TEKA. COMMISSION OF MOTORIZATION AND ENERGETICS IN AGRICULTURE – 2013, Vol. 13, No.3, 203-208

Internal geometry of active surfaces of teeth of cylindrical gear arch mixed gearing

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Received June 10.2013: accepted July 02.2013

S u m m a r y . In the article analytical dependences are got for determination of indexes of internal geometry of the arched indents, form an instrument with an asymmetrical initial contour that will allow to define the quality indexes of capacity of such transmissions.
K e y w o r d s . arched gearing, mixed gearing, initial contour, productive surface.

INTRODUCTION

Operability of machines in many branches is defined by quality of the mechanical drives entering into their composition [4]. The destiny of tooth gearings is thus great. Therefore enhancement of toothed drives will allow to increase operability of machines and consequently is the actual task. One of methods of enhancement of transmissions is geometry synthesis on quality indicators of working capacity [1, 2, 9, 10, 14, 16]. For implementation of such synthesis the dependences, connecting measure values with geometry of teeth or the tool are necessary. Such dependences for synthesis of geometry of the tool were received in work [7, 8, 18] for different types of transmissions, and in work [5, 19] it was for transmissions with arch teeths. Synthesized in the work [3, 6, 19] symmetric output circuits synthesized in operation have limited application in view of some technological shortcomings, but provide the powerful benefit on working capacity over traditional transmissions. The combination in one transmission of advantages of different types of linkages will allow to expand an area of their application [13, 21, 22].

OBJECTS AND PROBLEMS

Article purpose is to receive the equation for a measure definition of internal geometry of the arch teeths formed by the asymmetrical output circuit.

In case of determination of criteria of operability of tooth gearings with arch teeths the equations of surfaces of teeths, and also indexes of internal geometry of these surfaces are used.

The equation of the active surfaces can be received [18, 19], writing coordinates of surfaces of a linkage in coordinate systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$ which are connected to a gearwheel and a wheel. Realizing transition from fixed coordinate system XYZ to systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$, we will receive:

1) the equations of surfaces of fingers of arch wheels in systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$ for the convex side:

$$\vec{r}_{1f} = \begin{cases} x_1 = (f_1 + R_1)\cos\varphi_1 + \Omega_{1f}\cos\beta\sin\varphi_1, \\ y_1 = (f_1 + R_1)\sin\varphi_1 - \Omega_{1f}\cos\beta\cos\varphi_1, \\ z_1 = z_0 - f_2\sin\beta; \end{cases}$$

(gearwheel head); (1)

$$\vec{r}_{1\Phi} = \begin{cases} x_1 = (\Phi_1 + R_1)\cos\varphi_1 + \Omega_{1\Phi}\cos\beta\sin\varphi_1, \\ y_1 = (\Phi_1 + R_1)\sin\varphi_1 - \Omega_{1\Phi}\cos\beta\cos\varphi_1, \\ z_1 = z_0 - \Phi_2\sin\beta; \end{cases}$$

(gearwheel pinch); (2)

$$\vec{r}_{2f}=\begin{cases}x_2=(f_1-R_2)\cos\varphi_2-\Omega_{1f}\cos\beta\sin\varphi_2,\\ y_2=-(f_1-R_2)\sin\varphi_2-\Omega_{1f}\cos\beta\cos\varphi_2,\\ z_2=z_0-f_2\sin\beta;\end{cases}$$

(wheel head); (3)

$$\vec{r}_{2\Phi}=\begin{cases}x_2=(\Phi_1-R_2)\cos\varphi_2-\Omega_{1\Phi}\cos\beta\sin\varphi_2,\\ y_2=-(\Phi_1-R_2)\sin\varphi_2-\Omega_{1\Phi}\cos\beta\cos\varphi_2,\\ z_2=z_0-\Phi_2\sin\beta;\end{cases}$$

(wheel pinch); (4)

2) the equations of surfaces of fingers of arch wheels in systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$ for the concave side:

$$\vec{r}_{1f}=\begin{cases}x_1=(f_1+R_1)\cos\varphi_1-\Omega_{1f}\cos\beta\sin\varphi_1,\\ y_1=(f_1+R_1)\sin\varphi_1+\Omega_{1f}\cos\beta\cos\varphi_1,\\ z_1=z_0+f_2\sin\beta;\end{cases}$$

(gearwheel head); (5)

$$\vec{r}_{1\Phi}=\begin{cases}x_1=(\Phi_1+R_1)\cos\varphi_1-\Omega_{1\Phi}\cos\beta\sin\varphi_1,\\ y_1=(\Phi_1+R_1)\sin\varphi_1+\Omega_{1\Phi}\cos\beta\cos\varphi_1,\\ z_1=z_0+\Phi_2\sin\beta;\end{cases}$$

