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Numerical solution of the problem of spatial movement of a loose mixture in a vibrolot

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Summary. A computational solution of the spatial motion problem of a loose mixture in a vibrating tray by finite-difference method is obtained in the article.

The essence of the method consists in the fact that the differential operators regarding the spatial variables are replaced by finite-difference operators in the mesh points. In that time, the finite-difference operator is to be approximate the differential one as accurately as it possible. Finite-difference operators with the second order of accuracy are used.

Boundary conditions introduce changes in the coefficients of difference operators. We assume that these changes are already have been taken into account in the equations which correspond a system of ordinary differential equations relative to the vector variable. The dominant terms of the dynamics equations reckon in the summands of equations. They affect the convergence speed of the approximate solutions to the exact one. To the right-hand members the terms, which contain mixed derivatives, are reckon in. In this form, the equations are prepared for the application of the difference splitting method.

In order to obtain a stationary solution of the problem, the following points are used: the presence of dissipation in the problem (forces of internal and external friction) leads to the fact that under constant external conditions (the boundary conditions do not depend on time), the flow of the loose mixture stabilizes with time and irrespective of the initial conditions reach its stationary state. Therefore, the initial-boundary-value problem with invariable boundary conditions is formulated further. And the stationary solution is considered as asymptotic.

The multidimensionality of the problem leads to the need to solve systems containing a large number of equations. In addition, when solving the latter, numerical instability of the algorithm may arise is possible. One way of solving the equations is to move from differential equations to a system of algebraic equations by replacing the time-differentiation operator with finite difference. In the following, an implicit difference scheme corresponding to the Crank-Nicholson method with a double step in time, with a second order of accuracy, both in time and in spatial variables, is applied.

Keywords: spatial motion, bulk mixture, finite-difference method, differential operators**.**

INTRODUCTION

The research of the loading process of the working organs of grain separators requires consideration of the spatial movement of the loose mixture along their surfaces. During the work [1] the equations system of spatial motion of a loose mixture flow is obtained along the inclined vibro-sieve, and the boundary conditions on the surfaces which restrict the volume of the loose mixture are made. However, the resulting system of equations is quite complex and requires numerical solution.

ANALYSIS OF RESEARCHES AND PUBLICATIONS

There are no ready-made algorithms for solution of equation systems of spatial motion of a loose mixture. Solved problems like this have not been found in the literature.

Numerical methods for physical problems solving are presented in articles [2-7]. They present the foundations of numerical methods for systems of linear and nonlinear equations, as well as differential and integral equations.

The solution method of nonlinear boundary problems with an unknown (free) boundary are considered in the article [8]. The basic computational methods for solving stationary problems for elliptic equations of the second and fourth order are given. Separately, a class of inverse problems with a free boundary is singled out. The possibilities of developed methods for numerical solution of applied problems are presented.

A universal and effective method for problems solving in mathematical physics is the finite difference method or the net-point method [9-12]. It allows to reduce the approximate solution of the partial differential equations to the solution of algebraic equations systems. The multi-net-point iterative method is considered in the articles [13, 14]. It is one of the most widely used methods at the present time for solving net-point boundary problems.

THE PURPOSE OF THE RESEARCH

Numerical solution of the problem of spatial motion of a loose mixture in a vibro-sieve by a finite-difference method.

THE RESULTS OF RESEARCH

The equations system of spatial motion of a loose mixture obtained in research [1] is reduced to the equations of the planned flow:

$$\frac{\partial}{\partial t}\gamma + u\frac{\partial}{\partial x}\gamma + \mathrm{v}\frac{\partial}{\partial y}\gamma + \gamma\frac{\partial}{\partial x}u + \gamma\frac{\partial}{\partial y}\mathrm{v} = 0, \quad (1)$$

$$\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u + \mathrm{v}\frac{\partial}{\partial y}u + \frac{g\cos\theta}{2}\frac{\partial}{\partial x}h + \frac{hg\cos\theta}{2\gamma}\frac{\partial}{\partial x}\gamma -$$
$$-\frac{2\mu h}{\gamma}\frac{\partial^2}{\partial x^2}u - \frac{\mu h}{\gamma}\frac{\partial^2}{\partial y^2}u - \frac{2\mu}{\gamma}\frac{\partial}{\partial x}h\frac{\partial}{\partial x}u - \quad (2)$$
$$-\frac{\mu}{\gamma}\frac{\partial}{\partial y}h\frac{\partial}{\partial y}u - \frac{\mu}{\gamma}\frac{\partial}{\partial y}\left(h\frac{\partial}{\partial x}\mathrm{v}\right) + \frac{C_s}{\gamma}u - g\sin\theta = 0,$$

$$\frac{\partial}{\partial t}\mathrm{v} + u\frac{\partial}{\partial x}\mathrm{v} + \mathrm{v}\frac{\partial}{\partial y}\mathrm{v} + \frac{g\cos\theta}{2}\frac{\partial}{\partial y}h + \frac{hg\cos\theta}{2\gamma}\frac{\partial}{\partial y}\gamma -$$
$$-\frac{\mu h}{\gamma}\frac{\partial^2}{\partial x^2}\mathrm{v} - \frac{2\mu h}{\gamma}\frac{\partial^2}{\partial y^2}\mathrm{v} - \frac{\mu}{\gamma}\frac{\partial}{\partial x}h\frac{\partial}{\partial x}\mathrm{v} - \quad (3)$$
$$-\frac{2\mu}{\gamma}\frac{\partial}{\partial y}h\frac{\partial}{\partial y}\mathrm{v} - \frac{\mu}{\gamma}\frac{\partial}{\partial x}\left(h\frac{\partial}{\partial y}u\right) + \frac{C_s}{\gamma}\mathrm{v} = 0.$$

where: u, v – are the components of the velocity vector of the motion of a continuous medium; x, y – coordinates of the Cartesian coordinate system; θ – angle of inclination of the sieve; γ is the particles density of the mixture; h – thickness of the layer, counted along the normal to the bottom of the tray down to the free surface; t – time; μ – the dynamic shear-viscosity coefficient; C_s – a phenomenological coefficient, analogous to the Shezi coefficient.

The three equations (1-3) contain four unknown functions h, γ, u, v. For the closure of this equations system, a kinematic boundary condition is involved (here $w = 0$):

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + \mathrm{v}\frac{\partial h}{\partial y} = 0. \quad (4)$$

The area of attribution of unknown functions is the surface $\Sigma_0 = \{0 < x < l, -l_1/2 < y < l_1/2\}$ (fig. 1).

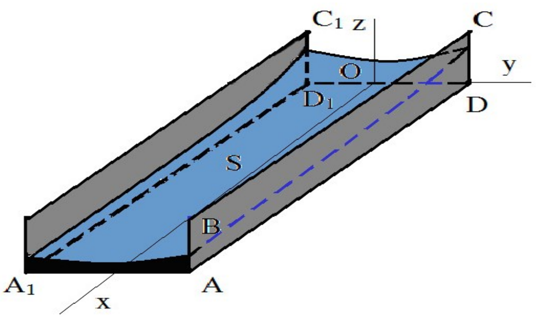

Fig. 1. Inclined vibrating tray

The boundary of this area consists of lines $L_1 = \{0 < x < l, y = -l_1/2\}$, $L_2 = \{0 < x < l, y = l_1/2\}$, $L_3 = \{x = 0, -l_1/2 < y < l_1/2\}$, $L_4 = \{x = l, -l_1/2 < y < l_1/2\}$,

Where l is the length of the tray, l_1 is the width of the tray.

Distributions are specified at the boundary L_3

$$h(t,0,y) = H^0(t,y), \quad \gamma(t,0,y) = G^0(t,y),$$
$$\mathrm{u}(t,0,y) = \mathrm{U}^0(t,y), \quad \mathrm{v}(t,0,y) = V^0(t,y). \quad (5)$$

Conditions:

$$\mathrm{v}(t,x,-l_1/2) = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=-l_1/2} - \left.\frac{C_s}{\mu}u\right|_{y=-l_1/2} = 0, \quad (6)$$

$$\mathrm{v}(t,x,l_1/2) = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=l_1/2} + \left.\frac{C_s}{\mu}u\right|_{y=l_1/2} = 0. \quad (7)$$

are fulfilled on lines L_1, L_2.

For numerical solution of the problem, let's use the finite-difference method [12, 15]. In the region Σ_0, let's introduce an analytical grid with constant pitch along $x: h_x = l/n_x$ and along $y: h_y = l_1/n_y$, where n_x, n_y - the number of nodes along the axis Ox, Oy, accordingly (Fig. 2).

