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THE INFLUENCE OF PLANT HEIGHT AND LEAF AREA ON THE PROCESS OF REAL EVAPOTRANSPIRATION OF BARLEY

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A b s t r a c t. During modelling the process of real evapotranspiration in basins used for agricultural purposes, there is a problem of differentiation of evapotranspiration value among the plants growing in a basin. Most of the time it is assumed, that meteorological factors (temperature, water vapour pressure, wind speed and sunshine duration) within a basin are the same and actually the only factors differing evopotranspiration among plants cultivated in basins are the plant height and the area of its foliage. These two factors were put by the author of this paper into the Shuttleworth-Wallace formula as function considering, this way, continuous growth of a plant and its leaf area.

K e y w o r d s: barley, evapotranspiration, plant height, logistic function, leaf area index

INTRODUCTION

In the research literature there are more and more papers in which the influence of leaf area on real evapotranspiration has been considered. Shuttleworth and Wallace modified the Penman-Monteith formula [5] introducing the leaf area index, L, to the formula (L is the area of all leaves above the unit area of the ground). The authors pointed out the components of the evapotranspiration process, in which one can observe the activity of leaves evaporting surface. They have not given, however, the form of the function which would consider the continuous changes of that index during the whole vegetation period. Therefore, this formula can only be used, when the specific value of leaf area index was known [4]. The author of this paper tried to describe this process taking into consideration the continuous changes of a plant height and the surface of its foliage. Leaf area index and plant height as growth functions, in which the argument is the next day of plant development, are put into the Shuttleworth-Wallace formula, considering the fact that leaf area changes along with plant growth. Logistic functions were used for the research because it is one of types of the growth function [2].

THEORY

Symbols used:

A (A_s)	total energy flux leaving the leafage, the substrate as sensible and latent heat per soil surface unit (W m^{-2})
H (H_s)	sensible heat flux from the leafage (substrate) (W m^{-2})
c_p	specific heat of the air (J kg^{-1}K^{-1})
d	height of the zero plane displacement (m)
d_o	vapour pressure deficit at the height of $d+z_o$ (hPa)
G	soil heat flux (W m^{-2})
h	plant cover height (m)
k	von Karman's constant (dimensionless)
K	diffusion coefficient (m^{2}s^{-1})
L	area of all leaves above the unit ground area - leaf area index (dimensionless)

r_a^a aerodynamic resistance in $(d+z_o\,; x)$ area (s m^{-1})
r_a^s aerodynamic resistance in $(0;\, d+z_o)$ area (s m^{-1})
r_a^c mean resistance of the plants leaves in the canopy boundary layer (s m^{-1})
r_b mean resistance of the canopy boundary layer (s m^{-1})
r_s^c total stomatal resistance of the canopy (s m^{-1})
r_s^s surface resistance of the substrate (s m^{-1})
r_{ST} mean stomatal resistance (s m^{-1})
$r_a^a(\alpha)$ value r_a^a, when $L = L_{\max}$
R_n radiation flux balance for the active surface (W m^{-2})
R_n^s radiation flux balance for the substrate (W m^{-2})
v wind speed at the height of x (m s^{-1})
v_* friction rate (m s^{-1})
x height of meteorological measurements (m)
z veriable height (m)
z_o roughness parameter of the plant crop (m)
z_o' roughness parameter of the bare surface (m)
γ psychrometric constant (hPa K^{-1})
λE latent heat flux from both: plant leafarea and the ground (W m^{-2})
E model values of barley evapotranspiration (mm)
λE_c latent heat flux from the plant canopy (W m^{-2})
δE_s latent heat flux from the soil (W m^{-2})
ρ air density (Kg m^{-3})
Δ $[e_W(T_X)-e_W(T_O)]/(T_X-T_O)$ (hPa K^{-1})
T_x air temperature at the height of x (^{o} C)
T_o air temperature at the height of $d+z_O$ (^{o} C)
$e_w(T)$ pressure of saturated vapour at the temperature of $T(T=T_X, T_O)$ (hPa)
ETR real evapotranspiration of barley (mm)
ETP potential evapotranspiration according to Penman (mm)

General model

The idea of Penman-Monteith equations is the method of writing the sensible heat flux H and latent heat flux λE relation to the energy available in other forms in the shape of equations. The total energy flux leaving the plant canopy equals:

$$A = H_c + \lambda E_c = R_n - G. \tag{1}$$

The energy available in the substrate equals:

$$A_s = H_s + \lambda E_s = R_n^s - G. \tag{2}$$

The total evaporation heat from plant canopy λE can be considered the sum of the evaporation heat from bath plants' leafage λE_c and substrate λE_s, i.e.:

$$\lambda E = \lambda E_c + \lambda E_s. \tag{3}$$

Each of these components can be derivated from the equations of Penman-Monteith type:

$$\lambda E_c = [\Delta(A-A_s)+\rho c_p d_o/r_a^c]\;[\Delta+\gamma\,(1+r_s^c/r_a^c)]^{-1} \tag{4}$$

$$\lambda E_s = [(\Delta A_s+\rho c_p d_o/r_a^s)]\;[\Delta+\gamma(1+r_s^s/r_a^s)]^{-1}. \tag{5}$$

Shuttleworth and Wallace [5] introduced the leaf area index to the Penman-Monteith formula pointing the processes, in which the activity of the evaporating surface of leafage is noticeable.