(gearwheel pinch); (6)

$$\vec{r}_{2f}=\begin{cases}x_2=(f_1-R_2)\cos\varphi_2+\Omega_{1f}\cos\beta\sin\varphi_2,\\ y_2=-(f_1-R_2)\sin\varphi_2+\Omega_{1f}\cos\beta\cos\varphi_2,\\ z_2=z_0+f_2\sin\beta;\end{cases}$$

(wheel head); (7)

$$\vec{r}_{2\Phi}=\begin{cases}x_2=(\Phi_1-R_2)\cos\varphi_2+\Omega_{1\Phi}\cos\beta\sin\varphi_2,\\ y_2=-(\Phi_1-R_2)\sin\varphi_2+\Omega_{1\Phi}\cos\beta\cos\varphi_2,\\ z_2=z_0+\Phi_2\sin\beta;\end{cases}$$

(wheel pinch). (8)

In the received equations the variables λ, μ, φ_i connected by ratios, the described equations of a linkage F_1^* and F_2^* [11, 19, 20].

Let's write also in mobile coordinate systems of a projection of unit vector of a normal [12, 19, 20]. In coordinate system $X_1Y_1Z_1$ we have projections of unit vector of a normal to a surface of teeths of a gearwheel (according to a head and a pinch):

$$\vec{e}_f=\begin{cases}e_{x1}=e_{xn}\cos\varphi_1-e_{yn}\sin\varphi_1,\\ e_{y1}=e_{xn}\sin\varphi_1+e_{yn}\cos\varphi_1,\\ e_{z1}=e_{zn};\end{cases}\qquad \vec{e}_\Phi=\begin{cases}e_{x1}=e_{xn}\cos\varphi_1-e_{yn}\sin\varphi_1,\\ e_{y1}=e_{xn}\sin\varphi_1+e_{yn}\cos\varphi_1,\\ e_{z1}=e_{zn}.\end{cases}\qquad (9)$$

In coordinate system $X_2Y_2Z_2$ we have projections of unit vector of a normal to a surface of teeths of a wheel (according to a head and a pinch):

$$\vec{e}_f=\begin{cases}e_{x2}=e_{xn}\cos\varphi_2+e_{yn}\sin\varphi_2,\\ e_{y2}=-e_{xn}\sin\varphi_2+e_{yn}\cos\varphi_2,\\ e_{z2}=e_{zn};\end{cases}\qquad \vec{e}_\Phi=\begin{cases}e_{x2}=e_{xn}\cos\varphi_2+e_{yn}\sin\varphi_2,\\ e_{y2}=-e_{xn}\sin\varphi_2+e_{yn}\cos\varphi_2,\\ e_{z2}=e_{zn}.\end{cases}\qquad (10)$$

In formulas (9), (10) e_{xn}, e_{yn}, e_{zn}, means:

$$\vec{e}_f=\begin{cases}e_{xn}=\dfrac{\pm f_2'}{n_f},\\ e_{yn}=-\dfrac{f_1'}{n_f}\cos\beta,\\ e_{zn}=\dfrac{f_1'}{n_f}\sin\beta;\end{cases},\quad \vec{e}_\Phi=\begin{cases}e_{xn}=\dfrac{\pm\Phi_2'}{n_\Phi},\\ e_{yn}=-\dfrac{\Phi_1'}{n_\Phi}\cos\beta,\\ e_{zn}=\dfrac{\Phi_1'}{n_\Phi}\sin\beta.\end{cases}\cdot\qquad (11)$$

Here:

$$n_f=\sqrt{(f_1')^2+(f_2')^2},\quad n_\Phi=\sqrt{(\Phi_1')^2+(\Phi_2')^2}.$$

The upper sign undertakes for the convex side, lower is for concave.

For determination of criteria of operability of tooth gearings it is necessary to have coefficients of the first and second quadratic forms. For determination of coefficients private derivatives of radius vectors and vectors of normals to the active of surfaces of teeths of a gearwheel ($n=1$) and wheel ($n=2$) are necessary.