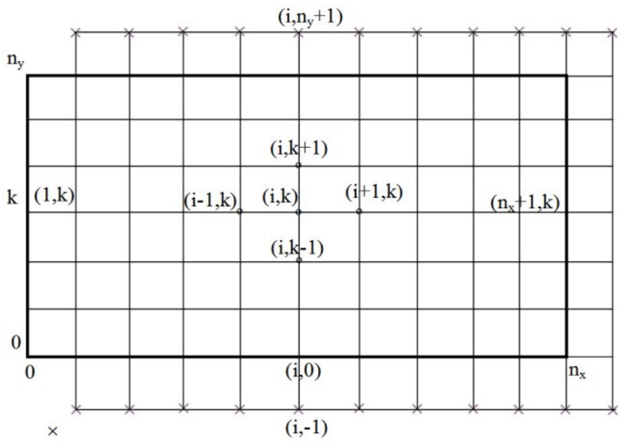

Fig. 2. Analytical grid

The nodes coordinate of the grid will be equal:

$$x_i = i\,h_x \ (i = \overline{0,n_x}), \ y_k = k\,h_y - l_1/2 \ (\mathrm{k} = \overline{0,n_y}).$$

The essence of the finite-difference method is that differential operators by spatial variables are replaced by finite-difference operators in the nodes of the grid. There are many ways of such substitution. And most importantly, the finite-difference operator is to be approximate differential one most accurately. Let's use finitely difference operators having the second order of accuracy:

$$\left(\frac{\partial u}{\partial x}\right)_{ik} \approx \frac{u_{i+1,k} - u_{i-1,k}}{2h_x}, \quad \left(\frac{\partial^2 u}{\partial x^2}\right)_{ik} \approx \frac{u_{i+1,k} - 2u_{ik} + u_{i-1,k}}{2h_x},$$

$$\left(\frac{\partial u}{\partial y}\right)_{ik} \approx \frac{u_{i,k+1} - u_{i,k-1}}{2h_y}, \quad \left(\frac{\partial^2 u}{\partial y^2}\right)_{ik} \approx \frac{u_{i,k+1} - 2u_{ik} + u_{i,k-1}}{2h_y}$$

The terms containing the mixed derivatives will be approximated as follows:

$$\left[\frac{\partial}{\partial y}\left(h\frac{\partial u}{\partial x}\right)\right]_{ik} \approx \frac{h_{i,k+1}\left(\mathrm{v}_{i+1,k+1} - \mathrm{v}_{i-1,k+1}\right) - h_{i,k-1}\left(\mathrm{v}_{i+1,k-1} - \mathrm{v}_{i-1,k-1}\right)}{4h_x h_y},$$

$$\left[\frac{\partial}{\partial x}\left(h\frac{\partial u}{\partial y}\right)\right]_{ik} \approx \frac{h_{i+1,k}\left(u_{i+1,k+1}-u_{i+1,k-1}\right)-h_{i-1,k}\left(u_{i-1,k+1}-u_{i-1,k}\right)}{4h_x h_y}.$$

The summands representing the convective derivative in equations (1-4) (the second and third summands in the left part of the equations which were specified) require a special approximation, what is related to the stability of finite-difference scheme. The authors of works [12, 16] propose the following approximation method, which we are going to consider exemplificative of a differential operator in the form of $u\partial u/\partial x$. If an actor $u \ge 0$, then it is recommended to take a difference operator sort of "back":

$$\left(u\frac{\partial u}{\partial x}\right)_{ik} \approx u_{ik}\frac{u_{ik}-u_{i-1,k}}{h_x}.$$

If the actor $u < 0$, then it is necessary to take a difference operator such sort of "forward":

$$\left(u\frac{\partial u}{\partial x}\right)_{ik} \approx u_{ik}\frac{u_{i+1,k}-u_{ik}}{h_x}.$$

In the case if u is the coefficient by a differential operator changes the sign from point to point, then it is necessary to represent this coefficient in the form of a sum of two summands $u^+ = \left(u+|u|\right)/2$ and $u^- = \left(u-|u|\right)/2$ corresponding to the positive and negative values of the function u, correspondingly. Then:

$$\left(u\frac{\partial u}{\partial x}\right)_{ik} = \left(u_{ik}^+ + u_{ik}^-\right)\left(\frac{\partial u}{\partial x}\right) = u_{ik}^+\left(\frac{\partial u}{\partial x}\right) + u_{ik}^-\left(\frac{\partial u}{\partial x}\right).$$

In this case also the first summand is approximated by the difference operator "back", and the second summand by the operator "forward":

$$\left(u\frac{\partial u}{\partial x}\right)_{ik} = u_{ik}^+\left(\frac{\partial u}{\partial x}\right) + u_{ik}^-\left(\frac{\partial u}{\partial x}\right) \approx u_{ik}^+\frac{u_{ik}-u_{i-1,k}}{h_x} + u_{ik}^-\frac{u_{i+1,k}-u_{i,k}}{h_x} = -\frac{u_{ik}^+}{h_x}u_{i-1,k} + \frac{|u_{ik}|}{h_x}u_{ik} + \frac{u_{ik}^-}{h_x}u_{i+1,k}. \quad (8)$$

Analogous actions can be committed for the operators of the form $\mathrm{v}\partial u/\partial y$:

$$\left(\mathrm{v}\frac{\partial u}{\partial y}\right)_{ik} = \mathrm{v}_{ik}^+\left(\frac{\partial u}{\partial y}\right) + \mathrm{v}_{ik}^-\left(\frac{\partial u}{\partial y}\right) \approx \mathrm{v}_{ik}^+\frac{u_{ik}-u_{i,k-1}}{h_y} + \mathrm{v}_{ik}^-\frac{u_{i,k+1}-u_{i,k}}{h_y} = -\frac{\mathrm{v}_{ik}^+}{h_y}u_{i-1,k} + \frac{|\mathrm{v}_{ik}|}{h_y}u_{ik} + \frac{\mathrm{v}_{ik}^-}{h_y}u_{i+1,k}. \quad (9)$$

It can be noted that the difference approximation of differential operators leads them to difference operators of the form:

$$u\frac{\partial u}{\partial x} \approx \overset{x}{a}_{ik}u_{i-1,k} + \overset{x}{c}_{ik}u_{ik} + \overset{x}{b}_{ik}u_{i+1,k}.$$

with coefficients:

$$\overset{x}{a}_{ik} = -\frac{u_{ik}^+}{h_x}, \quad \overset{x}{c}_{ik} = \frac{|u_{ik}|}{h_x}, \quad \overset{x}{b}_{ik} = \frac{u_{ik}^-}{h_x}.$$

etc.

Let's introduce an extended grid by adding nodes $\left\{x_i, y_{-1}\right\}_{i=1}^{n_x+1}$, $\left\{x_i, y_{n_y+1}\right\}_{i=1}^{n_x+1}$, $\left\{x_{n_x+1}, y_k\right\}_{k=-1}^{n_y+1}$ (fig. 2). Let's extend the equation (1-4) to the region. Thereby this attract attention to the additionally introduced nodes. In the approximation of differential operators at the boundary points $y = \pm l_1/2$, the functions in nodes extending beyond the region Σ_0^+ (on Fig. 2 they are marked by a cross) are defined as follows: h, γ expand evenly, v expand oddly, and u expand using boundary conditions (6, 7):

$$\begin{gathered} h_{i,-1} = h_{i,1}, \quad \gamma_{i,-1} = \gamma_{i,1}, \\ \mathrm{v}_{i,-1} = -\mathrm{v}_{i,1}, \quad u_{i,-1} = u_{i,1} - \frac{2h_y C_s}{\mu}u_{i0}, \\ h_{i,n_y+1} = h_{i,n_y-1}, \quad \gamma_{i,n_y+1} = \gamma_{i,n_y-1}, \\ \mathrm{v}_{i,n_y+1} = -\mathrm{v}_{i,n_y-1}, \quad u_{i,n_y+1} = u_{i,n_y-1} - \frac{2h_y C_s}{\mu}u_{n_y}, \\ \left(i = \overline{1, n_x+1}\right). \end{gathered} \quad (10)$$

For the boundary $x = l$, when determining the values of functions in nodes going beyond the region Σ_0^+, let's use the boundary conditions (10)

$$u_{n_x+1,k} = u_{n_x-1,k}, \quad \mathrm{v}_{n_x+1,k} = \mathrm{v}_{n_x-1,k} \quad \left(k = \overline{0, n_y}\right). \quad (11)$$

