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A MATHEMATICAL MODEL OF COCOA POD DEFORMATION BASED ON HERTZ THEORY

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A b s t r a c t. A mathematical model based on Hertz's theory of contact stress was developed for the analysis of cocoa pod deformation when subjected to a uni-axial compressive stress, in its lateral axis, between rigid parallel plates. A two-sample t-test comparison of the experimentally measured values of the stiffness modulus and those obtained from the initial Hertz theory equations indicated a very significant difference, which was observed to be due to the irregular shape of the pod, its furrowed circumference and internal hollowness, contrary to some basic assumptions of the theory. Correcting factors developed to account for these irregularities significantly improved the predictions of the modified Hertz relation. The shape factor, which expressed the deviation of the material from a true ellipsoid in terms of the material contact area with the loading plate, was found to be the most critical correction factor in terms of which a combined correction factor could be expressed to simplify necessary computations.

K e y w o r d s: Hertz theory, cocoa pod, agricproduct deformation, mechanical property

INTRODUCTION

One of the operations involved in the on-farm processing of cocoa is the breaking of the pods to extract the wet beans, which are subsequently fermented. Until now this is done manually, but efforts are being made by researchers [2,4,8] to develop a mechanical device for pod breaking. It is considered necessary to first characterize the pod's breaking behaviour, by carrying out compressive loading tests [1], to obtain its force-deformation characteristics.

Although agricultural materials are generally non-homogenous, isotropic or elastic it has been found possible to define an elastic range of behaviour within which their elastic parameters can be quantified. Mohsenin [7] observed that under small strains most agricultural materials exhibit extensive elasticity, to which the Hertz theory of contact stress is applicable.

An earlier work [5] on the mechanical properties of cocoa pods has shown that its breaking under parallel plate loading is a complex process, in which the stress and strain cannot be accurately measured because of the variation in the area under load with increased loading. Hence adopting the methods employed by Rehkugler [11] and Reece and Lot [10] in their studies on the breaking of eggs, the breaking force and the absolute deformation were measured to define a stiffness modulus, which is the ratio of the maximum failure load to the deformation at failure. The force-deformation curve was observed to be mostly linear.

The objective in this work is to adapt the Hertz theory of contact stress to predicting the stiffness modulus of a cocoa pod, from its independent physical properties and hence provide a better theoretical basis for understanding its breaking behaviour.

THEORETICAL DEVELOPMENT

The fundamental Hertz relations

In deriving Hertz relationships for contact stresses, certain fundamental assumptions are imperative. These are that the material is homogeneous, the load is static, the surfaces are smooth, and that the radii of curvature of the contacting bodies, by far, exceed the radius of the contact surface [7]. This latter assumption implies that only a normal (compressive) stress arises over the contact surface, the effect of tangential stress being negligible.

Timoshenko and Goodier [13] derived an expression relating the applied force to the radius of the contact surface as:

$$a = \left[\frac{3}{4}\pi F\,(\partial_1 + \partial_2)\,(\frac{R_1 R_2}{R_1 + R_2})\right]^{\frac{1}{2}} \quad (1)$$

where a - radius of the contact surface, F - applied force between contacting bodies, ∂_1 - material/property constant of first body (body 1), ∂_2 - material/property constant of the second body (body 2), R_1 - radius of curvature of body 1, R_2 - radius of curvature of body 2.

The total deformation of both bodies in contact was then expressed as:

$$D = \left[\frac{9}{16}\pi^2 F^2\,(\partial_1 + \partial_2)\,(\frac{R_1 R_2}{R_1 + R_2})\right]^{\frac{1}{3}} \quad (2)$$

Most fruits are convex, ellipsoidal or spheroidal in shape [7,9]. Mohsenin [7], considered the case of contact between two fruits, one having minimum and maximum radii of curvature R_1 and R_1', and the other, R_2 and R_2'. Their combined deformation was given as:

$$D = \frac{3K}{2}\left[\frac{F^2 A^2}{2}\,(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'})\right]^{\frac{1}{3}} \quad (3)$$

and

$$A = \frac{1 - \nu_1^2}{\pi E_1} + \frac{1 - \nu_2^2}{\pi E_2} \quad (4)$$

where K - shape constant, ν_1 and ν_2 - Poisson ratios of the bodies, E_1 and E_2 - Young moduli of elasticity of the bodies.

In Eq. (3), the contact area between the two bodies was considered eliptic rather than circular as indicated in the case of spherical fruits [3,12]. This accords well with the ellipsoidal shape of cocoa pods. Maduako and Faborode [6] evaluated the sphericity of cocoa pods to be about 51 %.