Differentiating the equation (1)-(8) on λ and φ, considering also the linkage equations, we have:

$$\vec{r}_{1f}^{\,\lambda}=\begin{cases}x_1^\lambda=f_1'\cos\varphi_1\pm(\Omega_{1f}\cos\beta)^\lambda\sin\varphi_1,\\ y_1^\lambda=f_1'\sin\varphi_1\mp(\Omega_{1f}\cos\beta)^\lambda\cos\varphi_1,\\ z_1^\lambda=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp(f_2\sin\beta)^\lambda;\end{cases}$$

$$\vec{r}_{1\Phi}^{\,\lambda}=\begin{cases}x_1^\lambda=\Phi_1'\cos\varphi_1\pm(\Omega_{1\Phi}\cos\beta)^\lambda\sin\varphi_1,\\ y_1^\lambda=\Phi_1'\sin\varphi_1\mp(\Omega_{1\Phi}\cos\beta)^\lambda\cos\varphi_1,\\ z_1^\lambda=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp(\Phi_2\sin\beta)^\lambda;\end{cases}$$

$$\vec{r}_{2f}^{\,\lambda}=\begin{cases}x_2^{\lambda}=f_1'\cos\varphi_2\mp\left(\Omega_{1f}\cos\beta\right)^{\lambda}\sin\varphi_2,\\ y_2^{\lambda}=-f_1'\sin\varphi_2\mp\left(\Omega_{1f}\cos\beta\right)^{\lambda}\cos\varphi_2,\\ z_2^{\lambda}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp\left(f_2\sin\beta\right)^{\lambda};\end{cases}$$

$$\vec{r}_{2\Phi}^{\,\lambda}=\begin{cases}x_2^{\lambda}=\Phi_1'\cos\varphi_2\mp\left(\Omega_{1\Phi}\cos\beta\right)^{\lambda}\sin\varphi_2,\\ y_2^{\lambda}=-\Phi_1'\sin\varphi_2\mp\left(\Omega_{1\Phi}\cos\beta\right)^{\lambda}\cos\varphi_2,\\ z_2^{\lambda}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp\left(\Phi_2\sin\beta\right)^{\lambda};\end{cases}$$

$$\vec{r}_{1f}^{\,\varphi}=\begin{cases}x_1^{\varphi}=-\left(f_1+R_1\right)\sin\varphi_1\pm\Omega_{1f}\cos\beta\cos\varphi_1\pm\\ \pm\Omega_{1f}\sin\varphi_1\left(\cos\beta\right)^{\varphi},\\ y_1^{\varphi}=\left(f_1+R_1\right)\cos\varphi_1\pm\Omega_{1f}\cos\beta\sin\varphi_1\mp\\ \mp\Omega_{1f}\cos\varphi_1\left(\cos\beta\right)^{\varphi},\\ z_1^{\varphi}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp f_2\left(\sin\beta\right)^{\varphi};\end{cases}$$

$$\vec{r}_{1\Phi}^{\,\varphi}=\begin{cases}x_1^{\varphi}=-\left(\Phi_1+R_1\right)\sin\varphi_1\pm\Omega_{1\Phi}\cos\beta\cos\varphi_1\pm\\ \pm\Omega_{1\Phi}\sin\varphi_1\left(\cos\beta\right)^{\varphi},\\ y_1^{\varphi}=\left(\Phi_1+R_1\right)\cos\varphi_1\pm\Omega_{1\Phi}\cos\beta\sin\varphi_1\mp\\ \mp\Omega_{1\Phi}\cos\varphi_1\left(\cos\beta\right)^{\varphi},\\ z_1^{\varphi}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp\Phi_2\left(\sin\beta\right)^{\varphi};\end{cases}$$

$$\vec{r}_{2f}^{\,\varphi}=\begin{cases}x_2^{\varphi}=-\left(f_1-R_2\right)\sin\varphi_2\mp\Omega_{1f}\cos\beta\cos\varphi_2\mp\\ \mp\Omega_{1f}\sin\varphi_2\left(\cos\beta\right)^{\varphi},\\ y_2^{\varphi}=-\left(f_1-R_2\right)\cos\varphi_2\pm\Omega_{1f}\cos\beta\sin\varphi_2\mp\\ \mp\Omega_{1f}\cos\varphi_2\left(\cos\beta\right)^{\varphi},\\ z_2^{\varphi}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp f_2\left(\sin\beta\right)^{\varphi};\end{cases}$$

$$\vec{r}_{2\Phi}^{\,\varphi}=\begin{cases}x_2^{\varphi}=-\left(\Phi_1-R_2\right)\sin\varphi_2\mp\Omega_{1\Phi}\cos\beta\cos\varphi_2\mp\\ \mp\Omega_{1\Phi}\sin\varphi_2\left(\cos\beta\right)^{\varphi},\\ y_2^{\varphi}=-\left(\Phi_1-R_2\right)\cos\varphi_2\pm\Omega_{1\Phi}\cos\beta\sin\varphi_2\mp\\ \mp\Omega_{1\Phi}\cos\varphi_2\left(\cos\beta\right)^{\varphi},\\ z_2^{\varphi}=\dot{z}_0\cdot\dfrac{d\mu}{d\lambda}\mp\Phi_2\left(\sin\beta\right)^{\varphi};\end{cases}$$

Here the upper sign corresponds to the convex side of tooth, lower is concave.