Let's write the difference equations for the node $\left\{x_i, y_k\right\}_{i=1,k=0}^{n_x,h_y}$ of the grid region corresponding to the equations (1-4)

$$\frac{d}{dt}h_{ik} + \overset{Ax}{a}_{ik}u_{i-1,k} + \overset{Ax}{c}_{ik}u_{i,k} + \overset{Ax}{b}_{ik}u_{i+1,k} + \overset{Ay}{a}_{ik}u_{i,k-1} + \overset{Ay}{c}_{ik}u_{i,k} + \overset{Ay}{b}_{ik}u_{i,k+1} = 0, \quad (12)$$

$$\frac{d}{dt}\gamma_{ik} + \overset{Ax}{a}_{ik}\gamma_{i-1,k} + \overset{Ax}{c}_{ik}\gamma_{i,k} + \overset{Ax}{b}_{ik}\gamma_{i+1,k} + \overset{Ay}{a}_{ik}\gamma_{i,k-1} + \overset{Ay}{c}_{ik}\gamma_{i,k} + \overset{Ay}{b}_{ik}\gamma_{i,k+1} + \overset{C2x}{a}_{ik}u_{i-1,k} + \overset{C2x}{c}_{ik}u_{i,k} + \overset{C2x}{b}_{ik}u_{i+1,k} + \overset{D2y}{a}_{ik}\mathrm{v}_{i,k-1} + \overset{D2y}{c}_{ik}\mathrm{v}_{i,k} + \overset{D2y}{b}_{ik}\mathrm{v}_{i,k+1} = 0, \quad (13)$$

$$\begin{gathered} \frac{d}{dt}u_{ik} + \overset{A3x}{a}_{ik}h_{i-1,k} + \overset{A3x}{c}_{ik}h_{i,k} + \overset{A3x}{b}_{ik}h_{i+1,k} + \overset{B3x}{a}_{ik}\gamma_{i-1,k} + \\ + \overset{B3x}{c}_{ik}\gamma_{i,k} + \overset{B3x}{b}_{ik}\gamma_{i+1,k} + \left(\overset{Ax}{a}_{ik} + \overset{C3x}{a}_{ik}\right)u_{i-1,k} + \\ + \left(\overset{Ax}{c}_{ik} + \overset{C3x}{c}_{ik} + \frac{C_s}{\gamma_{ik}}\right)u_{i,k} + \left(\overset{Ax}{b}_{ik} + \overset{C3x}{b}_{ik}\right)u_{i+1,k} + \\ + \left(\overset{Ay}{a}_{ik} + \overset{C3y}{a}_{ik}\right)u_{i,k-1} + \left(\overset{Ay}{c}_{ik} + \overset{C3y}{c}_{ik}\right)u_{i,k} + \\ + \left(\overset{Ay}{b}_{ik} + \overset{C3y}{b}_{ik}\right)u_{i,k+1} = F_{ik}^{(3)}, \end{gathered} \quad (14)$$

$$\frac{d}{dt}\mathrm{v}_{ik}+\overset{A4y}{a}_{ik}h_{i,k-1}+\overset{A4y}{c}_{ik}h_{i,k}+\overset{A4y}{b}_{i,k}h_{i,k+1}+\overset{B4y}{a}_{ik}\gamma_{i,k-1}+$$
$$+\overset{B4y}{c}_{ik}\gamma_{i,k}+\overset{B4y}{b}_{ik}\gamma_{i,k+1}+\left(\overset{Ax}{a}_{ik}+\overset{D4x}{a}_{ik}\right)\mathrm{v}_{i-1,k}+$$
$$+\left(\overset{Ax}{c}_{ik}+\overset{D4x}{c}_{ik}+\frac{C_s}{\gamma_{ik}}\right)\mathrm{v}_{i,k}+\left(\overset{Ax}{b}_{ik}+\overset{D4x}{b}_{ik}\right)\mathrm{v}_{i+1,k}+ \tag{15}$$
$$+\left(\overset{Ay}{a}_{ik}+\overset{D4y}{a}_{ik}\right)\mathrm{v}_{i,k-1}+\left(\overset{Ay}{c}_{ik}+\overset{D4y}{c}_{ik}\right)\mathrm{v}_{i,k}+$$
$$+\left(\overset{Ay}{b}_{ik}+\overset{D4y}{b}_{ik}\right)\mathrm{v}_{i,k+1}=F_{ik}^{(4)},$$

where :

$$\overset{Ax}{a}_{ik}=-\frac{u_{ik}^{+}}{h_x},\ \overset{Ax}{c}_{ik}=\frac{|u_{ik}|}{h_x},\ \overset{Ax}{b}_{ik}=\frac{u_{ik}^{-}}{h_x},$$
$$\overset{Ay}{a}_{ik}=-\frac{\mathrm{v}_{ik}^{+}}{h_x},\ \overset{Ay}{c}_{ik}=\frac{|\mathrm{v}_{ik}|}{h_x},\ \overset{Ay}{b}_{ik}=\frac{\mathrm{v}_{ik}^{-}}{h_x}.$$
$$\overset{C2x}{a}_{ik}=-\frac{u_{ik}^{+}}{h_x},\ \overset{C2x}{c}_{ik}=\frac{|u_{ik}|}{h_x},\ \overset{C2x}{b}_{ik}=\frac{u_{ik}^{-}}{h_x},$$
$$\overset{D2y}{a}_{ik}=-\frac{\mathrm{v}_{ik}^{+}}{h_x},\ \overset{D2y}{c}_{ik}=\frac{|\mathrm{v}_{ik}|}{h_x},\ \overset{D2y}{b}_{ik}=\frac{\mathrm{v}_{ik}^{-}}{h_x}.$$
$$\overset{A3x}{a}_{ik}=-\frac{g\cos\theta}{4h_x},\ \overset{A3x}{c}_{ik}=0,\ \overset{A3x}{b}_{ik}=\frac{g\cos\theta}{h_x},$$
$$\overset{B3x}{a}_{ik}=-\frac{h_{ik}g\cos\theta}{\gamma_{ik}h_x},\ \overset{B3x}{c}_{ik}=0,\ \overset{B3x}{b}_{ik}=\frac{h_{ik}g\cos\theta}{\gamma_{ik}h_x}.$$
$$\overset{C3x}{a}_{ik}=2\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},\ \overset{C3x}{c}_{ik}=-4\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},\ \overset{C3x}{b}_{ik}=2\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},$$
$$\overset{C3y}{a}_{ik}=\mu\frac{h_{ik}}{\gamma_{ik}h_x},\ \overset{C3y}{c}_{ik}=-2\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},\ \overset{C3y}{b}_{ik}=\mu\frac{h_{ik}}{\gamma_{ik}h_x^2}.$$
$$\overset{A4y}{a}_{ik}=-\frac{g\cos\theta}{4h_y},\ \overset{A4y}{c}_{ik}=0,\ \overset{A4y}{b}_{ik}=\frac{g\cos\theta}{4h_y},$$
$$\overset{B4y}{a}_{ik}=-\frac{h_{ik}g\cos\theta}{4\gamma_{ik}h_y},\ \overset{B4y}{c}_{ik}=0,\ \overset{B4y}{b}_{ik}=\frac{h_{ik}g\cos\theta}{4\gamma_{ik}h_y}.$$
$$\overset{D4x}{a}_{ik}=\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},\ \overset{D4x}{c}_{ik}=-2\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},\ \overset{D4x}{b}_{ik}=\mu\frac{h_{ik}}{\gamma_{ik}h_x^2},$$
$$\overset{D4y}{a}_{ik}=2\mu\frac{h_{ik}}{\gamma_{ik}h_y^2},\ \overset{D4y}{c}_{ik}=-4\mu\frac{h_{ik}}{\gamma_{ik}h_y^2},\ \overset{D4y}{b}_{ik}=2\mu\frac{h_{ik}}{\gamma_{ik}h_y^2}.$$

