SUBMODEL OF BYPASS FLOW IN CRACKING SOILS. PART 1- THEORY

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A b s t r a c t. The ACCESS-II hydrological sub-model is inherently one dimensional, it models a vertical water movement in the soil profile. Extension of this model to account for a bypass flow through cracks or macro-pores has been done using a horizontal flow model. The main problem is to incorporate a horizontal flow submodel into the ACCESS-II in order to take into account the most important features of bypass flow phenomenon.

K e y w o r d s: bypass flow, soil cracks, subsidence

INTRODUCTION

In this study we present two models of the crack creation based on the subsidence and crack geometry parameter measurements. We model a quick vertical water flow through cracks and macropores and redistribution to the horizontal soil layers. We employ the model of swelling-shrinking soil to model dynamical crack formation and water redistribution model based on the Green-Ampt approach.

METHODS

Soil shrinking - swelling characteristic

The water content change is the main cause of the soil shrinking and swelling process. The soil volume change is most significant at the soil surface where water content changes in the widest range. There are four ranges of soil volume change distinguished: structural, normal, residual, and zero [4,6,7].

The structural shrinking range starts from the saturation state. The volume of the soil changes less than the removed water volume due to inter-aggregate water removal.

Further drying leads to a range where the soil volume change is equal to the volume of water removed from the soil. This range is called normal shrinking range.

When an air enters the soil, the soil volume change becomes smaller than the removed water volume. This is called residual shrinking range.

At the end of drying process shrinkage stops and the soil volume does not change anymore, this range is called zero shrinking range.

The volume change process can be described as a function of the water content.

The soil shrinkage curve gives an information about the soil volume change (Fig. 1). The soil is not an isotropic medium and can change its dimensions in different way in different directions. We should describe the partition of total volume change over change in subsidence (layer thickness) and change in crack volume. Conversion of three-dimensional soil volume changes into cracking and surface subsidence is of great importance because cracking and surface subsidence have different and contrasting effects on water transport process in the soil.
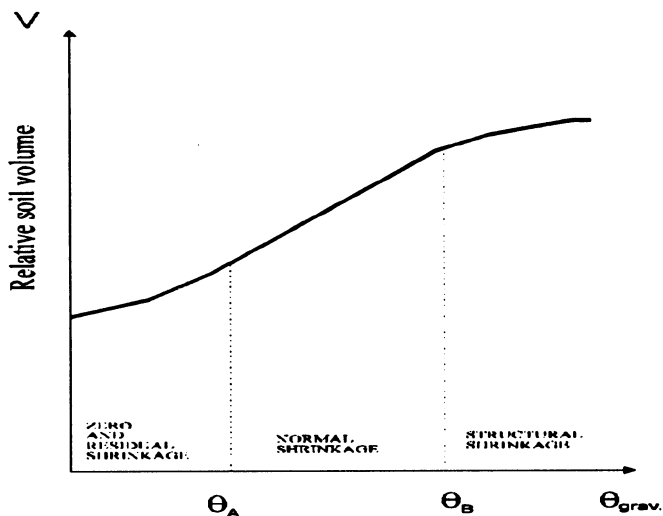
Fig. 1. Swelling - shrinking curve for soil.

Subsidence

Subsidence Δz can be measured *in situ*. Changes in layer thickness Δz can be converted into crack volume per layer (Fig. 2), and added giving the total crack volume within the soil profile. Bronswijk [1-3], proposes application of rotating disks immersed into shrinking soil at different depths to measure subsidence. If the assumption of the uniform shrinking of the soil is fulfilled, the surface subsidence gives complete information about crack volume. Bronswijk presents the following measured result (compiled here in Table 1).

