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THE FRACTAL CHARACTERISTICS OF LATTICE MEDIA. PART II

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A b s t r a c t. The starting point to fractal analysis can be the comparison of different dimensions. Fractal description can be used in the investigations of medium motions in slat systems as well as for the description of corn staying in a natural state.

K e y w o r d s: isotropic turbulence, fractal characteristics

INTRODUCTION

This paper is a continuation of the Part I [2] dealing with the description of a lattice. The dimension of figure of topological space - may be the least natural number *n*, of such a quality, that covering it with any small closed sets causes that each point of the figure belongs, at most, to (n+1) sets (Fig. 1). Since the component figures of the 'rectangular' are contained in it, so the dimension of component figures will not exceed the measure of 'rectangular'. As the 'rectangular' is the sum of component figures so its dimension will not exceed the greatest dimension of component figures. It means that particular components may be of a different size. For it may be so for physical reasons. For example, the parameters treated as co-ordinates can 'fall out' in the case of some figures. However, it should be emphasised that these figures will not then be homeomorphic.

THE EXAMPLES OF WAVE MOTION

The dimension of segments - of broken curves - amounts to 1 and this is the topological dimension w^T. For example, the Cantor set (Fig. 2) posses the similarity dimension D= ln2/ln3 $\approx$ 0.63 and the topological dimension w^T=0. Thus the topological dimension w^T is not equal to D ($w^T \neq D$). It can be seen therefore that the characteristics of figures can be better defined with the use of topological dimension. It should be emphasised that similarity dimension is the special case of Hausdorff dimension possible to determining for any set of the Euclidean space [1,3]. And so, the dimension theory and the metrix theory can be bound, with the use of a distance (length), with a curvature object. Relationship with the torsion object is also available, as the curvature object is expressed by the connection objects which can be divided into symmetrical and unsymmetrical parts. It is worth to mention that there exist some attractive sets, so called attractors, which are neither stable points nor periodical trajectories. Some of these sets are the strange attractors - near them apparently chaotic motion is obtained when iterating [3]. Attractor is supposed to be, in some points, the Cantor set multiplied cartesially by a segment, for example.

With in the cereal corn waving in a wind stalks are bended in the beginning and then they get straight causing the creation of the areas of disturbed motion (Fig. 3). Similarly, the participation of disturbed - turbulent motion

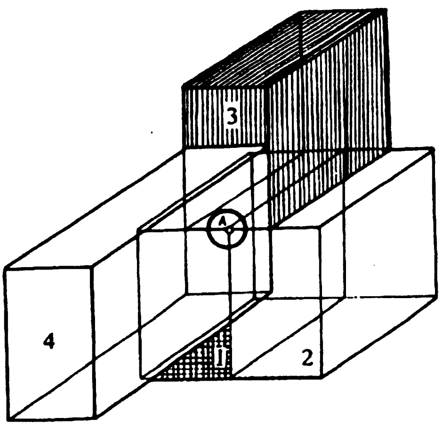

Fig. 1. The illustration of the definition of dimension (the sphere of the centre A belongs to four figures).

can be seen in the 'rectangular' of corn (between the points A and B, but not only).

Introducing the 'rectangular' into the crop anger of combine harvester as well as its further movement are like turbulence phenomenon. Large disturbances can be observed in the beginning, then their geometrical dimension decreases, to become finally laminar in the oblique conveyer. The same is also true of combine harvester threshing drum. In this case, an aeromechanics resembles univocally the isotropic turbulence, i.e., such (one) for which the whirls of given quantity can be generated with the same intensity, in any point of space.