$$F_{ik}^{(3)}=g\sin\theta+\frac{2\mu}{\gamma_{ik}}\frac{\left(h_{i+1,k-1}-h_{i-1,k}\right)\left(u_{i+1,k}-u_{i-1,k}\right)}{4h_x^2}+$$
$$+\frac{\mu}{\gamma_{ik}}\frac{\left(h_{i,k+1}-h_{i,k-1}\right)\left(u_{i,k+1}-u_{i,k-1}\right)}{4h_y^2}+ \tag{16}$$
$$+\frac{\mu}{\gamma_{ik}}\frac{h_{i,k+1}\left(\mathrm{v}_{i+1,k+1}-\mathrm{v}_{i-1,k+1}\right)-h_{i,k-1}\left(\mathrm{v}_{i+1,k-1}-\mathrm{v}_{i-1,k-1}\right)}{4h_xh_y},$$

$$F_{ik}^{(4)}=\frac{2\mu}{\gamma_{ik}}\frac{\left(h_{i,k+1}-h_{i,k-1}\right)\left(\mathrm{v}_{i,k+1}-\mathrm{v}_{i,k-1}\right)}{4h_y^2}+$$
$$+\frac{\mu}{\gamma_{ik}}\frac{\left(h_{i,k+1}-h_{i,k-1}\right)\left(\mathrm{v}_{i+1,k}-\mathrm{v}_{i-1,k}\right)}{4h_x^2}+ \tag{17}$$
$$+\frac{\mu}{\gamma_{ik}}\frac{h_{i+1,k}\left(u_{i+1,k+1}-u_{i+1,k-1}\right)-h_{i-1,k}\left(\mathrm{u}_{i-1,k+1}-u_{i-1,k-1}\right)}{4h_xh_y}.$$

The equations (12-15) contain summand: $\overset{Ax}{a}_{1k}h_{0k}=\overset{Ax}{a}_{1k}H_0(t,0,y_k)$, $\overset{Ax}{a}_{1k}\gamma_{0k}=\overset{Ax}{a}_{1k}G_0(t,0,y_k)$,

$$\left(\overset{Ax}{a}_{1k}+\overset{C3x}{a}_{1k}\right)u_{0k}=\left(\overset{Ax}{a}_{1k}+\overset{C3x}{a}_{1k}\right)U_0(t,0,y_k),$$
$$\left(\overset{Ax}{a}_{1k}+\overset{D4x}{a}_{1k}\right)\mathrm{v}_{0k}=\left(\overset{Ax}{a}_{1k}+\overset{D4x}{a}_{1k}\right)V_0(t,0,y_k),$$

which contain the values of the functions specified on the boundary L_3. Let's put these summands to the right side of the equations (12-15):

$$f_{ik}^{(1)}=-\overset{Ax}{a}_{1k}H_0(t,0,y_k),$$
$$f_{ik}^{(2)}=-\overset{Ax}{a}_{1k}G_0(t,0,y_k)-\overset{C2x}{a}_{1k}U_0(t,0,y_k),$$
$$f_{ik}^{(3)}=F_{ik}^{(3)}-\overset{A3x}{a}_{1k}H_0(t,0,y_k)-\overset{B3x}{a}_{1k}G_0(t,0,y_k)- \tag{18}$$
$$-\left(\overset{Ax}{a}_{1k}+\overset{C3x}{a}_{1k}\right)U_0(t,0,y_k),$$
$$f_{ik}^{(4)}=F_{ik}^{(4)}-\left(\overset{Ax}{a}_{1k}+\overset{D4x}{a}_{1k}\right)V_0(t,0,y_k).$$

The boundary conditions (10) introduce changes in the coefficients of difference operators. We assume that these changes are already have been taken into account in the equations. These equations are a system of ordinary differential equations relative to the vector variable:

$$Z=\begin{pmatrix}H\\G\\U\\V\end{pmatrix},\quad H=\{h_{ik}\}_{i=1,k=0}^{n_x,n_y},\quad G=\{\gamma_{ik}\}_{i=1,k=0}^{n_x,n_y}, \tag{19}$$
$$U=\{u_{ik}\}_{i=1,k=0}^{n_x,n_y},\quad V=\{\mathrm{v}_{ik}\}_{i=1,k=0}^{n_x,n_y},$$

and they have the form:

$$\frac{dZ}{dt}+\Omega Z=F, \tag{20}$$

Where the matrix Ω has a following structure:

$$\Omega=\begin{pmatrix}\overset{11}{\Omega}&0&0&0\\0&\overset{22}{\Omega}&\overset{23}{\Omega}&\overset{24}{\Omega}\\\overset{31}{\Omega}&\overset{32}{\Omega}&\overset{33}{\Omega}&0\\\overset{41}{\Omega}&\overset{42}{\Omega}&0&\overset{44}{\Omega}\end{pmatrix} \tag{21}$$

The right sides are equal:

$$F = \begin{pmatrix} \overset{1}{F} \\ \overset{2}{F} \\ \overset{3}{F} \\ \overset{4}{F} \end{pmatrix} \qquad (22)$$

$$\begin{aligned} &\overset{11}{\Omega} = A^{(x)} + A^{(y)} \quad \Omega^{22} = A^{(x)} + A^{(y)} \quad \Omega^{23} = C^{(2x)} \quad \Omega^{24} = D^{(2y)} \\ &\overset{31}{\Omega} = A^{(3x)} \quad \overset{32}{\Omega} = B^{(3x)} \quad \overset{33}{\Omega} = C^{(3x)} + C^{(3y)} \\ &\overset{41}{\Omega} = A^{(4y)} \quad \overset{42}{\Omega} = B^{(4y)} \quad \overset{44}{\Omega} = D^{(4x)} + D^{(4y)}, \end{aligned} \qquad (23)$$

$$\begin{aligned} &\overset{1}{F} = \left\{ \overset{1}{f}_{ik} \right\}_{i=1,k=0}^{n_x,n_y}, \quad \overset{2}{F} = \left\{ \overset{2}{f}_{ik} \right\}_{i=1,k=0}^{n_x,n_y}, \\ &\overset{3}{F} = \left\{ \overset{3}{f}_{ik} \right\}_{i=1,k=0}^{n_x,n_y}, \quad \overset{4}{F} = \left\{ \overset{4}{f}_{ik} \right\}_{i=1,k=0}^{n_x,n_y}. \end{aligned} \qquad (24)$$

The action of the matrices which enter into the right sides of relations (23) is determined by form

$$\begin{aligned} &\left(A^{(x)}H\right)_{ik} = \overset{Ax}{a}_{ik} h_{i-1,k} + \overset{Ax}{c}_{ik} h_{i,k} + \overset{Ax}{b}_{ik} h_{i+1,k}, \\ &\left(A^{(y)}H\right)_{ik} = \overset{Ay}{a}_{ik} h_{i,k-1} + \overset{Ay}{c}_{ik} h_{i,k} + \overset{Ay}{b}_{ik} h_{i,k+1}. \end{aligned} \qquad (25)$$

$$\begin{aligned} &\left(C^{(2x)}U\right)_{ik} = \overset{C2x}{a}_{ik} u_{i-1,k} + \overset{C2x}{c}_{ik} u_{i,k} + \overset{C2x}{b}_{ik} u_{i+1,k}, \\ &\left(D^{(2y)}V\right)_{ik} = \overset{D2y}{a}_{ik} \mathrm{v}_{i,k-1} + \overset{D2y}{c}_{ik} \mathrm{v}_{i,k} + \overset{D2y}{b}_{ik} \mathrm{v}_{i,k+1}. \end{aligned} \qquad (26)$$

$$\begin{aligned} &\left(A^{(3x)}H\right)_{ik} = \overset{A3x}{a}_{ik} h_{i-1,k} + \overset{A3x}{c}_{ik} h_{i,k} + \overset{A3x}{b}_{ik} h_{i+1,k}, \\ &\left(B^{(3x)}G\right)_{ik} = \overset{B3x}{a}_{ik} \gamma_{i,k-1} + \overset{B3x}{c}_{ik} \gamma_{i,k} + \overset{B3x}{b}_{ik} \gamma_{i,k+1} \\ &\left(C^{(3x}U\right)_{ik} = \overset{C3x}{a}_{ik} u_{i,k-1} + \overset{C3x}{c}_{ik} u_{i,k} + \overset{C3x}{b}_{ik} u_{i,k+1}, \\ &\left(C^{(3y}U\right)_{ik} = \overset{C3y}{a}_{ik} u_{i,k-1} + \overset{C3y}{c}_{ik} u_{i,k} + \overset{C3y}{b}_{ik} u_{i,k+1}. \end{aligned} \qquad (27)$$