In the compression testing of food and agricultural products, three loading elements can be used, namely, rigid parallel plates, smooth spherical indenter and cylindrical die. The first two cases are more relevant to this work.

For compression between rigid parallel plates, the radii of curvature of the flat plates are assumed to be infinite, so that $1/R_2=0$ and $1/R_2'=0$ in Eq. (3), giving;

$$D = \frac{3k}{2}\left[\frac{F^2 A^2}{2}\,(\frac{1}{R_1} + \frac{1}{R_1'})\right]^{\frac{1}{3}} \quad (5)$$

For the particular case of a rigid plate, made of steel, in terms of the elatic modulus E:

$$A = \frac{1-\nu^2}{\pi E_1}$$

and Eq. (5) becomes [7]:

$$E = \frac{0.338\,K^{\frac{3}{2}}\,F\,(1-\nu^2)}{D^{\frac{3}{2}}}\,(\frac{1}{R_1} + \frac{1}{R_1'})^{\frac{1}{2}} \quad (6)$$

For compression by means of spherical

indenter, the conditons are that $1/R_2 = 1/R_2'$ $= 2/d$, where d is the diameter of the indenter. Hence Eq. (3) becomes:

$$D = \frac{k}{2}\left[\frac{F^2(l-v^2)}{2\pi E^2}\left(\frac{1}{R_1}+\frac{1}{R_1'}+\frac{4}{d}\right)\right]^{\frac{1}{3}} \quad (7)$$

or

$$E = \frac{0.338\,K^{\frac{2}{3}}F(1-v^2)}{D^{\frac{3}{2}}}\left[\left(\frac{1}{R_1}+\frac{1}{R_1'}+\frac{4}{d}\right)\right]^{\frac{1}{3}} \quad (8)$$

Basic Hertz theory equations for a cocoa pod

In order to adapt the Hertz equation to the loading of a cocoa pod, the stiffness modulus is used in place of the elastic modulus.

Hence Eq. (6) translates to:

$$S = \frac{0.338\,K^{\frac{2}{3}}FA_0(1-v^2)}{2R_1 D^{\frac{3}{2}}}\left(\frac{1}{R_1}+\frac{1}{R_1'}\right)^{\frac{1}{2}} \quad (9)$$

where S - stiffness modulus, A_0 is the contact area on which the force is acting.

For loading with a spherical indenter, Eq. (8) similarly gives;

$$S = \frac{0.338\,K^{\frac{2}{3}}F(1-v^2)A_0}{2R_1 D^{\frac{3}{2}}}\left[\frac{1}{R_1}+\frac{1}{R_1'}+\frac{4}{d}\right]^{\frac{1}{2}} \quad (10)$$

Eqs (9) and (10) are the basic Hertz theory equations for a cocoa pod under parallel plate and spherical indenter loading respectively, based on the assumptions made earlier.

EXPERIMENTAL WORK

Experimental materials

Pod samples of the two common and commercially grown varieties of Nigerian cocoa, namely, Amelonado and F_3 Amazon, were harvested from the Obafemi Awolowo University Teaching and Research Farm in Ile-Ife, Nigeria. The samples were selected to ensure that various sizes of the different varieties were represented, and only ripe healthy pods were used.

Test equipment and methodology

Quasi-static, uniaxial parallel plate compression test was performed on whole pod samples of the two varieties. Loading was done on the lateral axis. The machine used was the universal testing machine (proving ring model ELE 1152-8082). Before the tests, the axial length and diameter of each pod was measured. After loading to breakage, the contact areas of the pod surface with the plates were determined from an impression stained on sheets of white paper placed in-between them. An analog planimeter (ARISTO model) was used for evaluating the area. The force-deformation curves obtained were subsequently analysed to obtain the stiffness modulus, and the toughness - as the area under the curve.

RESULTS AND DISCUSSION

The basic breaking characteristics of cocoa pods are presented in Table 1. The mean values of the maximum breaking force, the deformation at failure, the toughness and the stiffness modulus are 0.715 ±0.038 kN, 7.6 ±0.37 mm, 2.74 ±0.27 J and 94.1 ±6.3 kN/m respectively for F_3 Amazon cocoa pods. The corresponding values for Amelonado cocoa pods are 0.553 ±0.034 kN, 6.38 ±0.44 mm, 1.70 ±0.16 J and 89.8 ±6.44 kN/m. F_3 Amazon cocoa pods offer greater resistance to breakage than Amelonado pods. This must be related to the higher thickness of F_3 Amazon pod husks [6].