It is possible to assume after Mandelbrot [1] that the area of focused turbulent motion is the fractal. The velocity distribution of 'rectangular' being distored can be approximated with Cantor curve. Total energy will be distributed to the areas of disturbance according

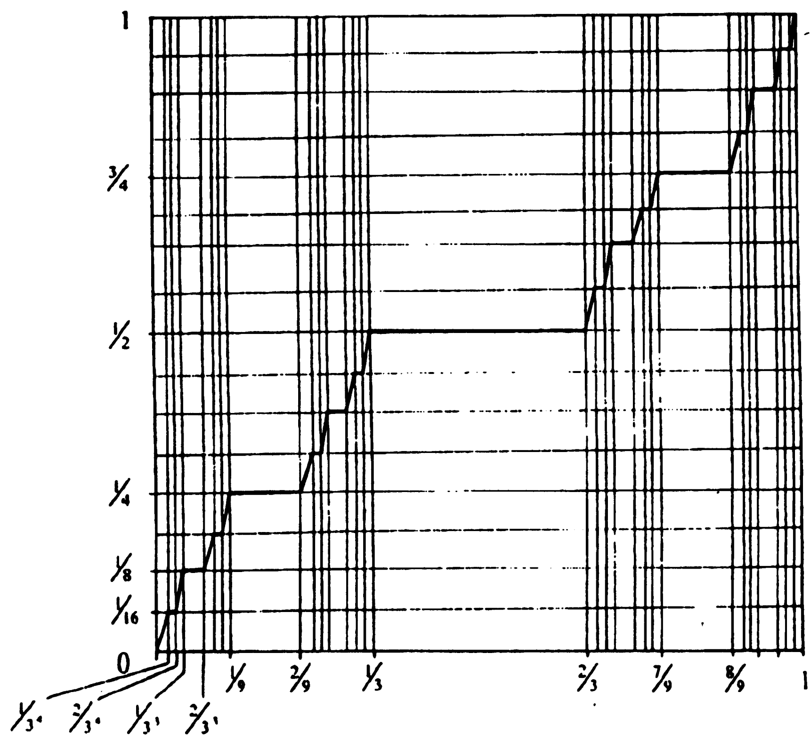

Fig. 2. The Cantor set

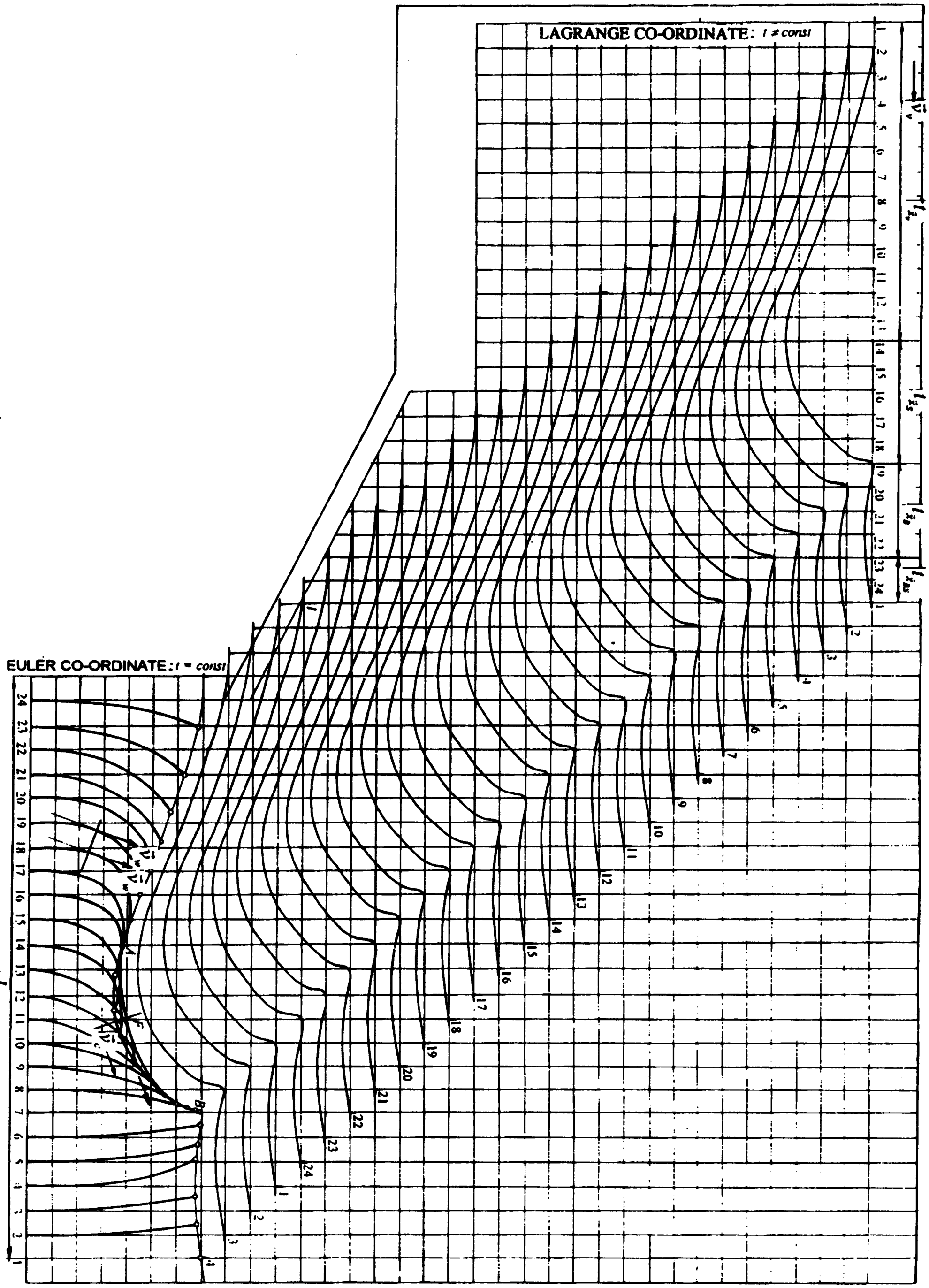

Fig. 3. The model of corn wave motion

to their diameters - the main measure l. The velocity in such an area will be given:

$$\vec{v}(\xi)=\sum_{p=1}^{\infty}\vec{a}_p \sin k_p\xi \qquad (1)$$

where k_p - frequency, a_p - amplitude.

The harmonic analysis of $\vec{v}(\xi)$ can be also carried out on the dimension l (Fig. 4). The part of energy falling to the frequency given, will be determined by the energy spectrum $E(k_p)$. Energy introducing into plant material is accomplished through the areas of the greatest main dimension (by the lowest frequences). On the other, the kinetic energy will be changed into heat, generally, in small areas (by higher frequences). These areas of turbulence prove to be mainly the areas of contact points concentration to which an energy is delivered.

Let the areas of concentration of dimension l_o will be taken to the areas of size:

$$l_1=\frac{1}{r}l_o, l_2=\frac{1}{r}l_1=$$

$$\left(\frac{1}{r}\right)^2 l_o,\dots,l_n=\left(\frac{1}{r}\right)^n l_o \qquad (2)$$