$$\begin{aligned} &\left(A^{(\mathrm{A}4y)}H\right)_{ik} = \overset{\mathrm{A}4y}{a}_{ik} h_{i-1,k} + \overset{\mathrm{A}4y}{c}_{ik} h_{i,k} + \overset{\mathrm{A}4y}{b}_{ik} h_{i+1,k}, \\ &\left(B^{(\mathrm{B}4y)}G\right)_{ik} = \overset{B4y}{a}_{ik} \gamma_{i,k-1} + \overset{B4y}{c}_{ik} \gamma_{i,k} + \overset{B4y}{b}_{ik} \gamma_{i,k+1} \\ &\left(D^{(D4x}V\right)_{ik} = \overset{D4x}{a}_{ik} \mathrm{v}_{i,k-1} + \overset{D4x}{c}_{ik} \mathrm{v}_{i,k} + \overset{D4x}{b}_{ik} \mathrm{v}_{i,k+1}, \\ &\left(D^{(D4y}V\right)_{ik} = \overset{D4y}{a}_{ik} \mathrm{v}_{i,k-1} + \overset{D4y}{c}_{ik} \mathrm{v}_{i,k} + \overset{D4y}{b}_{ik} \mathrm{v}_{i,k+1}. \end{aligned} \qquad (28)$$

$$\begin{aligned} &\overset{Ax}{a}_{ik} = -\frac{u_{ik} + |u_{ik}|}{h_x}, \quad \overset{Ax}{c}_{ik} = \frac{|u_{ik}|}{h_x}, \quad \overset{Ax}{b}_{ik} = \frac{u_{ik} - |u_{ik}|}{h_x}, \\ &\overset{Ay}{a}_{ik} = \frac{\mathrm{v}_{ik} - |\mathrm{v}_{ik}|}{h_y}, \quad \overset{Ay}{c}_{ik} = \frac{\mathrm{v}_{ik} - |\mathrm{v}_{ik}|}{h_y}, \quad \overset{Ay}{b}_{ik} = \frac{\mathrm{v}_{ik} - |\mathrm{v}_{ik}|}{h_y}. \end{aligned} \qquad (29)$$

$$\begin{aligned} &\overset{C2x}{a}_{ik} = -\frac{\gamma_{ik}}{2h_x}, \quad \overset{Ax}{c}_{ik} = 0, \quad \overset{Ax}{b}_{ik} = \frac{\gamma_{ik}}{2h_x}, \\ &\overset{D2y}{a}_{ik} = -\frac{\gamma_{ik}}{2h_y}, \quad \overset{D2y}{c}_{ik} = 0, \quad \overset{D2y}{b}_{ik} = \frac{\gamma_{ik}}{2h_y}. \end{aligned} \qquad (30)$$

$$\begin{aligned} &\overset{A3x}{a}_{ik} = -\frac{g\cos\theta}{2h_x}, \; \overset{A3x}{c}_{ik} = 0, \; \overset{A3x}{b}_{ik} = \frac{g\cos\theta}{2h_x}, \\ &\overset{B3x}{a}_{ik} = -\frac{h_{ik} g\cos\theta}{2h_y\gamma_{ik}}, \; \overset{B3x}{c}_{ik} = 0, \; \overset{B3x}{b}_{ik} = \frac{h_{ik} g\cos\theta}{2h_y\gamma_{ik}}, \\ &\overset{C3x}{a}_{ik} = 2\mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \; \overset{C3x}{c}_{ik} = -4\mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \; \overset{C3x}{b}_{ik} = 2\mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \\ &\overset{C3y}{a}_{ik} = \mu\frac{h_{ik}}{h_y^2\gamma_{ik}}, \; \overset{C3y}{c}_{ik} = -2\mu\frac{h_{ik}}{h_y^2\gamma_{ik}}, \; \overset{C3y}{b}_{ik} = \mu\frac{h_{ik}}{h_y^2\gamma_{ik}}. \end{aligned} \qquad (31)$$

$$\begin{aligned} &\overset{A4y}{a}_{ik} = -\frac{g\cos\theta}{2h_y}, \; \overset{A4y}{c}_{ik} = 0, \; \overset{A4y}{b}_{ik} = \frac{g\cos\theta}{2h_y}, \\ &\overset{B4x}{a}_{ik} = -\frac{h_{ik} g\cos\theta}{2h_y\gamma_{ik}}, \; \overset{B4y}{c}_{ik} = 0, \; \overset{B4y}{b}_{ik} = \frac{h_{ik} g\cos\theta}{2h_y\gamma_{ik}}, \\ &\overset{D4x}{a}_{ik} = \mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \; \overset{D4x}{c}_{ik} = -2\mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \; \overset{D4x}{b}_{ik} = \mu\frac{h_{ik}}{h_x^2\gamma_{ik}}, \\ &\overset{D4y}{a}_{ik} = 2\mu\frac{h_{ik}}{h_y^2\gamma_{ik}}, \; \overset{D4y}{c}_{ik} = -4\mu\frac{h_{ik}}{h_y^2\gamma_{ik}}, \; \overset{D4y}{b}_{ik} = \mu\frac{h_{ik}}{h_y^2\gamma_{ik}}. \end{aligned} \qquad (32)$$

The structure of equation (20) is chosen in compliance with the recommendations of the authors of papers [16, 17]. Here, the main summands of the dynamics equations, which influence on rapidity of convergence of approximate solutions to the exact solution, are related to the summand ΩZ. Summands that contain mixed derivatives are related to the right sides of equations. In this form, the equations have been prepared for the application of the difference splitting method.

In order to obtain a stationary solution of the problem it is possible to use the following considerations: the presence of dissipation in the sum (of internal and external friction forces) leads to the fact that under constant external conditions (the boundary conditions do not depend on time), the system (flow CC) with time stabilizes. And irrespective of the initial conditions with time it comes to its stationary state. Therefore, the initial-boundary-value problem with boundary conditions is defined further. And the stationary solution is considered as asymptotic with $t \to \infty$.

It should be noted that the motion of a thin layer along a solid surface can be unstable, as it occurs in the case of a flowing thin film of water [18].

The multidimensionality of the problem $(n = 2,3)$ result in the necessity of systems solution containing a large number of equations of the form (20). Moreover, when solve the latest, a numerical instability of the algorithm may occur [12, 15]. One of the methods for solution of equations (20) consists in the transition from differential equations to a system of algebraic equations by replacing the time-differentiation operator with finite-difference one. Functions $Z = Z(t)$ are considered at some finite interval of time $[0,T]$. This interval is divided into small subintervals $[t^j, t^{j+1}]$ with a certain pitch in time $\tau^j = t^{j+1} - t^j$. This pitch can be constant. In this case let's not specify the top index $\tau^j \equiv \tau$. Time differentiation is replaced by a difference operator:

$$\frac{dZ}{dt}\approx\frac{Z^{i+1}-Z^{j}}{\tau},$$

and equation (20) is replaced by the system of finite-difference equations:

$$\frac{Z^{j+1}-Z^{j}}{\tau}+\tilde{\Omega}Z=\tilde{F}\cdot \tag{33}$$

The operator $\tilde{\Omega}$ and the right side $\tilde{F}$ are chosen so that the approximation of equation (20) would have a maximal order, and the sequence determination process Z^{j} would be stable. Here explicit and implicit difference schemes can be applied, when:

$$\tilde{\Omega}Z=\Omega Z\Big|^{t=t^{j}},\quad \tilde{F}=F\Big|^{t=t^{j}}, \tag{34}$$¶

$$\tilde{\Omega}Z=\Omega Z\Big|^{t=t^{j+1}},\quad \tilde{F}=F\Big|^{t=t^{j+1}}. \tag{35}$$¶

In the case of an explicit scheme, equation (33) is reduced to a recurrence relation:

$$Z^{j+1}=Z^{j}+\tau\left(F^{j+1}-\Omega^{j+1}Z^{j}\right),\ Z^{0}=Z(0)\ (j=0,1,...), \tag{36}$$

which gives the simplest way of solution of difference equations. However, the algorithm in this case is conditionally stable. There must be some relation between time pitch τ and spatial pitch h_x, h_y, which results in the necessity of making very small pitches in time. In case of violation of the indicated mentioned relation, the numerical instability of the algorithm occurs. The implicit scheme, as a rule, is stable. Calculations can be carried out with a sufficiently great time pitch. These assertions are based on the available theorems of the convergence and stability of the concerned algorithms. Unfortunately, these theorems are proved only for linear equations of the standard type (hyperbolic, parabolic). In the case of nonlinear equations of non-standard type, we have to apply the proper algorithms by touch, conducting the proper numerical experiments. In this case, the justification of the application of one or another algorithm is the "plausibility" of the obtained result, which is heuristic in nature.