Differentiating the equation (9), (10) on λ and φ, we have private derivatives of vectors of normals to a surface of teeths of a gearwheel ($n=1$) and wheels ($n=2$) for a head and a pinch respectively:

$$\vec{e}_{1f}^{\,\lambda}=\begin{cases}e_{x1}^{\lambda}=e_{xn}^{\lambda}\cos\varphi_1-e_{yn}^{\lambda}\sin\varphi_1,\\ e_{y1}^{\lambda}=e_{xn}^{\lambda}\sin\varphi_1+e_{yn}^{\lambda}\cos\varphi_1,\\ e_{z1}^{\lambda}=e_{zn}^{\lambda};\end{cases}$$

$$\vec{e}_{1\Phi}^{\,\lambda}=\begin{cases}e_{x1}^{\lambda}=e_{xn}^{\lambda}\cos\varphi_1-e_{yn}^{\lambda}\sin\varphi_1,\\ e_{y1}^{\lambda}=e_{xn}^{\lambda}\sin\varphi_1+e_{yn}^{\lambda}\cos\varphi_1,\\ e_{z1}^{\lambda}=e_{zn}^{\lambda};\end{cases}$$

$$\vec{e}_{2f}^{\,\lambda}=\begin{cases}e_{x2}^{\lambda}=e_{xn}^{\lambda}\cos\varphi_2+e_{yn}^{\lambda}\sin\varphi_2,\\ e_{y2}^{\lambda}=-e_{xn}^{\lambda}\sin\varphi_2+e_{yn}^{\lambda}\cos\varphi_2,\\ e_{z2}^{\lambda}=e_{zn}^{\lambda};\end{cases}$$

$$\vec{e}_{2\Phi}^{\,\lambda}=\begin{cases}e_{x2}^{\lambda}=e_{xn}^{\lambda}\cos\varphi_2+e_{yn}^{\lambda}\sin\varphi_2,\\ e_{y2}^{\lambda}=-e_{xn}^{\lambda}\sin\varphi_2+e_{yn}^{\lambda}\cos\varphi_2,\\ e_{z2}^{\lambda}=e_{zn}^{\lambda};\end{cases}$$

$$\vec{e}_{1f}^{\,\varphi}=\begin{cases}e_{x1}^{\varphi}=\left(e_{xn}^{\varphi}-e_{yn}\right)\cos\varphi_1-\left(e_{xn}+e_{yn}^{\varphi}\right)\sin\varphi_1,\\ e_{y1}^{\varphi}=\left(e_{xn}^{\varphi}-e_{yn}\right)\sin\varphi_1+\left(e_{xn}+e_{yn}^{\varphi}\right)\cos\varphi_1,\\ e_{z1}^{\varphi}=e_{zn}^{\varphi};\end{cases}$$

$$\vec{e}_{1\Phi}^{\,\varphi}=\begin{cases}e_{x1}^{\varphi}=\left(e_{xn}^{\varphi}-e_{yn}\right)\cos\varphi_1-\left(e_{xn}+e_{yn}^{\varphi}\right)\sin\varphi_1,\\ e_{y1}^{\varphi}=\left(e_{xn}^{\varphi}-e_{yn}\right)\sin\varphi_1+\left(e_{xn}+e_{yn}^{\varphi}\right)\cos\varphi_1,\\ e_{z1}^{\varphi}=e_{zn}^{\varphi};\end{cases}$$

$$\vec{e}_{2f}^{\,\varphi}=\begin{cases}e_{x2}^{\varphi}=\left(e_{xn}^{\varphi}+e_{yn}\right)\cos\varphi_2+\left(e_{yn}^{\varphi}-e_{xn}\right)\sin\varphi_2,\\ e_{y2}^{\varphi}=-\left(e_{xn}^{\varphi}+e_{yn}\right)\sin\varphi_2+\left(e_{yn}^{\varphi}-e_{xn}\right)\cos\varphi_2,\\ e_{z2}^{\varphi}=e_{zn}^{\varphi};\end{cases}$$

$$\vec{e}_{2\Phi}^{\,\varphi}=\begin{cases}e_{x2}^{\varphi}=\left(e_{xn}^{\varphi}+e_{yn}\right)\cos\varphi_2+\left(e_{yn}^{\varphi}-e_{xn}\right)\sin\varphi_2,\\ e_{y2}^{\varphi}=-\left(e_{xn}^{\varphi}+e_{yn}\right)\sin\varphi_2+\left(e_{yn}^{\varphi}-e_{xn}\right)\cos\varphi_2,\\ e_{z2}^{\varphi}=e_{zn}^{\varphi}.\end{cases}$$