Leaf area index L in the Shuttleworth-Wallace formula

Leaf area index L equal to the area of all leaves which are above unit surface area, occurs in all the processes in which the activity of plant cover, as a separate source of energy, is included.

It has been experimentally stated [3] that radiation reaching the ground surface, R_n^s, derived using the Beer's correlation, is a function of L:

$$R_n^s = R_n \exp(-0.7\,L). \tag{6}$$

Surface resistances - mean resistance of the boundary layer of leafage r_a^c and total stomatal resistance of leafage r_s^c are surface resistances acting through the surface area of plant canopy. They change in inverse proportion to the total leaf area:

$$r_s^c = r_{sT}/2L \quad (7)$$

$$r_a^c = r_b/2L \quad (8)$$

where r_{sT} is the mean stomatal resistance of 400 s m^{-1} [6], and r_b is the mean resistance of the boundary layer equal to 25 s m^{-1} [1].

Leafage surface also affects the value of aerodynamic resistance, which in this paper is separately derived in the layer between the ground and the height of $d+z_o$ (marked r_a^s) as well as in the layer from $d+z_o$ to the measure height of x, as r_a^a:

$$r_a^a = L_{max}^{-1} L r_a^a(\alpha) + L_{max}^{-1}(L_{max} - L) r_a^a(0) \quad (9)$$

$$r_a^s = L_{max}^{-1} L r_a^s(\alpha) + L_{max}^{-1}(L_{max} - L) r_a^s(0) \quad (10)$$

where

$$r_a^s(0) = \ln(x/z_o^{'})\ln[(d+z_o)/z_o^{'}]/k^2 u \quad (11)$$

$$r_a^a(0) = \ln^2(x/z_o^{'})/k^2 u - r_a^s(0). \quad (12)$$

Plant height as in the Shuttleworth-Wallace formula

In the S-W formula, the index of turbulent diffusion is the function of height:

$$K = [k^2 v(h-d)]^{-1} \ln[(x-d)/z_o] \exp[-n(1-z/h)] \quad (13)$$

$$d = 0.63h, \; z_o = 0.13h. \quad (14)$$

Aerodynamic resistances for open substrate are functions of plant height:

$$r_a^s(0) = \ln(x/z_o^{'})\ln[(d+z_o)/z_o^{'}]/k^2 v \quad (15)$$

$$r_a^a(0) = \ln^2(x/z_o{'})/k^2 v - r_a^s(0). \quad (16)$$

Plant height and leaf area as functions of growth

It is assummed that plant height is a function of time t in the form:

$$f(t) = a(1+b)\exp(-ct)^{-1} \quad (17)$$

where a, b, c are unknown parameters of the function $f(t)$ so-called logistic function. It is one of the so-called growth curves, chosen to describe the height increase during vegetation period on the basis of previous attempts to match curves of different type. The shape of the logistic function indicates that at the moment of t_o=0 (the moment of the start of measures) the mean height was equal to $a(1+b)^{-1}$. Intensive height increase lasted till $t_p = \ln(b)/c$ (t_p) is the point of curving on the logistic curve $f(t)$. Plant height reaches a critical value being the horizontal asymptote of the curve. The unknown parameters a, b, c are estimated according to the measurements of plant height $w_1, w_2, ..., w_n$ and time moments t_1 =1, t_2 =2,...,t_n= n, assumming the 10-day time step. According to the least squares method of estimation of the parameters, the a, b, and c minimize the sum of deviation squares of observations from the curve, defined as:

$$S(a,b,c) = \sum_{j=1}^{n} \left(w_j - \frac{a}{1+be^{-cj}} \right)^2. \quad (18)$$

Considering the fact that the leafage surface changes along with plant height, it is assummed that the leaf area index L, during the next day of vegetation period is the same as the function:

$$L(t) = a \frac{1+b}{1+b\exp(-kt)} \quad (19)$$

where a, b, k have positive values.

The analysis of the trend of 24-h values of real evapotranspiration of spring barley in the years between 1984-1987 stimulates some specific conditions, which should be ensured by the function describing the changes in leafage surface area. The trend of this process for spring barley can be devided into three periods.

Period I, including 80 days and beginning with seeding, is the period when the values of barley evapotranspiration increase. Also, it is the period, in which the most intensive growth of the plant can be observed, which indicates the supposition that it is also the period of the most intensive growth of the leafage surface area.

Period II, a very short period, during which the values of evapotranspiration do not change, barley reaches the maximum height, which also means - the maximum leaf area.