In the future we will apply the implicit difference scheme which is proper to the Crank-Nicholson method with a double time pitch:

$$\frac{Z^{j+1}-Z^{j-1}}{2\tau}+\frac{1}{2}\left[(\Omega Z)\Big|^{j+1}+(\Omega Z)\Big|^{j-1}\right]=F(t_j+\tau), \tag{37}$$

having a second order of accuracy by the time and spatial variables. Here the vector Z^{j+1} is unknown and the system is nonlinear. Linearization of systems is carried out. Each summand in the operator $(\Omega Z)^{j+1}$ has the form of a product of some function of these unknowns (of coefficient) on a finite-difference operator of numerical differentiation. So this coefficient is calculated by the way of the functions which are specified at the time station t^{j} These functions are determined by the way of one time pitch τ per an explicit scheme ("zero" pitch) according to the recurrence relation (36):

$$\tilde{Z}^{j}=Z^{j-1}+\tau\left(F^{j-1}-\Omega^{j-1}Z^{j-1}\right). \tag{38}$$¶

Let's agreed notation:¶

$$\Omega^{j}=\Omega\left(t^{j},\tilde{Z}^{j}\right),\quad F^{j}=F\left(t^{j},\tilde{Z}^{j}\right).$$¶

If we solve equation (37) relative to an unknown vector Z^{j+1}:

$$\left(E+\tau\Omega^{j}\right)Z^{j+1}=\left(E-\tau\Omega^{j}\right)Z^{j-1}+2\tau F^{j}. \tag{39}$$

We will obtain the equations which, according to (21-24), take on form:

$$\left(E^{(1)}+\tau\overset{11}{\Omega}{}^{j}\right)H^{j+1}=\left(E^{(1)}-\tau\overset{11}{\Omega}{}^{j}\right)H^{j-1}+2\tau\overset{1}{F}{}^{j}, \tag{40}$$

$$\left(\overset{1}{E}+\tau\overset{11}{\Omega}{}^{j}\right)H^{j+1}=\left(\overset{1}{E}-\tau\overset{11}{\Omega}{}^{j}\right)H^{j-1}+2\tau\overset{1}{F}{}^{j},$$

$$\left(\overset{2}{E}+\tau\overset{22}{\Omega}{}^{j}\right)G^{j+1}+\tau\overset{23}{\Omega}{}^{j}U^{j+1}+\tau\overset{24}{\Omega}{}^{j}V^{j+1}=$$

$$=\left(\overset{2}{E}-\tau\overset{22}{\Omega}{}^{j}\right)H^{j-1}+\tau\overset{23}{\Omega}{}^{j}U^{j-1}+\tau\overset{24}{\Omega}{}^{j}V^{j-1}+2\tau\overset{1}{F}{}^{j},$$

$$\left(\overset{3}{E}+\tau\overset{22}{\Omega}{}^{j}\right)G^{j+1}+\tau\overset{23}{\Omega}{}^{j}U^{j+1}+\tau\overset{24}{\Omega}{}^{j}V^{j+1}= \tag{41}$$

$$=\left(\overset{2}{E}-\tau\overset{22}{\Omega}{}^{j}\right)H^{j-1}+\tau\overset{23}{\Omega}{}^{j}U^{j-1}+\tau\overset{24}{\Omega}{}^{j}V^{j-1}+2\tau\overset{1}{F}{}^{j},$$

$$\tau\overset{41}{\Omega}{}^{j}H^{j+1}+\tau\overset{42}{\Omega}{}^{j}G^{j+1}+\left(\overset{4}{E}+\tau\overset{44}{\Omega}{}^{j}\right)V^{j+1}=$$

$$=\tau\overset{41}{\Omega}{}^{j}H^{j-1}+\tau\overset{42}{\Omega}{}^{j}U^{j-1}+\left(\overset{4}{E}-\tau\overset{44}{\Omega}{}^{j}\right)V^{j-1}+2\tau\overset{1}{F}{}^{j}.$$

where: $\left(\overset{1}{E}H\right)_{ik}=h_{ik}$.

To avoid the necessity to solve systems (39) of high-order, let's use the two-cyclic multicomponent splitting method [12]. Let's represent a matrix Ω in the form of a sum:

$$\Omega=\overset{1}{\Omega}+\overset{2}{\Omega}+\overset{3}{\Omega}+\overset{4}{\Omega}, \tag{42}$$

where,

$$\overset{1}{\Omega}=\begin{pmatrix} A^{(x)} & 0 & 0 & 0\\ 0 & A^{(x)} & & 0\\ 0 & 0 & A^{(x)}+C^{(3x)}+C_sI & 0\\ 0 & 0 & 0 & A^{(x)}+D^{(4x)}\end{pmatrix}, \tag{43}$$

$$\overset{2}{\Omega}=\begin{pmatrix} A^{()y} & 0 & 0 & 0\\ 0 & A^{(y)} & & 0\\ 0 & 0 & A^{(y)}+C^{(3y)} & 0\\ 0 & 0 & 0 & A^{(y)}+D^{(4y)}+C_sI\end{pmatrix}, \tag{44}$$

$$\overset{3}{\Omega}=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & & C^{(2x)} & 0\\ A^{(3x)} & B^{(3x)} & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}, \tag{45}$$

$$\overset{4}{\Omega}=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & & & D^{(4y)}\\ 0 & 0 & 0 & 0\\ A^{(4y)} & B^{(4y)} & 0 & 0\end{pmatrix}, \tag{46}$$

$$\left(C_s IU\right)_{ik}=\frac{C_s}{\gamma_{ik}}u_{ik},\quad \left(C_s IV\right)_{ik}=\frac{C_s}{\gamma_{ik}}\mathrm{v}_{ik}. \qquad (47)$$

As a zero pitch, let's consider the definition of the vector $\tilde{Z}$ by the way of relation (36). Then we divide the interval $\left[t^{j-1},t^{j+1}\right]$ into equal subinterval ("fractional pitch") $\left[t^{j-4/5},t^{j-1}\right]$, $\left[t^{j-3/5},t^{j-4/5}\right]$, $\left[t^{j-2/5},t^{j-3/5}\right]$, $\left[t^{j-1/5},t^{j-2/5}\right]$, $\left[t^{j+1/5},t^{j-1/5}\right]$, $\left[t^{j+2/5},t^{j+1/5}\right]$, $\left[t^{j+3/5},t^{j+2/5}\right]$, $\left[t^{j+4/5},t^{j+3/5}\right]$, $\left[t^{j+1},t^{j+4/5}\right]$ and on in each subinterval we will solve the following corresponding equations

$$\left(E+\frac{\tau}{2}\overset{1}{\Omega}\right)Z^{j-4/5}=\left(E-\frac{\tau}{2}\overset{1}{\Omega}\right)Z^{j-1}\ \text{(first pitch)} \qquad (48)$$

$$\left(E+\frac{\tau}{2}\overset{2}{\Omega}\right)Z^{j-3/5}=\left(E-\frac{\tau}{2}\overset{2}{\Omega}\right)Z^{j-4/5}\ \text{(second pitch)} \qquad (49)$$

$$\left(E+\frac{\tau}{2}\overset{3}{\Omega}\right)Z^{j-2/5}=\left(E-\frac{\tau}{2}\overset{3}{\Omega}\right)Z^{j-3/5}\ \text{(third pitch)} \qquad (50)$$

$$\left(E+\frac{\tau}{2}\overset{4}{\Omega}\right)Z^{j-1/5}=\left(E-\frac{\tau}{2}\overset{4}{\Omega}\right)Z^{j-2/5}\ \text{(fourth pitch)} \qquad (51)$$

$$Z^{j+1/5}=Z^{j-1/5}+2\tau F^{j}\ \text{(fifth pitch)} \qquad (52)$$

$$\left(E+\frac{\tau}{2}\overset{4}{\Omega}\right)Z^{j+2/5}=\left(E-\frac{\tau}{2}\overset{4}{\Omega}\right)Z^{j+1/5}\ \text{(sixth pitch)} \qquad (53)$$

$$\left(E+\frac{\tau}{2}\overset{3}{\Omega}\right)Z^{j+3/5}=\left(E-\frac{\tau}{2}\overset{3}{\Omega}\right)Z^{j+2/5}\ \text{(seventh pitch)} \qquad (54)$$

$$\left(E+\frac{\tau}{2}\overset{2}{\Omega}\right)Z^{j+4/5}=\left(E-\frac{\tau}{2}\overset{2}{\Omega}\right)Z^{j+3/5}\ \text{(eighth pitch)} \qquad (55)$$

$$\left(E+\frac{\tau}{2}\overset{1}{\Omega}\right)Z^{j+1}=\left(E-\frac{\tau}{2}\overset{1}{\Omega}\right)Z^{j+4/5}\ \text{(ninth pitch)} \qquad (56)$$

It is proved that the system of equations (48-56) is equivalent to equations (37) accurate within member of order $O\left(\tau^2\right)$ [12].