Table 2 summarizes the application of Hertz theory to cocoa pod deformation. First the measured values (S_e), and the values obtained from the basic Hertz theory (S_c) were compared, using a two sample t-test. Stiffness

Table 1. Some breaking characteristics of cocoa pods in parallel plate compression test (crosshead speed = 4 mm/min)

F3 Amazon				Amelonado			
Breaking force F_{max}, (kN)	Deformation at failure D (mm)	Toughness (J)	Stiffness modulus (kN/m)	Breaking force F_{max} (kN)	Deformation at failure D (mm)	Toughness (J)	Stiffness modulus (kN/m)
0.89	7.5	3.34	118	0.48	5.5	1.32	87.3
0.87	7.5	-	116	0.30	7.0	1.05	42.9
0.69	7.5	2.58	92.0	0.42	4.6	0.97	91.3
0.45	5.0	1.13	90.0	0.63	6.0	1.89	105
0.69	7.5	2.59	92.0	0.68	6.0	2.04	113
0.96	6.5	3.12	148	0.36	6.0	1.08	60.0
0.57	7.5	2.14	76.0	0.47	8.0	1.88	58.8
0.96	8.5	-	113	0.53	4.5	1.19	118
0.60	7.5	2.25	80.0	0.54	8.0	2.16	67.5
0.60	6.5	1.95	92.3	0.66	6.5	2.15	102
0.71	12.5	4.44	56.8	0.59	5.5	1.62	106
0.54	6.0	1.62	90.0	0.35	7.5	1.31	46.7
0.63	7.0	2.21	90.0	0.47	4.0	0.94	116
0.86	9.0	4.05	100.0	0.74	5.5	1.96	140
0.90	7.5	-	60.0	0.77	7.0	2.70	110
0.46	7.0	-	65.7	0.75	9.0	3.38	83.3
0.88	10.0	4.40	88.0	0.51	5.0	1.28	102
0.90	5.5	2.48	164.0	0.84	12.5	-	67.2
0.63	6.5	-	96.9				
0.51	9.5	-	53.7				
x* 0.715	7.6	2.74	94.1	0.553	6.38	1.70	89.8
e* 0.038	0.37	0.27	6.3	0.034	0.44	0.16	6.44

x* - mean; e* - standard error.

modulus values S_c were obtained from Eq. (9), by substituting the various values of the pods physical dimensions, radii of curvature at the point of contact with loading plates, contact area at failure, the deformation of the pod, and the maximum load, and using a Poisson ratio of 0.30.

The tests show that the difference between the pairs of value was significant at the 99 % confidence level. The difference could be attributed to some of the assumptions of Hertz which were not quite justified for cocoa pods. Firstly, a cocoa pod is not a homogeneous isotropic material. Rather, it has been shown to be a composite fruit consisting of three distinct constituents, namely the beans, the central placenta with mucilage and the enclosing pod husk [6]. Secondly, the surface of the pod is rough and contoured with ridges and furrows, especially the F_3 Amazon variety, for which the greater disparity in measured and predicted values were recorded. Based on these considerations some indices were introduced to account for these factors.

Modification of the basic Hertz's equation

1. A thickness index/factor, C_t, was introduced to account for the hollow nature of the pod, since only the pod husk appears to offer any appreciable resistance to breakage [5]:

$$C_t = \frac{2t}{d} \tag{11}$$

where t - thickness of pod husk, d - diameter of pod at contact point.

2. A pod shape factor, C_a, was to take care of the deviation of the contact surface between the pod and the loading plates from the theoretical elliptical shape it was assumed to be on the basis of the pods being true ellipsoids:

$$C_a = \frac{\text{theoretical elliptic contact area}}{\text{actual contact area measured}} \tag{12}$$

3. A surface roughness index C_r, was introduced to account for the corrugation of the pod surface with furrows and ridges, and is defined as:

$$C_r = 1 - \frac{t_r - t_f}{R} \tag{13}$$

where t_r - thickness of the husk at the ridge, t_f - thickness of the husk at the furrow, R - radius of pod at contact point.