Period III, evapotranspiration values decrease, which is related to drying of the barley leaves.

Considering the conditions mentioned above, the parameters a, b, k and L_{max} of the logistic function are derived from the following conditions:

$$L(t=85)=L_{max} \quad (20)$$

$$0<a<1 \quad L(0)=a \quad (21)$$

$$\sum_{t=1}^{85}\left[E\left(t;\, a, b, k, L_{max}\right)-ETR(t)\right]^2=\min \quad (22)$$

where t is the next day of vegetation period.

According to the assumption of the least square method, the estimation of the a, b, k and L_{max} parameters minimizes the sum of squares of model values deviations from empiric values of evapotranspiration.

Compatibility measures of model values to empiric data

The measures of model values E conformity to empiric data ETR are as follow:

1) Standard deviation of the remainder (non-weight estimation of standard deviation or the mean error of estimation):

$$S=\left[\frac{\sum_{t=1}^{n}[ETR(t)-E(t)]^2}{n-3}\right] \quad (23)$$

where $ETR(t)$ and $E(T)$ are values of ETR and E, respectively in day t.

The standard deviation of the remainder indicates how the empiric data ETR differ from respective theoretical values of E.

2) Relative, mean error of approximation in per cent:

$$V=\frac{\frac{1}{n}\sum_{t=1}^{n}[(ETR(t)-E(t)]}{\overline{ETR}}100. \quad (24)$$

3) Remainder variability coefficient in per cent:

$$V_u=\frac{S}{\overline{ETR}}100. \quad (25)$$

RESULTS

The empiric data used for this paper are from the Agro- and Hydrometeorology Observatory of the University of Agriculture in Wrocław. 24 h values of meteorological elements for the years between 1984-1990 as well as measured values of real evaoptranspiration of spring barley from the investigated years have been used for numeric calculations.

In Table 1, the real values of barley evapotranspiration for the months of May and June with the values derived on the basis of the proposed model are compared. Differences in the values found indicate that the model matches reality. The values of Penman's potential evapotranspiration which, according his concept, is the estimation of the higher limit of evaporation from only plant surface with unlimited water storage, are also compared in Table 1. Analysing the derived values we can state that Penman's concept is true only for shorter plants (May). Towards the end of June the plant reaches the maximum height, which also means - the maximum leafage surface area. The differences between potential and real evapotranspiration significantly decrease, sometimes the real evapotranspiration is even higher than the potential (June 1986, 1987).

Table 1. The comparison of monthly sums of real evapotranspiration of barley (*ETR*) with model values (*E*) and with potential evapotranspiration (*ETP*)

Year	Month	*ETR*	*E*	*ETP*
1984	May	53.90	49.59	80.44
	June	96.70	95.19	97.00
1985	May	77.10	73.08	105.01
	June	79.40	80.44	90.57
1986	May	124.60	103.77	110.71
	June	144.60	143.21	117.15
1987	May	66.20	75.17	88.19
	June	151.10	133.04	93.15

Table 2. Parameters a, b, k, L_{max} of logistic function describing the increase of leafage surface area and measures of confirmity of real and model values

Year	L_{max}	a	b	k	S	V	V_u
1984	6.0	0.5	31.2	0.04	0.111	1.5%	4.5%
1985	2.3	0.2	10.5	0.15	0.083	3.1%	3.2%
1986	4.2	0.1	41.0	0.37	0.225	8.3%	5.1%
1987	10.0	0.1	99.4	0.14	0.202	9.7%	6.0%

In Table 2 the values of a, b and k parameters of logisitc function, which describes the growth of the leafage area, are given. These parameters were determined using the least squares method. The table also contains the maximum values of the leaf area index for barley L_{max} as well as the measures of conformity of real and model values.

CONCLUSIONS

On the basis of the research data the following conclusions can be drawn:

1. Maximum value of leaf area (L_{max}) reached by spring barley during vegetation period varies every year.

2. The conformity of empiric values with the values derived from the model contains minimum errors and proves good agreement of the model with reality.

3. Good confirmity of the model's values with empiric data proves the validity of using the logisitc function describing continuous changes of leafage surface area.

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WPŁYW WYSOKOŚCI ROŚLINY I POWIERZCHNI JEJ LISTOWIA NA PROCES EWAPOTRANSPIRACJI RZECZYWISTEJ JĘCZMIENIA

Przy modelowaniu procesu ewapotranspiracji rzeczywistej jęczmienia uwzględnia się dwa czynniki, które wpływają na wielkość ewapotranspiracji roślin uprawianych w zlewni użytkowanej rolniczo. Są to wysokość rośliny i powierzchnia jej listowia. Do modelu wprowadza się je w postaci funkcji logistycznych.

Słowa kluczowe: wysokość roślin, wskaźnik powierzchni liści, funkcja logistyczna, jęczmień, ewapotranspiracja.