First and second fractional pitches

If the index notation of the quantities is used, then equation (48) can be written in the form

$$\begin{aligned}
&\frac{\tau}{2}\overset{Ax}{a}_{ik}\,h_{i-1,k}^{j-4/5}+\left(1+\frac{\tau}{2}\overset{Ax}{c}_{ik}\right)h_{i,k}^{j-4/5}+\frac{\tau}{2}\overset{Ax}{b}_{ik}\,h_{i+1,k}^{j-4/5}=\\
&=-\frac{\tau}{2}\overset{Ax}{a}_{ik}\,h_{i-1,k}^{j-1}+\left(1-\frac{\tau}{2}\overset{Ax}{c}_{ik}\right)h_{i,k}^{j-1}-\frac{\tau}{2}\overset{Ax}{b}_{ik}\,h_{i+1,k}^{j-1},\\
&\frac{\tau}{2}\overset{Ax}{a}_{ik}\,\gamma_{i-1,k}^{j-4/5}+\left(1+\frac{\tau}{2}\overset{Ax}{c}_{ik}\right)\gamma_{i,k}^{j-4/5}+\frac{\tau}{2}\overset{Ax}{b}_{ik}\,\gamma_{i+1,k}^{j-4/5}=\\
&=-\frac{\tau}{2}\overset{Ax}{a}_{ik}\,\gamma_{i-1,k}^{j-1}+\left(1-\frac{\tau}{2}\overset{Ax}{c}_{ik}\right)\gamma_{i,k}^{j-1}-\frac{\tau}{2}\overset{Ax}{b}_{ik}\,\gamma_{i+1,k}^{j-1}.
\end{aligned} \qquad (57)$$

$$\begin{aligned}
&\frac{\tau}{2}\left(\overset{Ax}{a}_{ik}+\overset{C3x}{a}_{ik}\right)u_{i-1,k}^{j-4/5}+\left(1+\frac{\tau}{2}\left(\overset{Ax}{c}_{ik}+\overset{C3x}{c}_{ik}+\frac{C_s}{\gamma_{ik}^{j}}\right)\right)u_{i,k}^{j-4/5}+\\
&+\frac{\tau}{2}\left(\overset{Ax}{b}_{ik}+\overset{C3x}{b}_{ik}\right)u_{i+1,k}^{j-4/5}=-\frac{\tau}{2}\left(\overset{Ax}{a}_{ik}+\overset{C3x}{a}_{ik}\right)u_{i-1,k}^{j-1}+\\
&+\left(1-\frac{\tau}{2}\left(\overset{Ax}{c}_{ik}+\overset{C3x}{c}_{ik}+\frac{C_s}{\gamma_{ik}^{j}}\right)\right)u_{i,k}^{j-1}-\frac{\tau}{2}\left(\overset{Ax}{b}_{ik}+\overset{C3x}{b}_{ik}\right)u_{i+1,k}^{j-1},\\
&\frac{\tau}{2}\left(\overset{Ax}{a}_{ik}+\overset{D4x}{a}_{ik}\right)\mathrm{v}_{i-1,k}^{j-4/5}+\left(1+\frac{\tau}{2}\left(\overset{Ax}{c}_{ik}+\overset{D4x}{c}_{ik}\right)\right)\mathrm{v}_{i,k}^{j-4/5}+\\
&+\frac{\tau}{2}\left(\overset{Ax}{b}_{ik}+\overset{D4x}{b}_{ik}\right)\mathrm{v}_{i+1,k}^{j-4/5}=-\frac{\tau}{2}\left(\overset{Ax}{a}_{ik}+\overset{D4x}{a}_{ik}\right)\mathrm{v}_{i-1,k}^{j-1}+\\
&+\left(1-\frac{\tau}{2}\left(\overset{Ax}{c}_{ik}+\overset{D4x}{c}_{ik}\right)\right)\mathrm{v}_{i,k}^{j-1}-\frac{\tau}{2}\left(\overset{Ax}{b}_{ik}+\overset{D4x}{b}_{ik}\right)\mathrm{v}_{i+1,k}^{j-1}.
\end{aligned} \qquad (58)$$

Here $k=\overline{0,n_y}$, and for each k we get a system of linear algebraic equations of $\left(n_x\times n_x\right)$ order relative to unknowns $\left\{h_{i,k}^{j-4/5}\right\}_{i=1}^{n_x}$, $\left\{\gamma_{i,k}^{j-4/5}\right\}_{i=1}^{n_x}$, $\left\{u_{i,k}^{j-4/5}\right\}_{i=1}^{n_x}$, $\left\{\mathrm{v}_{i,k}^{j-4/5}\right\}_{i=1}^{n_x}$ with tridiagonal matrix. there is an effective method of solution for solving such systems - the sweep method [12].

For the second pitch, we also deal with a system of algebraic equations with a three-diagonal matrix, which is solved by the sweep method relative to unknown $\left\{h_{i,k}^{j-3/5}\right\}_{k=0}^{n_y}$, $\left\{\gamma_{i,k}^{j-3/5}\right\}_{k=0}^{n_y}$, $\left\{u_{i,k}^{j-3/5}\right\}_{k=0}^{n_y}$, $\left\{\mathrm{v}_{i,k}^{j-3/5}\right\}_{k=0}^{n_y}$ for all $i=\overline{1,n_x}$

$$\begin{aligned}
&\frac{\tau}{2}\overset{Ay}{a}_{ik}\,h_{i,k-1}^{j-3/5}+\left(1+\frac{\tau}{2}\overset{Ay}{c}_{ik}\right)h_{i,k}^{j-3/5}+\frac{\tau}{2}\overset{Ay}{b}_{ik}\,h_{i,k+1}^{j-3/5}=\\
&=-\frac{\tau}{2}\overset{Ay}{a}_{ik}\,h_{i,k-1}^{j-4/5}+\left(1-\frac{\tau}{2}\overset{Ay}{c}_{ik}\right)h_{i,k}^{j-4/5}-\frac{\tau}{2}\overset{Ay}{b}_{ik}\,h_{i,k+1}^{j-4/5},\\
&\frac{\tau}{2}\overset{Ay}{a}_{ik}\,\gamma_{i,k-1}^{j-3/5}+\left(1+\frac{\tau}{2}\overset{Ay}{c}_{ik}\right)\gamma_{i,k}^{j-3/5}+\frac{\tau}{2}\overset{Ay}{b}_{ik}\,\gamma_{i,k+1}^{j-3/5}=\\
&=-\frac{\tau}{2}\overset{Ay}{a}_{ik}\,\gamma_{i,k-1}^{j-4/5}+\left(1-\frac{\tau}{2}\overset{Ay}{c}_{ik}\right)\gamma_{i,k}^{j-4/5}-\frac{\tau}{2}\overset{Ay}{b}_{ik}\,\gamma_{i,k+1}^{j-4/5}.
\end{aligned} \qquad (59)$$