Using everything higher explained, and also elements of differential geometry [15, 17], we will receive coefficients of the quadratic form of a surface of teeth of a gearwheel (for a head):

$$E_1=\left(f_1'\right)^2+\left(\left(\Omega_{1f}\cos\beta\right)^{\lambda}\right)^2+\left(\left(z_1\right)^{\lambda}\right)^2,$$

$$F_1=\pm f_1'\Omega_{1f}\cos\beta\mp\left(\Omega_{1f}\cos\beta\right)^{\lambda}\left(\left(f_1+R_1\right)\mp\Omega_{1f}\left(\cos\beta\right)^{\varphi}\right)+z_1^{\lambda}z_1^{\varphi},$$

$$G_1=\left(\left(f_1+R_1\right)\mp\Omega_{1f}\left(\cos\beta\right)^{\varphi}\right)^2+\left(\Omega_{1f}\cos\beta\right)^2+\left(z_1^{\varphi}\right)^2,$$

$$L_1=-f_1'e_{xn}^{\lambda}\pm\left(\Omega_{1f}\cos\beta\right)^{\lambda}e_{yn}^{\lambda}-z_1^{\lambda}e_{zn}^{\lambda},$$

$$2M_1=-f_1'\left(e_{xn}^{\varphi}-e_{yn}\right)\pm\left(\Omega_{1f}\cos\beta\right)^{\lambda}\left(e_{xn}+e_{yn}^{\varphi}\right)-z_1^{\lambda}e_{zn}^{\varphi}\mp$$
$$\mp\Omega_{1f}\cos\beta\cdot e_{xn}^{\lambda}-\left(\left(f_1+R_1\right)\mp\Omega_{1f}\left(\cos\beta\right)^{\varphi}\right)e_{yn}^{\lambda}-z_1^{\varphi}e_{zn}^{\lambda},$$

$$N_1 = \mp\Omega_{1f}\cos\beta\left(e_{xn}^{\varphi} - e_{yn}\right) - \left((f_1 + R_1) \mp \Omega_{1f}(\cos\beta)^{\varphi}\right)\left(e_{xn} + e_{yn}^{\varphi}\right) - z_1^{\varphi} e_{zn}^{\varphi}. \tag{12}$$

Similarly for a pinch of a gearwheel it is had:

$$E_1 = \left(\Phi_1'\right)^2 + \left((\Omega_{1\Phi}\cos\beta)^{\lambda}\right)^2 + \left((z_1)^{\lambda}\right)^2,$$

$$F_1 = \pm\Phi_1'\Omega_{1\Phi}\cos\beta \mp (\Omega_{1\Phi}\cos\beta)^{\lambda}\left((\Phi_1 + R_1) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right) + z_1^{\lambda} z_1^{\varphi},$$

$$G_1 = \left((\Phi_1 + R_1) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right)^2 + (\Omega_{1\Phi}\cos\beta)^2 + \left(z_1^{\varphi}\right)^2,$$

$$L_1 = -\Phi_1' e_{xn}^{\lambda} \pm (\Omega_{1\Phi}\cos\beta)^{\lambda} e_{yn}^{\lambda} - z_1^{\lambda} e_{zn}^{\lambda},$$

$$2M_1 = -\Phi_1'\left(e_{xn}^{\varphi} - e_{yn}\right) \pm (\Omega_{1\Phi}\cos\beta)^{\lambda}\left(e_{xn} + e_{yn}^{\varphi}\right) - z_1^{\lambda} e_{zn}^{\varphi} \mp \mp\Omega_{1\Phi}\cos\beta \cdot e_{xn}^{\lambda} - \left((\Phi_1 + R_1) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right) e_{yn}^{\lambda} - z_1^{\varphi} e_{zn}^{\lambda},$$

$$N_1 = \mp\Omega_{1\Phi}\cos\beta\left(e_{xn}^{\varphi} - e_{yn}\right) - \left((\Phi_1 + R_1) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right)\left(e_{xn} + e_{yn}^{\varphi}\right) - z_1^{\varphi} e_{zn}^{\varphi}. \tag{13}$$

