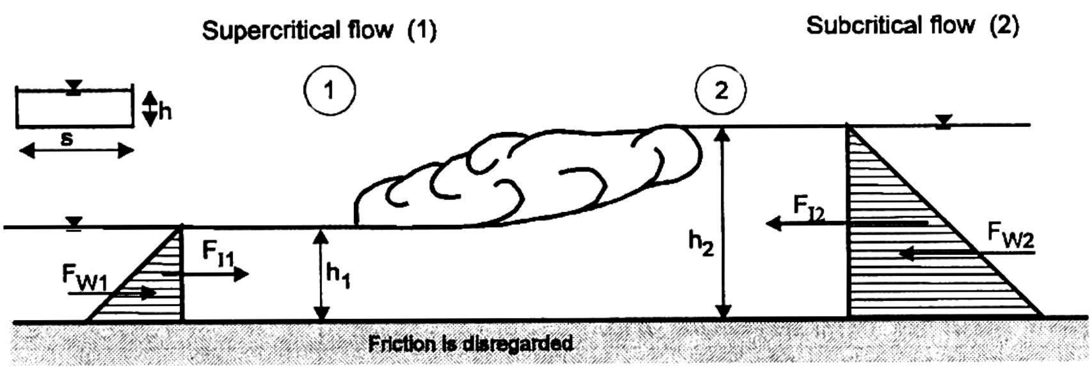

Fig. 8. Formulation for the momentum principle for the hydraulic jump equation

$$h_{iA} = h_{gr} \cdot \frac{0.7}{\sqrt{\frac{(H+z)}{h_{gr}} + 0.5}} \qquad (8)$$

Determination of $h_i = h_{iD}$ from the Tailwater Conditions

The hydraulic jump equation derives from the formulation for the momentum principle $S = F_W + F_I$ (fig. 8) with the requirement of equilibrium $S_i = S_ö$ and $F = v / \sqrt{g \cdot h}$:

$$h_1 = \frac{h_2}{2} \cdot [-1 + \sqrt{8 \cdot F_2^2 + 1}\ ] \qquad (9)$$

Height H_c of the Drop Structure for Type c

From *Eq.* (7) follows after transformation with $z = 0$ and $h_{iA} = h_{iD}$:

$$h = \frac{h_{gr}}{2} \cdot \left[\left(\frac{h_{gr}}{h_{iD}} \right)^2 - 1 \right]$$

or (10)

$$H = \frac{h_{gr}}{2} \cdot \left(F_2^{4/3} - 1 \right)$$

Hydraulic Loss Coefficient ζ_A at the Drop Type c

The hydraulic loss at the straight drop is:

$$h_{vA} = \zeta_A \cdot \frac{v_i^2}{2g}$$

with

$$H_E = h_i + h_{vi} + \zeta_A \cdot h_{vi} =$$

$$= h_i + (1 + \zeta_A) \cdot h_{vi}$$

and

$$h_{vi} = \frac{v_i^2}{2g}$$

becomes:

$$\zeta_A = \frac{H_E - h_i}{h_{vi}} - 1 \qquad (11)$$

With $H_E = H + H_{\min}$, H according *Eq.* (10) and $H_{\min} = 3/2 \cdot h_{gr}$ further $h_i = = h_{gr} \cdot F_1^{-2/3}$ can be computed:

$$\zeta_A = \frac{H + H_{\min} - h_i}{F_1^2 \cdot g \cdot \frac{h_i}{2g}} - 1 =$$

$$= \frac{\frac{h_{gr}}{2} (F_1^{4/3} - 1) + \frac{3}{2} h_{gr} - \frac{2}{2} h_{gr} \cdot F_1^{-2/3}}{F_1^2 \cdot \frac{h_i}{2}} - 1$$

$$\zeta_A = \frac{(F_1^{4/3} - 1) + 3 - 2\ F_1^{-2}}{F_1^2 \cdot F_1^{-2/3}} - 1 =$$

$$= 1 - \frac{1}{F_1^{4/3}} + \frac{3}{F_1^{4/3}} - 2\, F_1^{-2} - 1$$

$$\zeta_A = 2 \quad (F_1^{-4/3}\, F_1^{-2}) \qquad (12)$$

The plot of the function for the hydraulic loss coefficient ζ_A at the drop type c with respect to the Froude number