The sum of concentration areas will be smaller than the volume of the 'rectangular'. It can be assumed that the concentration area of rank q will be taken to Q subareas of rank Q+1, where $Q<r^3$(on Fig. 5 r=3 have been assumed, for example). Then the concentration areas of rank (n+1) will cover the fraction $\gamma=\frac{Q}{r^3}$ of the space being occupied by the area of rank n. So that the areas of rank n will occupy the fraction γ^n of the volume being occupied by the area of rank zero. Such a conception may be accepted assuming that the volumes, in which contact points concentration takes place, lessen themselves expotentially during a working process. The concentration areas of the least volume form the fractal set of number D= -ln Q ln r. The velocity of changes $\vec{v}_n$ can be attributed to the concentration areas of rank n. The kinetic energy falling to the unit of mass, comprised in the areas of concentration, may be expressed by the dependence:

$$E_{k(n)}\approx\gamma^n v_{(n)}^2. \qquad (3)$$

Then, assuming that during the course of an 'elementary deformation' - e.g., an impact and the like, the significant part of energy will be transfered from the volume of area of rank n to the volume of the area of rank (n+1). The duration time of this 'elementary deformation' will amount to:

$$t_{(n)}\approx\frac{l_{(n)}}{v_{(n)}}. \qquad (4)$$

The velocity of energy transmission will amount to:

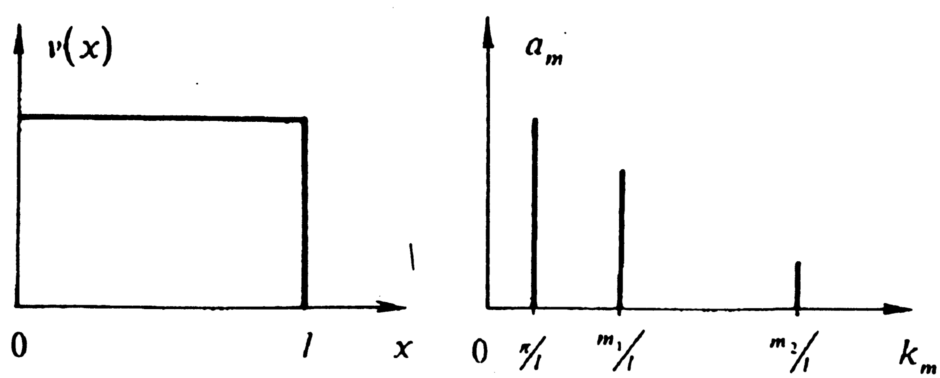

Fig. 4. The distribution of velocity on the dimension l and its harmonic analysis

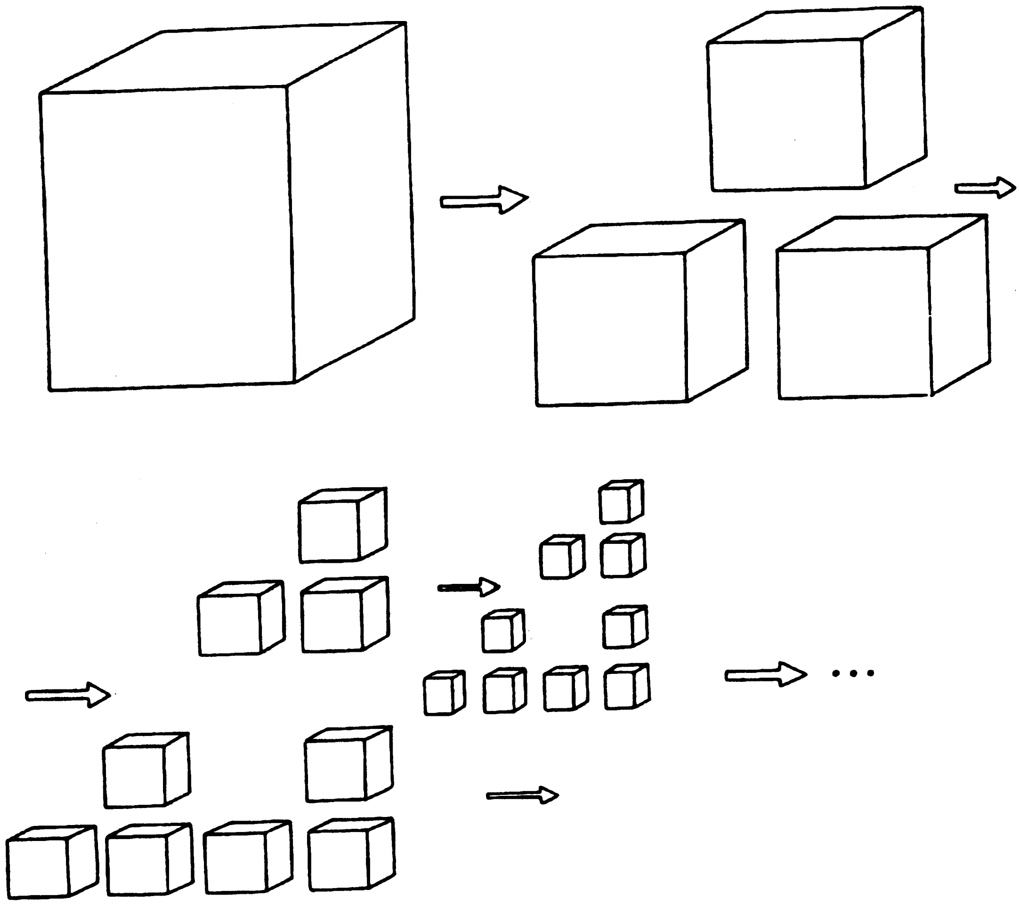

Fig. 5. The scheme of concentration areas disintegration

$$w_{(n)} = \frac{E_{k(n)}}{t_{(n)}} \approx \frac{\gamma^{n} v_{(n)}^{3}}{l_{(n)}}. \qquad (5)$$

In the absence of other energy sources the velocity of its flowing $w_{(n)}$ will be near the speed of energy delivery to the concentration area of the rank given, denoted by w. In this case the velocity w can be interpreted as the velocity of the impact.

Then it will be:

$$v_{(n)} \cong \frac{l_{(n)}^{1/3} w^{1/3}}{\gamma^{n/3}},$$

$$E_{(n)} \cong w^{\frac{2}{3}} l_{(n)}^{\frac{2}{3}} \gamma^{\frac{n}{3}} = w^{\frac{2}{3}} k_{(n)}^{-\frac{2}{3}} \gamma^{\frac{n}{3}}. \qquad (6)$$

Regarding that $Q = r^{D}, \gamma = \frac{Q}{r^{3}}, l_{n} = \frac{1}{k_{(n)}}$, $D = \frac{\ln Q}{\ln r}$, where r=1,2,3, ... , $k(n)$ - frequency of monochromatic wave transmitting an energy, we obtain:

$$E_{(n)} \cong w^{\frac{2}{3}} k_{(n)}^{-\frac{2}{3}} \left(k_{(n)} l_{o}\right)^{-\frac{1}{3}(3-D)} =$$

$$w^{\frac{2}{3}} k_{(n)}^{-\frac{2}{3}} \left(k_{(n)} l_{o}\right)^{-\frac{1}{3}(3+\ln Q \ln r)} \qquad (7)$$

As a matter of fact, we should say about the density of the distribution of quantity r. Since kinetic energy is the integral of the following shape:

$$E_{(n)} = \int_{k_{(n)}}^{k_{(n+1)}} E(k)dk \qquad (8)$$

so, after differentiation with respect to k the expression (7) will become the shape:

$$\frac{dE_{(n)}}{dk} =$$

