Int. Agrophysics 1996, 10, 123-129

FINITE ELEMENT MODELLING OF HEAT TRANSFER IN AVOCADOS

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Accepted March 11, 1996

A b s t r a c t. A two-dimensional finite element model was developed to predict temperature distribution in individual avocados subjected to air precooling. The proposed model, suitable for axisymmetrical shaped fruit, takes into account heat generation due to respiration, evaporative cooling effect due to transpiration as well as convection and radiation transfer on the fruit surface. The developed model was then applied to the cooling tests of avocado samples to estimating convection coefficient from experimentally measured temperature-time data for two locations within the fruit. An optimization procedure based on the minimization of the differences between experimental and calculated temperatures was used. The mean convection coefficients obtained for the two varieties (Fuerte, Hass) were not found to be significantly different (P=0.05). The values obtained using the finite element model were lower than those measured by an analytical method involving an aluminium avocado shaped model.

K e y w o r d s: precooling, heat transfer, modelling, finite element, avocado

INTRODUCTION

Over the last ten years, consumption of fresh tropical fruits, such as avocados, has rapidly increased in Western European countries and particularly in France. At present, temperature management is one of the essential factors required to extend the postharvest life of commodities and to supply distant markets with sound and attractive products.

For climacteric tropical fruits with a high respiration rate, such as avocado, it is essential to initiate the cold chain as soon as possible after picking. Fruits should be rapidly precooled to the recommended temperature to remove field heat and to reduce their metabolism. Avocados are cooled individually using either forced air cooling method or hydrocooling method. It is generally admitted that the greater the cooling rate, the more effective the precooling process. However, high cooling rates can lead to a significant loss of fruit quality caused by an increased development of chilling injury symptoms [16]. In the same way, chilling injury may occur during air cooling especially at low air humidities. Under these environmental conditions, the surface temperature of fruit can be significantly lower than ambient due to the evaporative cooling effect at the product surface [11,15]. Consequently, in order to maintain fruit quality during precooling, heat transfer rates and temperature distribution within the fruit and especially on its surface must be accurately controlled.

Many unsteady-state theoretical analyses of the cooling of fresh horticultural products have been proposed in the literature for individual products [4,8,13,14,18] as well as for fruits or vegetables in bulk [1,3,5,12]. In particular, the increasing use of accurate and powerful numerical methods, such as finite differences and finite elements, has led a number of researchers to develop complex but more realistic models. Such models can easily take into account heat generation due to respiration, evaporative cooling due to transpiration, as well as convection and radiation transfer at the surface. Nevertheless, even when such numerical methods are implemented, the accuracy of predicted temperature profiles may be limited because of the lack of thermal properties data or imprecise knowledge of cooling conditions [6].

For avocado, some experimental values and predictive equations have been published in the literature on thermal properties for stone and avocado pulp [17,19,20]. On the other hand, only limited information is available on transpiration and radiation coefficient as well as on convective heat transfer coefficient for anomalous shaped fruits such as avocado.

The objectives of this study were: (a) to develop a two-dimensional finite element model for simulating the transient temperature distribution in individual avocado subjected to air cooling, (b) to calculate the convection heat transfer coefficient from experimental temperature-time data using this model, and (c) to assess the convection coefficient estimation method by comparing the results with those obtained using a widely applied analytical method.

MATERIALS AND METHODS

Avocados

Cooling tests were carried out using mature fruits of cvs. Hass and Fuerte, obtained from the wholesale market. Average weight of fruit was 265.3 and 244.4 g, respectively for the two varieties. Fruit used for cooling tests were selected based on pulp firmness and axisymmetrical shape.

Cooling equipment

Cooling tests were achieved in a 222 L cooling chamber (LMS). Cooling air temperature was kept constant at 10 °C within ±0.4 °C. To increase the evaporation effect at the fruit surface, tests were carried out at about 20 % relative humidity. The partial water vapour pressure of the cooling air was measured using a gas analyzer (Brüel & Kjaer model 1302). The local air velocity was measured using an air flow sensor (TSI). The average value was 0.45 ms^{-1} with a standard deviation of 0.15 ms^{-1}.

Sample preparation and experimental procedure

To suspend the fruit sample in the cooling air stream, a small needle (0.5 mm diameter) was inserted along the revolution axis from the apical end of the whole fruit to the stone. Two sub-miniature thermocouple probes (Type K, 0.5 mm diameter) were then carefully placed in a longitudinal plane section, one in the pulp and the other in the stone. Lastly, to obtain an initial uniform temperature within the fruit prior to testing, each fruit sample was successively immersed in a constant temperature water bath (30 °C) for 3 h, rapidly drained and stored in a controlled temperature chamber at 30 °C to remove the residual moisture from the product surface.