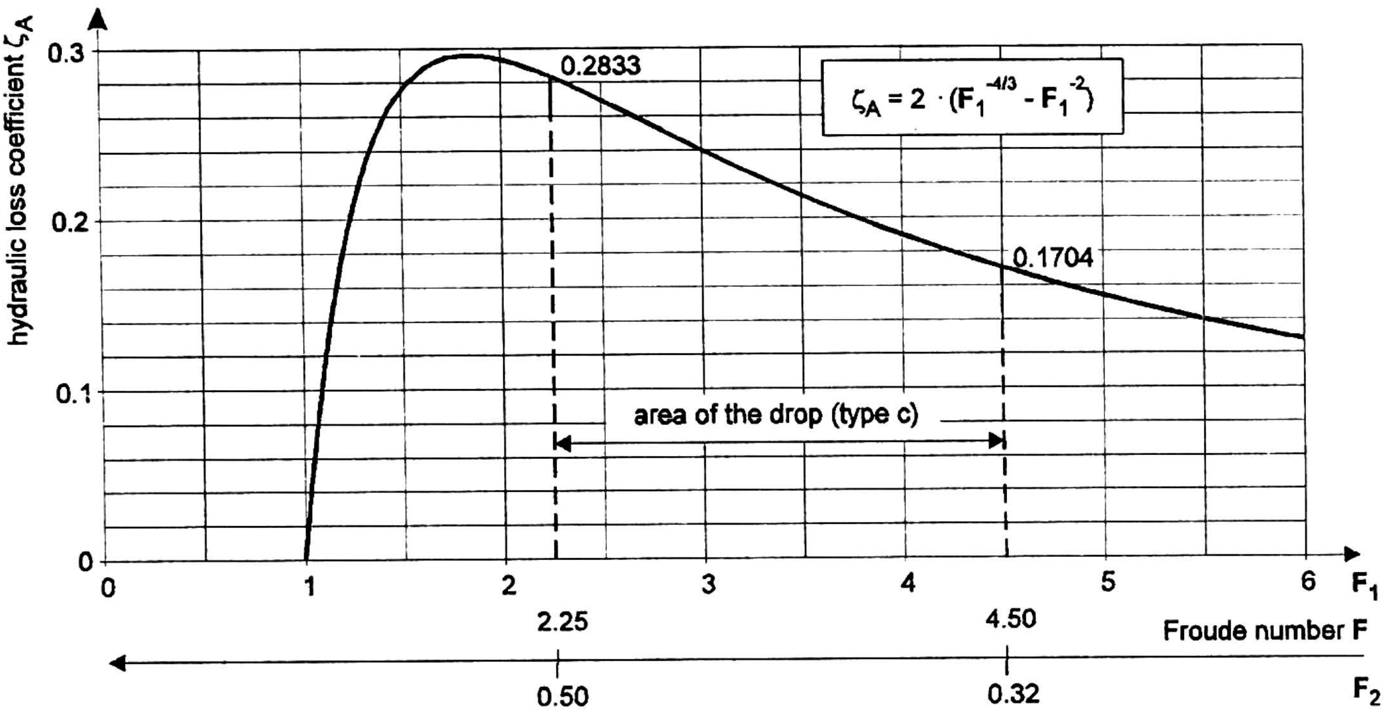

Fig. 9. Plot of the function for the hydraulic loss coefficient $\zeta_A = \phi\,(F)$ for type c

shows figure 9. The maximum hydraulic loss coefficient is 0.2963, which occurs at F = 1.837 (0.584). The coefficient ranges for $0.32 \leq F \leq 0.5$ between $0.17 \leq \zeta_A \leq 0.28$.

Example: Calculation of the Straight Drop as Discharge Type c

Below, the dimensions required are illustrated by giving an example. At the same time, the corresponding stretches needing protection will be determined. Only flow type c (standing jump) is suitable for calculation; in addition the minimum necessary hydraulically effective height H_W of the straight drop can be calculated. Then with these limits it is possible to ascertain flow type b (weak jump) and flow type d (steady jump), which occur when the height of the drop or the level of the streaming tailwater changes.

Example: Rectangular discharge area, bottom width s = 4.0 m, discharge $Q = 7.0\ m^3/s$, the corresponding Froude numbers: in the shooting area $F = 4.50$ and in the streaming area $F = 0.315$ (0.32).

Requirements: Hydraulically effective straight drop, height H of the drop when $h_{iA} = h_{iD}$, energy head, loss in height of the drop and of the roller, length of the stretches to be protected in the headwater and tailwater, as well as a comparison of the flow types occurring with different heights of the straight drop.