It is visible that the ratio of numbers from the third column to the first column, placed in the fourth column of Table 1, is close to 2/3. This means that the assumption of the uniform shrinking is fulfilled. In this Bronswijk [1]

proposed a dimensionless geometry factor r_s for the subsidence - volume change description defined in the equation:
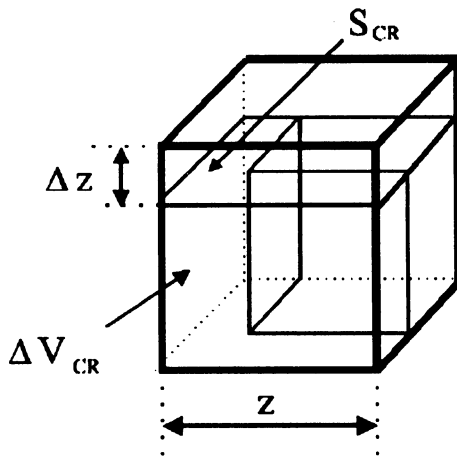

Fig. 2. Subsidence and crack volume change.

T a b l e 1. Measured values of subsidence and volume change for the soil

$\Delta V / z^2$ (mm)	Δz (mm)	$\Delta V_{CR} / z^2$ (mm)	$\Delta V_{CR} / \Delta V$
2.0	0.7	1.3	0.650
49.2	17.2	32.0	0.650
74.0	26.2	47.8	0.646
35.7	12.5	23.2	0.650
4.0	1.4	2.6	0.650
3.5	1.2	2.3	0.657

$$\Delta V = \left[1 - \left(1 - \frac{\Delta z}{z}\right)^{r_s}\right] V_o \qquad (1)$$

where V_o - saturated soil volume, ΔV - soil volume change, z - original height of the soil cube, Δz - surface subsidence.

Two limit cases for r_s value are defined following: subsidence without cracking, exists when $r_s = 1$(only vertical shrinking occurs), and cracking without subsidence, exists when $r_s \rightarrow \infty$ (only horizontal shrinking occurs).

For all other r_s values, both cracking and subsidence coexist ($r_s=3$: isotropic shrinkage; $1<r_s<3$: subsidence superiority; $r_s<3$: cracking superiority).

Using Eq.(1) for a giving r_s it is possible to calculate the relative crack area. We can rewrite this equation as follows:

$$\frac{\Delta V}{V_o} = 1 - \left(1 - \frac{\Delta z}{z}\right)^{r_s} \qquad (2)$$

where ΔV the soil volume change is defined by:

$$\Delta V = \Delta V_{CR} + \Delta V_s \qquad (3)$$

where ΔV_{CR} - volume of created cracks, $-\Delta V_s$ volume change of the soil caused by subsidence.

Using Eqs(2) and (3) one can write:

$$\frac{\Delta V_{CR} + \Delta V_S}{V_o} = 1 - \left(1 - \frac{\Delta z}{z}\right)^{r_s} . \qquad (4)$$

Assuming that $V_o = S_o z$, the relative area of crack $\sigma_{CR} = \dfrac{S_{CR}}{S_o}$ can be calculated as follows:

$$\sigma_{CR} = 1 - \left(1 - \frac{\Delta z}{z}\right)^{r_s - 1} \qquad (5)$$

where S_o - initial surface area of the soil cube, S_{CR} - crack area.

When the maximal subsidence Δz_{max} is measured, the Eq.(5) gives us the maximal value of the relative crack area $\sigma_{CR max}$:

$$\sigma_{CR max} = 1 - \left(1 - \frac{\Delta z_{max}}{z}\right)^{r_s - 1} . \qquad (6)$$

Using Eq.(2) with $r_s=3$, it means that soil cube of the unitary volume shrinks isotropically one can obtain the following equation describing the maximal relative crack area:

$$\sigma_{CR max} = 1 - (1 - V_{CR max})^{\frac{2}{3}} \qquad (7)$$

where $(1-V_{CR max})$ - minimal volume of unitary soil sample (after shrinkage), $(1-V_{CR max})^{2/3}$ - horizontal cross-section surface area of shrinked sample.

Crack geometry

In the model utilizing the measured subsidence for crack development description, crack dimensions are estimated indirectly using foregoing soil volume change model.