Coefficients of the quadratic form of a surface of teeth of a wheel (head):

$$E_2 = \left(f_1'\right)^2 + \left((\Omega_{1f}\cos\beta)^{\lambda}\right)^2 + \left((z_2)^{\lambda}\right)^2,$$

$$F_2 = \pm f_1'\Omega_{1f}\cos\beta \mp (\Omega_{1f}\cos\beta)^{\lambda}\left((f_1 - R_2) \mp \Omega_{1f}(\cos\beta)^{\varphi}\right) + z_2^{\lambda} z_2^{\varphi},$$

$$G_2 = \left((f_1 - R_2) \mp \Omega_{1f}(\cos\beta)^{\varphi}\right)^2 + (\Omega_{1f}\cos\beta)^2 + \left(z_2^{\varphi}\right)^2,$$

$$L_2 = -f_1' e_{xn}^{\lambda} \mp (\Omega_{1f}\cos\beta)^{\lambda} e_{yn}^{\lambda} - z_2^{\lambda} e_{zn}^{\lambda},$$

$$2M_2 = -f_1'\left(e_{xn}^{\varphi} + e_{yn}\right) \mp (\Omega_{1f}\cos\beta)^{\lambda}\left(e_{yn}^{\varphi} - e_{xn}\right) - z_2^{\lambda} e_{zn}^{\varphi} \mp \mp\Omega_{1f}\cos\beta \cdot e_{xn}^{\lambda} + \left((f_1 - R_2) \mp \Omega_{1f}(\cos\beta)^{\varphi}\right) e_{yn}^{\lambda} - z_2^{\varphi} e_{zn}^{\lambda},$$

$$N_2 = \mp\Omega_{1f}\cos\beta\left(e_{xn}^{\varphi} + e_{yn}\right) + \left((f_1 - R_2) \mp \Omega_{1f}(\cos\beta)^{\varphi}\right)\left(e_{yn}^{\varphi} - e_{xn}\right) - z_2^{\varphi} e_{zn}^{\varphi}. \tag{14}$$

Similarly for a pinch of a wheel it is had:

$$E_2 = \left(\Phi_1'\right)^2 + \left((\Omega_{1\Phi}\cos\beta)^{\lambda}\right)^2 + \left((z_2)^{\lambda}\right)^2,$$

$$F_2 = \pm\Phi_1'\Omega_{1\Phi}\cos\beta \mp (\Omega_{1\Phi}\cos\beta)^{\lambda}\left((\Phi_1 - R_2) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right) + z_2^{\lambda} z_2^{\varphi},$$

$$G_2 = \left((\Phi_1 - R_2) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right)^2 + (\Omega_{1\Phi}\cos\beta)^2 + \left(z_2^{\varphi}\right)^2,$$

$$L_2 = -\Phi_1' e_{xn}^{\lambda} \mp (\Omega_{1\Phi}\cos\beta)^{\lambda} e_{yn}^{\lambda} - z_2^{\lambda} e_{zn}^{\lambda},$$

$$2M_2 = -\Phi_1'\left(e_{xn}^{\varphi} + e_{yn}\right) \mp (\Omega_{1\Phi}\cos\beta)^{\lambda}\left(e_{yn}^{\varphi} - e_{xn}\right) - z_2^{\lambda} e_{zn}^{\varphi} \mp \mp\Omega_{1\Phi}\cos\beta \cdot e_{xn}^{\lambda} + \left((\Phi_1 - R_2) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right) e_{yn}^{\lambda} - z_2^{\varphi} e_{zn}^{\lambda},$$

$$N_2 = \mp\Omega_{1\Phi}\cos\beta\left(e_{xn}^{\varphi} + e_{yn}\right) + \left((\Phi_1 - R_2) \mp \Omega_{1\Phi}(\cos\beta)^{\varphi}\right)\left(e_{yn}^{\varphi} - e_{xn}\right) - z_2^{\varphi} e_{zn}^{\varphi}. \tag{15}$$