Basic Characteristics of the Straight Drop

Hydraulically Effective Straight Drop

$$H_W \geq [0.15 + 1.1\,((0.315^{-4/3} + 2 \cdot 0.315^{2/3} - 2)^{1/2} - 1)] \cdot 0.68 \geq \geq 0.77\ \text{m} \qquad \text{[of (6)]}$$

Depths of Water in the Headwater Area

Both Froude numbers (shooting and streaming) are related to the corresponding water depths $[h = h_{gr} \cdot F^{-2/3}]$.

$$h_{gr} = \sqrt[3]{\frac{\alpha \cdot Q^2}{g \cdot s^2}} =$$

$$= \sqrt[3]{\frac{1.0 \cdot 7.0^2}{9.81 \cdot 4.0^2}} \; 0.67 \text{ m}$$

$$h_i = 0.67 \cdot 4.5^{-2/3} = 0.25 \text{ m} = h_{iD}$$

$$h_{ö} = 0.67 \cdot 0.315^{-2/3} = 1.47 \text{ m} = h_u$$

Height of Straight Drop

$$H = \frac{h_{gr}}{2} \cdot \left[\left(\frac{h_{gr}}{h_{iD}}\right)^2 - 1\right] =$$

$$= \frac{0.68}{2} \cdot \left[\left(\frac{0.68}{0.25}\right)^2 - 1\right] = 2.18 \text{ m}$$

Velocity Head below the Drop

$$h_{vgr} = \left(\frac{Q}{s \cdot h_{gr}}\right)^2 \cdot \frac{1}{2g} = 0.34 \text{ m}$$

$$h_{vi} = \frac{v_i^2}{2g} = \left(\frac{Q}{s \cdot h_i}\right)^2 \cdot \frac{1}{2g} = 2.52 \text{ m}$$

$$h_{vu} = \frac{v_u^2}{2g} = \left(\frac{Q}{s \cdot h_u}\right)^2 \cdot \frac{1}{2g} = 0.07 \text{ m}$$

Loss in the Drop

For $F_2 = 0.315$ or $F_1 = 4.5$ becomes:

$$\zeta_A = 2 \cdot (F_1^{-4/3} - F_1^{-2}) =$$

$$= 2 \cdot (4.5^{-4/3} - 4.5^{-2}) = 0.171$$

$$h_{vA} = \zeta_A \cdot \frac{v_i^2}{2g} =$$

$$= 0.171 \cdot 2.52 = 0.43 \text{ m}$$

Loss in the Roller

$$h_{vD} = \frac{(h_u - h_i)^3}{4 \cdot h_u \cdot h_i} =$$

$$= \frac{(1.47 - 0.25)^3}{4 \cdot 1.47 \cdot 0.25} = 0.24 \text{ m}$$

Checking the Energy Head

$$H_E = H + H_{\min} = 2.18 +$$

$$+ 3/2 \cdot 0.68 = 3.20 \text{ m}$$

$$H_E = h_u + h_{vu} + h_{vA} + h_{vD} =$$

$$= 1.47 + 0.07 + 0.43 + 1.24 = 3.20 \text{ m}$$

Stretches to be protected

Headwater with Shooting Flow:

$$l_1 = 5 \cdot h_{gr} = 5 \cdot 0.68 =$$

$$= 3.40 \text{ m} \cong 3.50 \text{ m}$$

Length of the Drop:

$$l_2 = 4.3 \left(\frac{h_{gr}}{H}\right)^{1/3} \cdot (h_{gr} \cdot H)^{1/2} =$$

$$= 4.3 \cdot \left(\frac{0.68}{2.18}\right)^{1/3} \cdot (0.68 \cdot 2.18)^{1/2} =$$

$$= 3.55 \text{ m}$$

Length of the Jump:

$$l_4 = 5 \cdot (h_u - h_i) =$$

$$= 5 \cdot (1.47 - 0.25) = 6.10 \text{ m}$$

Length of the Apron:

$L_a = l_2 + l_4 = 3.55 + 6.10 =$

$= 9.55 \text{ m} \cong 10.0 \text{ m}$

Tailwater with Disturbed Flow:

$l_5 \approx l_4 = 6.00 \text{ m}$

Total Length:

$L = l_1 + l_2 + l_4 + l_5 =$

$= 3.40 + 3.55 + 6.10 + 6.10 =$

$= 19.15 \text{ m} \approx 20.0 \text{ m}$

Types of Flow with Different Heights of the Straight Drop

Taking the calculation example for discharge flow type c, the following areas with hydraulically effective heights of the drop result:

flow type a:
$H_a < 0.77$ m (ineffective)
– subcritical or undular,

flow type b:
0.77 m< $H_b < 2.18$ m
– weak,

flow type c:
$H_c = 2.18$ m
– standing,

flow type d:
$H_d > H_c > 2.18$ m
– steady or supercritical.