Crack parameters (width, depth, surface ratio) can be measured directly and used, with the measured soil properties, for the description of crack structure and crack evolution.

Maximal value of crack width is used as parameter in crack geometry models. We consider three models of crack network: triangular rectangular and hexagonal (see Fig. 3).
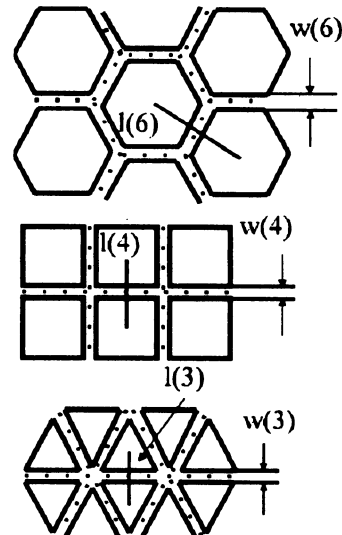

Fig. 3. Three types of crack network.

We model the crack network with the uniform tessellation (triangular, tetragonal, hexagonal). The relative crack area σ_{CR} depends on moisture, but does not depend on the crack network structure. We calculate geometry factors for each assumed geometry separately. For three types of networks (triangular, tetragonal and hexagonal) we calculate structure index $l(i)$ which is dependent on parameter ρ - the number of structure elements per unit surface area.

For triangular network:

$$l(3) = \frac{2}{\sqrt{3\sqrt{3}\,\rho}} \quad . \tag{8}$$

For rectangular network:

$$l(4) = \frac{1}{\sqrt{\rho}} \quad . \tag{9}$$

For hexagonal network:

$$l(6) = \sqrt{\frac{2}{\sqrt{3}\,\rho}} \tag{10}$$

where $l(i)$ - distance between centers of two neighbour structure elements.

The relative crack area σ_{CR} for each structure (triangular, tetragonal and hexagonal) fulfills the following equation:

$$w(i)^2 - 2l(i)w(i) + \sigma_{CR}l(i)^2 = 0 \quad . \tag{11}$$

Calculating σ_{CR} from this equation we get:

$$\sigma_{CR} = 1 - \left(1 - \frac{w(i)}{l(i)}\right)^2 \tag{12}$$

where $l(i)$ - distance between two structure elements, $w(i)$ - crack width, σ_{CR} - relative area of cracks.

For the field condition we propose to measure geometry parameters when the soil is dry (maximal value of σ_{CR}). In this case we can rewrite the Eq.(12) in the form:

$$\sigma_{CR\,max} = 1 - \left(1 - \frac{w(i)_{max}}{l(i)}\right)^2 \quad . \tag{13}$$

One can measure a maximal width of cracks and a mean distance between centers of two structure elements or number of structure elements per unit surface area. The Eq.(13) allows to calculate the third parameter knowing two of them.

The physics of water movement and redistribution

The shrinking and swelling process, causing soil cracks development, plays major role in the quick water transport into depth which cannot be reached due to conventional Darcian flow. The gravity rather than suction is the main force causing this type of transport. Cracks and macro-pores form the way for preferential flow of the water when the condition close to saturation occurs. This way of the water transport is extremely important during intensive rain or irrigation.

The amount of water transported preferentially changes the dynamics of water redistribution in the soil profile leading to quick saturation of the bottom part of the soil profile.

The simple model for the swelling-shrinking process description assumes linear changes of soil relative volume as function of moisture content (Fig. 4) [8].

In the range of residual and structural shrinking the volume change is relatively unimportant. The most important range is the normal shrinking range (Θ_A, Θ_B), where the
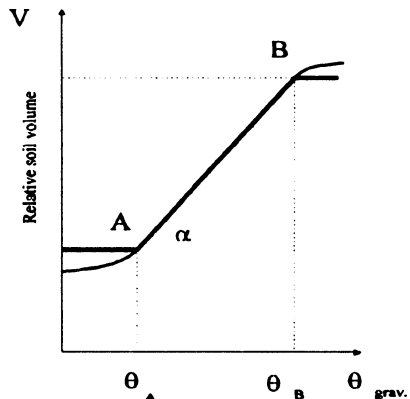

Fig. 4. Linear model of shrinking - swelling process.

volume change is highest and crack development is most likely. In the normal shrinking range there is more than 50 % of soil volume change [6].