In the received ratios for coefficients of quadratic forms upper the sign undertakes for the convex side, the lower sign is for concave. Value e_{xn}, e_{yn}, e_{zn} are undertaked from received before formulas with the upper sign for the convex side of tooth of a wheel (gearwheel), with the lower is for the concave side respectively. Others, not defined above a parameter value, taking into account the equations of a linkage have the following appearance (for a head and a pinch respectively):

$$(\cos\beta)^{\varphi} = \frac{R_i K}{1 \pm \Omega_{2f} K}, \quad (\cos\beta)^{\varphi} = \frac{R_i K}{1 \pm \Omega_{2\Phi} K},$$

$$(\sin\beta)^{\varphi} = -\frac{K R_i ctg\beta}{1 \pm \Omega_{2f} K}, \quad (\sin\beta)^{\varphi} = -\frac{K R_i ctg\beta}{1 \pm \Omega_{2\Phi} K},$$

$$\frac{d\mu}{d\varphi} = \frac{R_i}{\dot{y}_0\left(1 \pm \Omega_{2f} K\right)}, \quad \frac{d\mu}{d\varphi} = \frac{R_i}{\dot{y}_0\left(1 \pm \Omega_{2\Phi} K\right)},$$

$$\frac{d\mu}{d\lambda} = \frac{\mp\Omega_{2f}'\cos\beta}{\dot{y}_0\left(1 \pm \Omega_{2f} K\right)}, \quad \frac{d\mu}{d\lambda} = \frac{\mp\Omega_{2\Phi}'\cos\beta}{\dot{y}_0\left(1 \pm \Omega_{2\Phi} K\right)},$$

$$(\Omega_1\cos\beta)^{\lambda} = \Omega_1'\cos\beta \mp \frac{\Omega_{2f}'\Omega_1 K\cos\beta}{1 \pm \Omega_{2f} K}, \tag{16}$$

$$(\Omega_2\cos\beta)^{\lambda} = \Omega_2'\cos\beta \mp \frac{\Omega_{2\Phi}'\Omega_2 K\cos\beta}{1 \pm \Omega_{2\Phi} K},$$

$$e_{xn}^{\varphi} = 0, \; e_{yn}^{\varphi} = -\frac{f_1'}{n} \cdot \frac{R_i K}{1 \pm \Omega_{2f} K}, \; e_{yn}^{\varphi} = -\frac{\Phi_1'}{n} \cdot \frac{R_i K}{1 \pm \Omega_{2\Phi} K},$$

$$e_{zn}^{\varphi} = -\frac{f_1'}{n} \cdot \frac{R_i K\, ctg\beta}{1 \pm \Omega_{2f} K}, \quad e_{zn}^{\varphi} = -\frac{\Phi_1'}{n} \cdot \frac{R_i K\, ctg\beta}{1 \pm \Omega_{2\Phi} K}.$$

In the private derivatives given here for gearwheel teeth $i = 1$, for wheel teeth $i = 2$, the upper sign corresponds to the convex side of tooth, lower is for concave; formulas for e_{xn}^{λ}, e_{yn}^{λ}, e_{zn}^{λ} were given above. Besides, designations take place: $\Omega_{2f} = \Omega_{1f} + f_2$, $\Omega_{2\Phi} = \Omega_{1\Phi} + \Phi_2$.

Using coefficients of the quadratic form (12)-(15), it is possible to define normal curvatures of surfaces of teeths of a gearwheel and a wheel [19, 20]. For this purpose we use the ratio received earlier for determination of curvature of the forming surface in which instead of $d\mu$ it is necessary to add $d\varphi$, instead of E_n, F_n, G_n, L_n, M_n, N_n are appropriate coefficients for a gearwheel (12), (13) and wheels (14), (15).

Normal curvatures of surfaces of teeths of a gearwheel and wheel in the direction of contact lines $\varphi_i = const$ in case of will be equal:

$$K_{\varphi i} = \frac{L_i}{E_i}.$$

Normal curvatures if $\lambda = const$ (along screw lines) are equal:

$$K_{\lambda i} = \frac{N_i}{G_i}.$$

From the last ratio it is visible that if $K = 0$, $\beta = 0$, normal curvatures $K_{\lambda i}$ along screw lines are equal to zero.

Using the linkage equations for convex (the upper sign) and bent (the lower sign) the tooth sides, we will receive if $\mu = const$ (for a head and a pinch respectively):

$$\left(\frac{d\varphi}{d\lambda}\right)_{\mu} = \pm\frac{\Omega_{2f}{}'\cos\beta}{R_i}, \left(\frac{d\varphi}{d\lambda}\right)_{\mu} = \pm\frac{\Omega_{2\Phi}{}'\cos\beta}{R_i}. \quad (17)$$

Then in the normal section of teeth the normal curvature if $\mu = const$ looks like:

$$K_{\mu i} = \frac{L_i\, d\lambda^2 + 2M_i\, d\lambda\, d\varphi + N_i\, d\varphi^2}{E_i\, d\lambda^2 + 2F_i\, d\lambda\, d\varphi + G_i\, d\varphi^2}. \quad (18)$$