Design of the Straight Drop Structure (fig. 10)

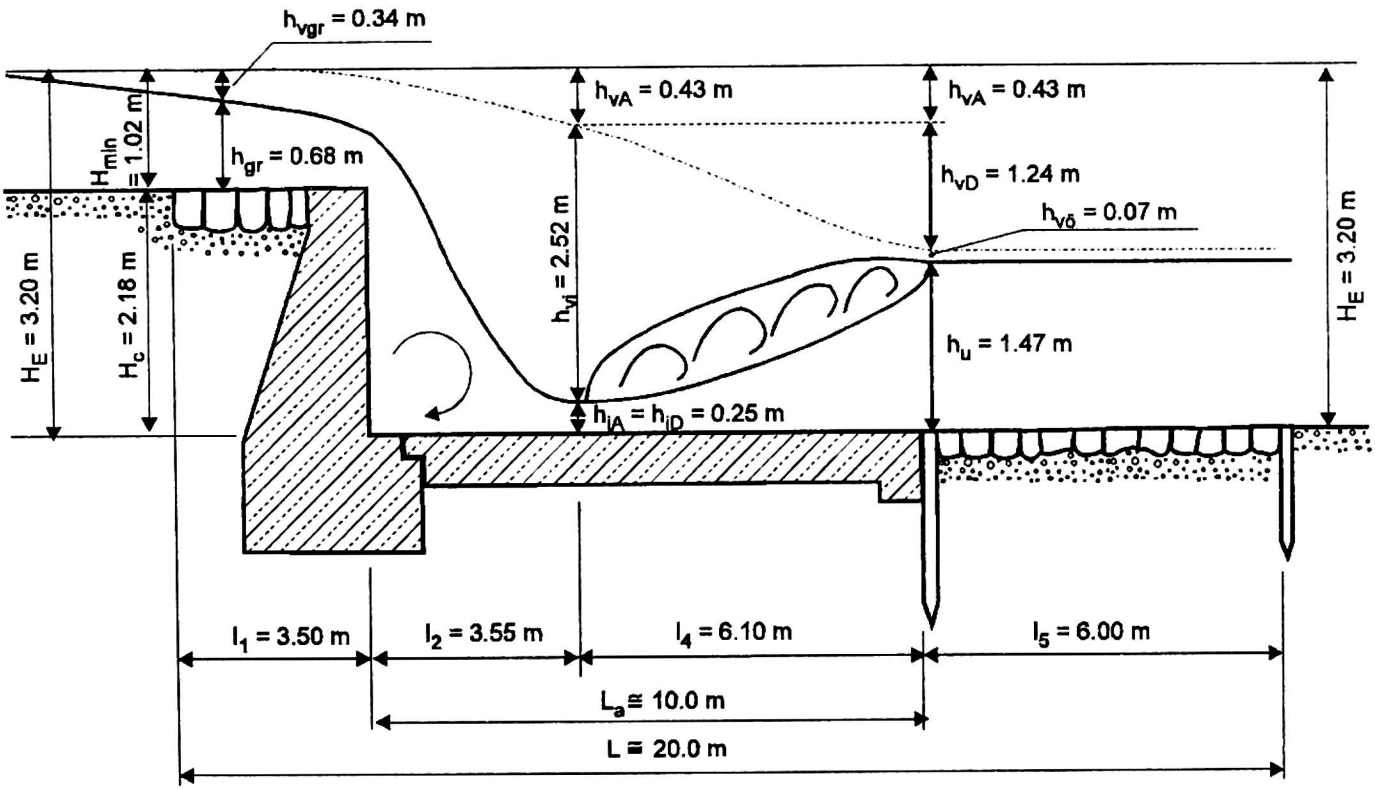

Fig. 10. Example of a calculation for type c [height doubled (i.e. 1:2)]

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