On the basis of the straight line approximation (Fig. 4) the crack surface changes as the water content varies are described. A linear volume change is assumed to describe volume - moisture dependence.

The soil relative volume change causes the soil to subside and to crack in the same time. The volume of cracks depends on the shrinkage details. If the volume change is linear there is only one parameter needed for crack volume description during drying process. For isotropic shrinking the crack volume change is also a linear function of the soil moisture.

The crack volume change in the drying process is, for each compartment of soil, described by:

$$\delta V_{CR} = \sigma_{CR}\, \delta h \qquad (14)$$

where δh - depth of the compartment, σ_{CR} - relative crack surface.

For small values of δh we assume that the crack walls are vertical and parallel to each other. For this case the relative crack area changes linearly with the moisture. The equa-

Fig. 5. Water redistribution from the crack.

tion of the relative crack area as a function of moisture has the following form:

$$\sigma_{CR}(\Theta) = \sigma_{CR\max} \frac{\Theta_B - \Theta}{\Theta_B - \Theta_A}. \qquad (15)$$

Water redistribution description within subsidence model

We assume that the following conditions are valid:

Cracks at the unit surface area of the soil are represented by relative surface area covered $\sigma_{CR}(\Theta)$.

Equivalent volume of the cracks per unit surface area of the soil is equal to the sum of crack volumes at different depths.

Total volume of a water able to enter the crack can be described by the equation:

$$W = W_1 + W_2 \qquad (16)$$

where W_1 - the volume of water that comes to the crack by run-off, it can be expressed by the equation:

$$W_1 = P\, dt - (\Theta_{sat1} - \Theta_1)\, dz_1 \qquad (17)$$

where P - daily precipitation, Θ_{sat1} - saturated water content of the first layer, Θ_1 - actual water content of the first layer, dz_1 - thickness of the first layer, dt - time step, W_2 - amount of water coming directly to the crack, it is equal to:

$$W_2 = \sigma_{CR1}(\Theta)\, P\, dt \qquad (18)$$

where $\sigma_{CR1}(\Theta)$ - relative crack surface area.

In the simplest case we assume also that the water entering the crack redistributes in the time much shorter than the time scale under consideration. The water content changes in each layer by addition the amount of water passing through the crack.

Water redistribution for crack geometry model

We calculate the redistribution of water that fills cracks using the following assumptions Fig. 5.

Water fills crack giving the hydrostatic pressure distribution at the crack wall which is
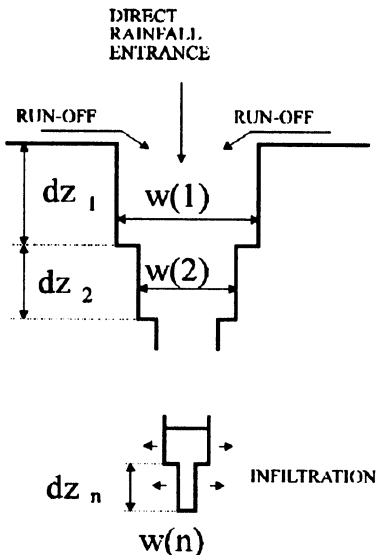
used as the boundary condition for the water infiltration into the soil.

Initial moisture for each time step of horizontal infiltration is assumed to be constant in space.

We use a Green-Ampt approach for the horizontal infiltration description.

Each layer is considered as an independent homogenous horizontal profile (from the point of view of the bypass flow sub-model).

Geometrical factors of cracks are calculated as, water content dependent using the foregoing reasoning.

The amount of water in the crack is calculated as:

$$W = W_1 + W_2 + W_3 \qquad (19)$$

where W_1 and W_2 are defined as previously, W_3 is an amount of water remaining from the last time step.