The fruit sample was then suspended in the test section and experimental temperatures were recorded every minute using a digital data acquisition system (Campbell 21.X) until the stone temperature reached approximately 12 °C (dimensionless temperature of 0.1). After cooling, the fruit was longitudinally cut to determine both the precise geometric distribution of stone and pulp and the exact location of the tips of the thermocouples inserted within each of these two components.

Determination of physical properties

Since avocado is a non-homogeneous fruit, the physical properties of stone and pulp were determined separately. Stone and pulp density data were obtained by measuring successively the weight and the volume of the whole stone and the two unpeeled halves of the avocado. Specific heat and thermal conductivities of the two components were estimated using predictive equations proposed for avocado fruit and based on water and oil contents [19]. Respiration heat generation was calculated from the following equation based on experimental data obtained for avocados: $Qv = 513 - 142\,T + 12\,T^2 - 0.22\,T^3$ (W m^{-3}) [21]. The transpiration coefficient value used in the model was taken to be 0.96 10^{-9} kg $m^{-2}s^{-1}$ Pa^{-1} [21]. Water activity of avocado peel was assumed to be constant during the cooling process and equal to 0.98. The peel emissivity value was taken to be 0.9 [11].

Mathematical model

The partial differential equation governing heat conduction in a nonhomogeneous axisymmetric body, which generates internal heat due to respiration, can be written as follows:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + Q_v(T) \quad (1)$$

The initial and boundary conditions used for solving Eq.(1) are:

- uniform initial condition:

$$T = T_0 \text{ at } t = 0 \text{ and for all } z \text{ and } r \quad (2)$$

- symmetry boundary condition:

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0 \text{ for all } z \text{ and } t \quad (3)$$

- heat flux continuity condition at the pulp-stone interface:

$$\lambda_{\text{stone}} \frac{\partial T}{\partial n} = \lambda_{\text{pulp}} \frac{\partial T}{\partial n} \quad (4)$$

- and lastly, at the fruit-air interface:

$$-\lambda \frac{\partial T}{\partial n} = h(T_s - T_a) + \sigma F \varepsilon\,[(T_s + 273.15)^4 - (T_a + 273.15)^4] + K\,(Ps\,a_w - P_a)\,L(T_s) \quad (5)$$

where K is the overall transpiration coefficient [22] given by:

$$\frac{1}{K} = \frac{1}{K_F} + \frac{1}{K_a} \quad . \quad (6)$$

The mass transfer coefficient K_a due to the boundary layer is estimated from h assuming $F(Le)$=1 [7].

Numerical study

Finite element method

The theory and practice of finite element computation, an engineering tool of wide application, are well known [9]. The goal of the method is to reduce the complex problem of calculating the continuous field of temperature in the avocado to a discrete, and therefore, simpler one. Approximate values of the temperature are computed analytically at given points of the geometry: the nodes. Interpolation defines the field throughout the entire avocado when needed.

The different steps of the method are shortly summarized:

a) discretization of the domain into small interconnected subregions of simple geometry (triangles in our study) called finite elements;
b) choice of interpolating functions ('shape functions') in agreement with the type of finite elements;
c) finite element discretization of the transient equation of heat transfers, including boundary conditions.
The classical matrix form can be written:

$$[C]\left\{\frac{\partial T}{\partial t}\right\} + [\lambda]\{T\} = \{Q\} \quad . \quad (7)$$

d) The time discretization is a finite-difference scheme applied to the vector $\left\{\frac{\partial T}{\partial t}\right\}$ with the semi-implicit Euler method.

Finite element computations

The finite element code, developed in Pascal language, was used to build the geometrical model representing the quartered avocado. It was then discretized by the same code into 64 3-node triangular finite elements, the grid of 46 nodes taking into consideration the stone and flesh distribution (Fig. 1). The calculations were performed using a 10 s time step.

Estimation of convection heat transfer coefficient

Convection coefficient was determined by minimizing the difference between predicted

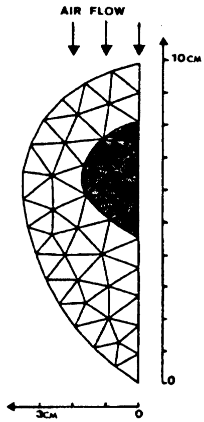

Fig. 1. Finite element discretization of the quartered avocado into triangular elements: dark grey and white elements represent stone and pulp tissues, respectively.