The effect of all three correcting indices on the Hertz's equation was analysed and a combined correction factor (C) was developed to improve the Hertz's relation:

$$C = \frac{C_a}{C_r (1 - C_t)^{\frac{1}{2}}} \tag{14}$$

The modified Hertz equation

The final form of the Hertz's relation for cocoa pod is as follows:

$$S = \frac{0.242\, C_a A_o \left(\frac{1}{R_1} + \frac{1}{R_1'}\right)^{\frac{1}{2}}}{R_1 D^{\frac{3}{2}} C_r (1 - C_t)^{\frac{1}{2}}} \tag{15}$$

or

$$S = \frac{0.242\, C A_o}{R_1 D^{\frac{2}{3}}} \left(\frac{1}{R_1} + \frac{1}{R_1'}\right)^{\frac{1}{2}} \tag{16}$$

All the symbols are as defined previously.

The values of the correcting indices and overall correcting factor as calculated are given in Table 2. The modified prediction of stiffness modulus is also summarized. Statistical analysis confirm the goodness of fit of the predicted values for both cocoa varieties at the 99 % confidence level.

An examination of the correcting indices indicate that C_t and C_r are fairly constant, being on the average, 0.273 ±0.005 and 0.972 ±0.003 respectively for F_3 Amazon, and 0.290 ±0.004 and 0.930 ±0.001 for the Amelonado variety. Using the mean values in Eq. (4), the combined correction factor (C) can be expressed in terms of only

Table 2. Comparison of measured and predicted values of stiffness modulus for F_3 Amazon and Amelonado cocoa pods

	Stiffness modulus (kN/m)		Correcting indices			Combined correcting factor, C	Improved calculated stiffness modulus, S_h (kN/m)
	Measured, S_e	Calculated, S_c	C_t	C_a	C_r		
				F_3 Amazon			
	118	64.7	0.29	1.60	0.99	1.92	124
	116	73.3	0.29	1.38	0.99	1.66	121
	92.0	59.2	0.29	1.31	0.98	1.58	93.6
	82.7	58.0	0.27	1.19	0.98	1.41	81.6
	113	71.9	0.28	1.41	0.98	1.69	122
	110	73.2	0.26	1.26	0.97	1.49	109
	58.8	42.1	0.23	1.17	0.97	1.36	57.2
	83.3	61.4	0.29	0.96	0.96	1.19	73.7
	67.2	36.4	0.28	1.52	0.95	1.72	68.7
	102	54.0	0.26	1.60	0.98	1.98	107
	90.0	53.5	0.28	1.45	0.98	1.75	93.6
	92.0	42.8	0.29	1.63	0.98	1.97	84.3
	148.0	59.4	0.29	1.95	0.97	2.39	142
	76.0	38.7	0.26	1.49	0.96	1.80	69.7
	47.1	16.3	0.24	2.12	0.95	2.56	41.7
x*	93.1	53.7	0.273	1.47	0.972		92.6
e*	6.7	4.1	0.005	0.08	0.003		7.3
				Amelonado			
	91.3	75.2	0.28	0.94	0.93	1.19	89.5
	88.0	42.2	0.27	1.49	0.92	1.90	80.2
	90.0	52.7	0.29	1.26	0.93	1.61	84.9
	87.3	68.8	0.32	0.91	0.92	1.20	82.5
	105	98.3	0.30	0.85	0.93	1.10	108
	113	83.2	0.30	0.97	0.93	1.38	114
	80.0	55.6	0.27	1.12	0.93	1.40	77.8
	90.0	54.0	0.29	1.35	0.94	1.72	92.8
	60.0	57.0	0.28	0.89	0.93	1.12	61.5
	60.0	36.1	0.28	1.36	0.93	1.72	62.2
	100	90.6	0.27	0.94	0.93	1.17	106
	60.0	33.2	0.29	1.43	0.93	1.83	60.5
	92.3	72.1	0.27	0.94	0.93	1.18	85.1
x*	85.9	63.0	0.290	1.11	0.930		85.0
e*	4.7	5.7	0.004	0.06	0.001		4.8

C_r - surface roughness index, C_t - thickness index, C_a - shape factor, x* and e* as in Table 1.

the shape factor C_a as follows:

for F_3 Amazon; $C=1.205\ C_a$

for Amelonado; $C=1.276\ C_a$.

This implies that only the shape factor need be determined thus reducing the measurements and computation needed to use the modified Hertz relation for analysis of cocoa pod deformation.

CONCLUSION

The Hertz contact stress theory was modified for use in the analysis of cocoa pod deformation by incorporating correcting indices which reflect the pods internal hollowness, surface roughness and irregular shape. The modified relation gave good prediction of the stiffness modulus of cocoa pods when subjected to compression between two rigid parallel plates. The shape factor was the most variable as the other two factors appear to be fairly constant for a given cocoa variety.

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