In order to take into account a finite time of water infiltration from the crack into the soil profile we utilize the simple model known as the Green-Ampt model [5].

The main assumptions of the Green and Ampt approach are:
- Wetting front is distinct and precisely defined.
- The matric potential at the wetting front remains effectively constant in time and position, it means that behind the wetting front, the soil is uniformly wet and of constant conductivity.

According to these assumptions the Darcy - - type equation can be written as follows:

$$\frac{dI}{dt} = K \frac{H_o - H_f}{L_f} \qquad (20)$$

where $\dfrac{dI}{dt}$ - flux into the soil and through the transmission zone, I - cumulative infiltration, K - hydraulic conductivity of the transmission zone, H_o - water potential at the entry surface, H_f - effective water potential at the wetting front, L_f - length of the wetted zone.

For the uniform wetting zone from the surface to the wetting front, the cumulative infiltration is equal to:

$$I = L_f (\Theta_t - \Theta_i) \qquad (21)$$

where Θ_t - transmission zone wetness during infiltration, Θ_i - initial profile wetness beyond the wetting front.

Using Eq.(20), after integration, one can obtain the following solution for cumulative infiltration (assuming $I(0)=0$):

$$I = \sqrt{2K(\Theta_t - \Theta_i)(H_o - H_f)t} \ . \qquad (22)$$

The water potential difference which exists between the crack surface of the wall and wetting front is equal to:

$$\Delta H_{CR} = H_o - H_f \qquad (23)$$

where H_o - is the difference between water surface level in the crack and compartment level where the infiltration occurs, this value is time dependent and changes after each time step; H_f - is the water potential existing in each compartment before infiltration starts, this value is modified after each time step.

Initial water content Θ_i in each compartment is time dependent value.

Saturated water content Θ_t is position dependent value.

For each compartment, we calculate the volume of the crack:

$$V_{CR}(\Theta) = \sigma_{CR}(\Theta) \Delta z \qquad (24)$$

where V_{CR} - volume of the soil cracks in the compartment, per unit surface area, σ_{CR} - is the relative crack area, Δz - thickness of the compartment.

The total length of one crack wall per unit surface area (0(i)) in each compartment for triangular, rectangular and hexagonal network is equal to:

$$O(3) = 3\sqrt{3}(l(3) - w)\rho \qquad (25)$$

$$O(4) = 4(l(4) - w)\rho \qquad (26)$$

$$O(6)=\frac{6}{\sqrt{3}}\left(l(6)-w\right)\rho \qquad (27)$$

where w - crack width, $l(i)$ - defined in Eqs(8), (9), and (10).

The actual crack width w corresponding to the water content can be calculated from equations:

$$w=l(i)\left(1-\sqrt{1-\sigma_{CR}(\Theta)}\right) \qquad (28)$$

and

$$\sigma_{CR}(\Theta)=\sigma_{CRmax}\frac{\Theta_B-\Theta}{\Theta_B-\Theta_A}. \qquad (29)$$

The amount of water passing from the crack to the soil profile in each compartment during each time step is calculated from:

$$V=\left[I(t+dt)-I(t)\right]O(i)\Delta z . \qquad (30)$$

This amount of water is added to the water remaining already in the soil compartment. After each time step it gives a new volumetric water content in each compartment. This value is returned then to the main subroutine.

CONCLUSIONS

The model of preferential flow in swelling soils can be adopted to describe the bypass flow caused by different phenomena. Using the crack relative area as the fit parameter one can model the bypass flow due to inter-aggregate pores presence or macro-pores of biological origin (earthworm holes, root channels etc.).

Proposed model can be used for bypass flow description if one can characterize macro-pores by the relative surface area and depth distribution. In the case of presented model the parameters are water content dependent but it is easy to use constant geometry pores or employ the dependence on other parameters. In any case parameters are specific for a given soil type and may vary in accordance to the actual conditions.

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