$$\frac{d}{dk}\left(w^{\frac{2}{3}} k_{(n)}^{-\frac{2}{3}-\frac{1}{3}(3-D)} l_o^{-\frac{1}{3}(3-D)} \right) = E(k)$$

and finally:

$$E(k) \cong$$

$$-\frac{1}{3}\left(5-\frac{\ln Q}{\ln r}\right) r^{\frac{2}{3}} k_{(n)}^{-\frac{1}{3}\left(8-\frac{\ln Q}{\ln r}\right)} l_o^{-\frac{1}{3}\left(3-\frac{\ln Q}{\ln r}\right)}. \qquad (9)$$

And now, for the given density of probability $p_\Gamma(r)$ the density of probability $p_E(E)$ can be calculated. To the kinetic energy calculated as above the internal energy - viscosity as well as the energy used for overcoming friction forces should be added. It of course, refers only to local bodies.

CONCLUSION

The relationships obtained in an empirical way are, as a rule, exponential of exponent being a fractional number, therefore certain associations with a fractal structure may be suggested. The cereal corn as a physical phenomenon posses to complicated structure to be investigated, at full its length, with the use of classical analysis. Therefore, the new comparative methods of analysis are necessary. And if they even prove to be rough at first, but being successively improved they will bring a model nearer to reality. The next stage of making these methods perfect may be the introduction of compound variable (e.g., July sets).

Last of all the remark of subjective nature fractals are the subject of great interest mainly for aesthetical reasons. The meeting of fractal theme with the description of waving cereal corn - having also its aesthetical values - makes a certain sign.

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