$$\begin{aligned}
&\frac{\tau}{2}\left(\overset{Ay}{a}_{ik}+\overset{C3y}{a}_{ik}\right)u_{i,k-1}^{j-3/5}+\left(1+\frac{\tau}{2}\left(\overset{Ay}{c}_{ik}+\overset{C3y}{c}_{ik}\right)\right)u_{i,k}^{j-3/5}+\\
&+\frac{\tau}{2}\left(\overset{Ay}{b}_{ik}+\overset{C3y}{b}_{ik}\right)u_{i,k+1}^{j-3/5}=-\frac{\tau}{2}\left(\overset{Ay}{a}_{ik}+\overset{C3y}{a}_{ik}\right)u_{i,k-1}^{j-4/5}+\\
&+\left(1-\frac{\tau}{2}\left(\overset{Ay}{c}_{ik}+\overset{C3y}{c}_{ik}\right)\right)u_{i,k}^{j-4/5}-\frac{\tau}{2}\left(\overset{Ay}{b}_{ik}+\overset{C3y}{b}_{ik}\right)u_{i,k+1}^{j-4/5},\\
&\frac{\tau}{2}\left(\overset{Ay}{a}_{ik}+\overset{D4y}{a}_{ik}\right)\mathrm{v}_{i-1,k}^{j-3/5}+\left(1+\frac{\tau}{2}\left(\overset{Ay}{c}_{ik}+\overset{D4y}{c}_{ik}+\frac{C_s}{\gamma_{ik}^{j}}\right)\right)\mathrm{v}_{i,k}^{j-3/5}+\\
&+\frac{\tau}{2}\left(\overset{Ay}{b}_{ik}+\overset{D4y}{b}_{ik}\right)\mathrm{v}_{i+1,k}^{j-3/5}=-\frac{\tau}{2}\left(\overset{Ay}{a}_{ik}+\overset{D4y}{a}_{ik}\right)\mathrm{v}_{i,k-1}^{j-4/5}+\\
&+\left(1-\frac{\tau}{2}\left(\overset{Ay}{c}_{ik}+\overset{D4y}{c}_{ik}+\frac{C_s}{\gamma_{ik}^{j}}\right)\right)\mathrm{v}_{i,k}^{j-4/5}-\frac{\tau}{2}\left(\overset{Ay}{b}_{ik}+\overset{D4y}{b}_{ik}\right)\mathrm{v}_{i,k+1}^{j-4/5}.
\end{aligned} \qquad (60)$$

The third and fourth fractional pitches

The third time pitch is connected with equation (50), which is equivalent to the system of equations:

$$\begin{gathered} H^{j-2/5} = H^{j-3/5}, \quad V^{j-2/5} = V^{j-3/5}, \\ G^{j-2/5} + \frac{\tau}{2}\overset{2x}{C}U^{j-2/5} = G^{j-3/5} - \frac{\tau}{2}\overset{2x}{C}U^{j-3/5}, \\ U^{j-2/5} + \frac{\tau}{2}\overset{3x}{A}U^{j-2/5} + \frac{\tau}{2}\overset{3x}{B}G^{j-2/5} = \\ = U^{j-3/5} - \frac{\tau}{2}\overset{3x}{A}U^{j-3/5} - \frac{\tau}{2}\overset{3x}{B}G^{j-3/5}. \end{gathered} \tag{61}$$

This system of matrix equations has a solution, which can be represented accurate within members of order in the form:

$$\begin{gathered} H^{j-2/5} = H^{j-3/5}, \quad V^{j-2/5} = V^{j-3/5}, \\ G^{j-2/5} = G^{j-3/5} - \tau\overset{2x}{C}U^{j-3/5}, \\ U^{j-2/5} = U^{j-3/5} - \tau\overset{3x}{A}H^{j-3/5} - \tau\overset{3x}{B}G^{j-3/5}. \end{gathered} \tag{62}$$

The fourth pitch corresponds to the equation (51):

$$\begin{gathered} H^{j-1/5} = H^{j-2/5}, \quad U^{j-1/5} = U^{j-2/5}, \\ G^{j-1/5} + \frac{\tau}{2}\overset{2y}{D}V^{j-1/5} = G^{j-2/5} - \frac{\tau}{2}\overset{2y}{D}V^{j-2/5}, \\ V^{j-1/5} + \frac{\tau}{2}\overset{4y}{A}H^{j-1/5} + \frac{\tau}{2}\overset{4y}{B}G^{j-1/5} = \\ = V^{j-2/5} - \frac{\tau}{2}\overset{4y}{A}H^{j-2/5} - \frac{\tau}{2}\overset{4y}{B}G^{j-2/5}. \end{gathered} \tag{63}$$

and has the corresponding solution:

$$\begin{gathered} H^{j-1/5} = H^{j-2/5}, \quad U^{j-1/5} = U^{j-2/5}, \\ G^{j-1/5} = G^{j-2/5} - \tau\overset{2y}{D}V^{j-2/5}, \\ V^{j-1/5} = V^{j-2/5} - \tau\overset{4y}{A}H^{j-2/5} - \tau\overset{4y}{B}G^{j-2/5}. \end{gathered} \tag{64}$$

From the relations (62, 64) it can be seen that for these two pitches finding of a solution add up to operations of multiplication of matrices and vectors.

The fifth pitch add up to calculate the recurrence relation (52), and the subsequent pitches correspond to 1-4 pitches, which are carried out in a reverse order.

Choice of the phenomenal coefficient of Shezy

There are no experimental data concerning of determination of the Shezy coefficient C_s in the literature, which is concerned to the dynamics of flows. Let's consider the one-dimensional steady-state Kuette motion of a viscous fluid within the channel of a width h (Fig. 3) [19, 20] that we have an idea of the order of this magnitude.

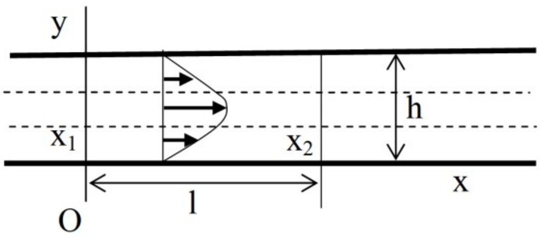

Fig.3. One-dimensional motion of a viscous liquid inside the channel

Let $u = u(y)$ is x component of velocity. The velocity profile is parabolic:

$$u = \frac{\Delta p}{l}\ \mathrm{y(h\text{-}y)},$$

where: $\Delta p = p_1 - p_2$ is the pressure difference at the distance l between the points x_1, x_2.

The average fluid velocity u_s in the channel is equal:

$$u_s = \frac{1}{h}\int_0^h u(y)dy = \frac{\Delta p\, h^2}{6l}.$$

Shearing stress T on a solid wall are conditioned by hydraulic resistance:
$C_s u_s$, i.e.

$$T \equiv \mu\left(\left.\frac{\partial u}{\partial y}\right|_{y=0} + \left.\frac{\partial u}{\partial y}\right|_{y=h}\right) \equiv 2\mu\frac{\Delta p\, h}{l} = C_s u_s \equiv C_s\frac{\Delta p\, h^2}{6l}.$$

From where follows:

$$C_s = \frac{12\mu}{h}.$$

Thus, the Shezy coefficient has the same order as the coefficient of dynamic shear viscosity μ.

CONCLUSION

A numerical solution of the spatial motion problem of a loose mixture in a vibrating tray is obtained by finite-difference method.

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ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧИ ПРОСТРАНСТВЕННОГО ДВИЖЕНИЯ СЫПУЧЕЙ СМЕСИ В ВИБРОЛОТКЕ

М. Пивень

Аннотация. В статье получено численное решение задачи пространственного движения сыпучей смеси в вибролотке конечноразностным методом.

Суть метода заключается в том, что дифференциальные операторы по пространственным переменным заменяются конечноразностными операторами в узлах сетки, при этом конечноразностный оператор должен как можно точнее аппроксимировать дифференциальный. Использованы конечно-разностные операторы, имеющие второй порядок точности.

Граничные условия вносят изменения в коэффициенты разностных операторов. Полагаем, что эти изменения уже учитываются в уравнениях, представляющих собой систему обыкновенных дифференциальных уравнений относительно векторной переменной. К слагаемым уравнений отнесены главные члены уравнений динамики, влияющие на быстроту сходимости приближенных решений к точному. К правым частям отнесены слагаемые, содержащие смешанные производные. В таком виде уравнения подготовлены к применению разностного метода расщепления.

Для получения стационарного решения задачи использованы следующие положения: наличие диссипации в задаче (сил внутреннего и внешнего трения) приводит к тому, что при неизменных внешних условиях (граничные условия не зависят от времени) поток сыпучей смеси со временем стабилизируется и не зависимо от начальных условий приходит в свое стационарное состояние. Поэтому далее формулируется начально-краевая задача с неизменными граничными условиями, а стационарное решение рассматривается как асимптотическое.

Многомерность задачи приводит к необходимости решения систем, содержащих большое число уравнений. К тому же при решении последних возможно возникновение численной неустойчивости алгоритма. Один из способов решения уравнений заключается в переходе от дифференциальных уравнений к системе алгебраических уравнений посредством замены оператора дифференцирования по времени конечноразностным. В дальнейшем применена неявная разностная схема, соответствующая методу Кранка-Николсона с удвоенным шагом по времени, имеющая второй порядок точности и по времени и по пространственным переменным.

Ключевые слова: пространственное движение, сыпучая смесь, конечноразностный метод, дифференциальные операторы.