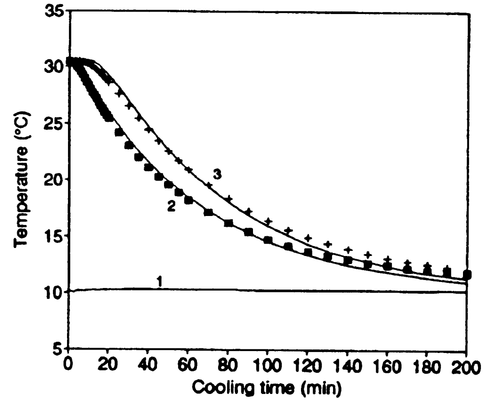

Fig. 2. Comparison of predicted and experimental temperatures within an avocado subjected to a cooling test at 10 °C and 20 % RH: 1) cooling air temperature; 2) pulp temperature (predicted —, experimental □); 3) stone temperature (predicted —, experimental +).

and experimental temperatures. A first solution from the model was calculated using an estimate coefficient value obtained from a Nu-Re-Pr correlation [11] for a sphere of the same volume. Then, various iterative coefficient values were used until the sum $S(h)$ of the squared residuals between experimental and calculated temperatures reached a minimum:

$$S(h)=\sum_i\sum_j(T_{i,j}^{\mathrm{exp}}-T_{i,j}^{\mathrm{calc}})^2$$

where i and j are related to the various temperature measurement points and to time steps, respectively.

Furthermore, to check the validity of the proposed method, the estimate value of h was compared against experimental data obtained for an aluminium object using a widely applied analytical method [2]. For this purpose, five replicate cooling tests were carried out, under the same condition as for the fresh fruit, for an axisymmetrical aluminium model of approximately the same shape and size as a typical avocado.

RESULTS AND DISCUSSION

Validity of the proposed model

Figure 2 shows a typical comparison between the experimentally measured temperatures and the predicted temperature-time curves obtained from the model for a Hass variety fruit. The composition and the thermal properties of the fruit are presented in Table 1. The calculated temperatures were obtained for an h value of 9.7 W m^{-2} $°C^{-1}$, giving the best agreement between predicted and experimental temperatures.

Analysis of the results reveals that: a) the numerical model leads to pulp temperature data higher than those actually measured, especially during the early stage of the cooling process, the maximum discrepancy not exceeding 1 °C for all cooling tests; b) a less marked difference is also observed for stone temperature (0.8 °C); c) the finite element model underestimates pulp and stone temperatures (0.8 °C) during the late stage of cooling process. The order of magnitude of these differences is close to the one reported for a very realistic and complex

Table 1. Composition and themal properties of a typical avocado

Sample characteristics	Pulp	Stone
Water content (%, w/w)	71.5	54.0
Oil content (%, w/w)	18.8	0
Thermal conductivity (Wm^{-1} $°C^{-1}$)	0.425	0.454
Specific heat (KJ kg^{-1} $°C^{-1}$)	3.460	2.765
Density (kg m^{-3})	977	1172

model developed for cooling of spherical horticultural products [13].

These differences may arise from the interplay of various factors not accounted for in the proposed model: (a) the effect of position varying convection coefficient precisely outlined for avocado [10]; (b) the effect of temperature on thermal properties, thermal conductivity of pulp decreasing by 7 % as temperature ranges from 30 to 10 °C; and (c) the effect of the accuracy of both the predictive equations used to estimate thermal properties (especially for specific heat and heat of respiration), and the data taken for transpiration coefficient and peel emissivity.

Estimation of convection coefficient

To assess the repeatability of the method for estimation convection coefficient, 5 consecutive cooling tests were run on the same fruit without changing the position of the thermocouple probes in the pulp or the stone. The mean convection coefficient value obtained for a sample of Hass variety was 9.6 W m^{-2} $°C^{-1}$, with a standard deviation of 0.6 W m^{-2} $°C^{-1}$. Since we used the same input parameters for simulating these cooling tests, the variability of the convection coefficient can only result from experimental conditions: errors of temperature measurement, poor humidity control especially at the beginning of the cooling test when the fruit is put into the cooling chamber. We also observed for some tests a non uniform initial temperature, although the maximum difference between the stone and the pulp was less than 0.4 °C.