Having separated numerator and a denominator of equality (18) on $d\lambda^2$, and using a ratio (17), we will receive:

$$K_{\mu i} = \frac{R_i^2 L_i \pm 2M_i R_i \Omega_{2f}{}'\cos\beta + N_i\left(\Omega_{2f}{}'\cos\beta\right)^2}{R_i^2 E_i \pm 2F_i R_i \Omega_{2f}{}'\cos\beta + G_i\left(\Omega_{2f}{}'\cos\beta\right)^2},$$

(for a head);

$$K_{\mu i} = \frac{R_i^2 L_i \pm 2M_i R_i \Omega_{2\Phi}{}'\cos\beta + N_i\left(\Omega_{2\Phi}{}'\cos\beta\right)^2}{R_i^2 E_i \pm 2F_i R_i \Omega_{2\Phi}{}'\cos\beta + G_i\left(\Omega_{2\Phi}{}'\cos\beta\right)^2}.$$

(for a pinch).

In edge sections of cogwheels $z_1 = const$, $z_2 = const$. Then from the last equations (1)-(4) for the active surfaces of arch teeth, considering that $z_0 = const$ we will receive (for a head and a pinch respectively):

$$dz_i = \mp\left(f_2{}'\sin\beta\right)d\lambda \mp f_2\cdot\frac{\partial\sin\beta}{\partial\mu}d\mu = 0, \quad (19)$$

$$dz_i = \mp\left(\Phi_2{}'\sin\beta\right)d\lambda \mp \Phi_2\cdot\frac{\partial\sin\beta}{\partial\mu}d\mu = 0. \quad (20)$$

Using the linkage equation for the convex and concave sides of arch teeth, we have respectively for a head and a pinch:

$$dF_i = \dot{y}_0 d\mu - R_i d\varphi \pm \Omega_{2f}{}'\cos\beta\, d\lambda \pm \Omega_{2f}\cdot\frac{\partial\cos\beta}{\partial\mu}d\mu = 0 \quad (21)$$

$$dF_i = \dot{y}_0 d\mu - R_i d\varphi \pm \Omega_{2\Phi}{}'\cos\beta\, d\lambda \pm \Omega_{2\Phi}\cdot\frac{\partial\cos\beta}{\partial\mu}d\mu = 0 \quad (22)$$

Deciding systems of equations (19) and (21) for a head, (20) and (22) for a pinch, we find for edge sections of cogwheels (for a head and a pinch respectively):

$$\left(\frac{d\varphi}{d\lambda}\right)_{Ti} = \frac{f_2{}'\sin\beta\; tg\beta\left(1\pm\Omega_{2f}K\right)\pm f_2K\Omega_{2f}{}'\cos\beta}{f_2K\,R_i},$$

$$\left(\frac{d\varphi}{d\lambda}\right)_{Ti} = \frac{\Phi_2{}'\sin\beta\; tg\beta\left(1\pm\Omega_{2\Phi}K\right)\pm \Phi_2K\Omega_{2\Phi}{}'\cos\beta}{\Phi_2K\,R_i}.$$

Normal curvature of surfaces of teeths will be equal in edge section:

$$K_T = \frac{L_i + 2M_i\left(\frac{d\varphi}{d\lambda}\right)_{Ti} + N_i\left(\frac{d\varphi}{d\lambda}\right)_{Ti}^2}{E_i + 2F_i\left(\frac{d\varphi}{d\lambda}\right)_{Ti} + G_i\left(\frac{d\varphi}{d\lambda}\right)_{Ti}^2}.$$

CONCLUSIONS

1. Received analytical dependences for coefficients of the first and second quadratic forms are necessary for determination of criteria of operability of tooth gearings.
2. The received values of normal curvatures of surfaces of teeth in the characteristic directions can be used in case of synthesis of transmissions with arch tooth by these criteria.

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ВНУТРЕННЯЯ ГЕОМЕТРИЯ АКТИВНЫХ ПОВЕРХНОСТЕЙ ЗУБЦОВ ЦИЛИНДРИЧЕСКИХ АРОЧНЫХ ПЕРЕДАЧ СМЕШАННОГО ЗАЦЕПЛЕНИЯ

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Аннотация. В статье получены аналитические зависимости для определения показателей внутренней геометрии арочных зубцов, образованных инструментом с несимметричным исходным контуром, который позволит определить качественные показатели работоспособности таких передач.

Ключевые слова. Арочные зубчатые передачи, исходный контур, производящая поверхность, инструментальная рейка, система координат, коэффициенты квадратичных форм, нормальная кривизна, главные кривизны.