The results obtained from two sets of 5 fruits of Hass and Fuerte varieties are shown in Table 2. It will be noted the variability observed for the Hass variety is higher than that observed for the Fuerte variety. This probably results from the differences in the roughness of the tested Hass fruit. This factor, in fact, increases the fruit's actual exchange area and thereby leads to an overestimation of the convection coefficient for the roughest fruits. In addition, accurate measurement of the real contours of the quartered avocado is impeded by the roughness of the peel, thereby affecting the accuracy of the cooling simulation. The mean convection coefficient of Hass and Fuerte varieties were 5 and 12.5 % lower, respectively, than the mean value obtained from the aluminium avocado shaped model. For this sample, both numeric and analytical methods led to similar values.

Better results could be obtained with fruits more similar in shape and in size if the thermal properties of the avocado's skin, pulp and stone and their variation with temperature were better known. In the same way, it would be interesting to classify the order of magnitude of the uncertainties in measuring convection coefficient derived from errors in the positioning of temperature probes or from the simplifying axisymmetric hypothesis.

Table 2. Estimation of convection coefficient for 3 objects subjected to a cooling treatment

Sample	Number of tests	Fruit weight (g)		Pulp total solids (%, w/w)		Pulp oil content (%, w/w)		Convection coefficient (W m^{-2} $°C^{-1}$)		Method used
		mean	(S.D.)*	mean	(S.D.)	mean	(S.D.)	mean	(S.D.)	
Hass	5	265.3	(14.3)	27.4	(1.2)	17.2	(1.2)	9.9	0.9	F.E.
Fuerte	5	244.4	(9.7)	27.1	(3.0)	17.5	(2.9)	9.1	0.6	F.E.
Avocado shaped model	5**	...	...	...	...	...	...	10.3	0.4	Analytical
Avocado shaped model	5**	...	...	...	...	...	...	10.5	0.4	F.E.

(*) - standard deviation; (**) - number of repetitions; F.E. - finite element.

CONCLUSIONS

This study shows the potential use of the finite element method for avocado precooling modelling. The proposed model independently takes into account convection, radiation and evaporation effect on the heat flow at the fruit surface. Besides, the model also takes into consideration the internal heat generation due to respiration. The method is suitable for axi-symmetrical shaped fruits, homogeneous or not in their makeup.

Temperature predictions of the model are in good agreement with the experimental values, the maximum discrepancy not exceeding 1 °C. The model is also used to estimate the convection coefficient from experimental temperature-time profiles for two avocado varieties. The convection coefficient values estimated by the numerical method from the fresh fruits are lower than those calculated using an aluminium avocado shaped model.

Such an approach outlines the strong need for improving the knowledge of the thermal properties of the studied fruit, including its peel emissivity, and its physiological behaviour to propose reliable data regarding transpiration coefficient and heat of respiration. In further work, we will study the sensitivity of the model for all the main input parameters in order to classify the different sources of error in estimating the convection coefficient with the proposed method.

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NOTATION

a_w	Water activity of avocado pulp
Cp	Specific heat capacity (J kg^{-1}°C^{-1})
$[C]$	Capacitance matrix
F	Shape factor (assumed equal to 1)
$F(Le)$	Lewis function
h	Convection heat transfer coefficient (W m^{-2} °C^{-1})
K	Overall mass transfer (kg m^{-2} s^{-1} °C^{-1})
K_F	Fruit transpiration coefficient (kg m^{-2} s^{-1} °C^{-1})
K_a	Mass transfer coefficient due to bonudary layer (kg m^{-2} s^{-1} °C^{-1})
L	latent heat of evaporation (J kg^{-1})
n	Vector normal to the surface
P_a	Partial water vapour pressure of cooling air (Pa)
P_s	Saturated water vapour pressure at the fruit surface (Pa)
Q_v	Heat generation due to respiration (W m^{-3})
$\{Q\}$	Vector of elementary volumic sources
r	Radial coordinate
$S(h)$	Sum of squared residuals between experimental and predicted temperatures (°C^2)
t	Time (s)
T	Product temperature (°C)
T_0	Initial temperature of the product (°C)
T_a	Cooling air temperature (°C)
T_s	Product surface temperature (°C)
$\{T\}$	Vector of unknown temperatures (nodal values)
z	Axial coordinate in Z direction (m)
ε	Emissivity of the product surface
λ	Thermal conductivity (W m^{-1}°C^{-1})
$[\lambda]$	Conductivity matrix
ρ	Density (kg m^{-3})
σ	Stefan-Boltzmann constant (W m^{-2} K^{-4})