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Fourier transformation – an important tool in vibroacoustic diagnostics

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Summary. The objective feature of the reflection allows for using measurable physical quantities that characterize the processes accompanying the operation of a mechanical device, as signals carrying encoded information. In order to decode this information, the signal has to be retrieved and converted into a characteristic, usually determined in frequency domain. Using the DFT procedure, the computer allows for calculations of the estimates of frequency characteristics. For the effective use of the numerical methods, one needs to know how the information encoded in the signal is generated during its processing. In order to investigate these problems, a model that reflects the A/C conversion and the periodization of the fragment of retrieved signal in a closed time frame was used. The investigation of this model has shown that the DFT procedure generates stripes representing the harmonic waves of a signal only for admissible frequencies, equal to the total multiplicity of the inverse of signal retrieval time. To present the wave of another frequency, the DFT procedure generates a substitute spectrum. Consequently, the discrete spectrums are a result of a superposition of waves representing the harmonic signal components of frequencies that belong to the admissible set as well as waves of frequencies that do not belong to this set and form substitute spectrums.

Key words: vibroacoustic diagnostics, numerical signal processing, Fourier transformation.

INTODUCTION

For construction and operation of mechanical devices information related to its technical conditions, its properties and processes taking place is necessary. In reference to mechanical devices the obtainment of such knowledge requires a realization of a research process consisting in acquiring and interpretation of information. This is the task of technical diagnostics [4, 5, 9, 10, 11, 14].

A reliable source of information on a mechanical device is research. It is known that 'the matter-characteristic, objective feature of the reflection is shown through generating and conveying information on the conditions of objects of the material world' [17]. This feature allows using measurable physical quantities that characterize the processes accompanying the operation of a device as diagnostic signals carrying encoded information. In order to decode this information the signal has to be retrieved and converted into a characteristic.

Most of the processes occurring in a device in operation that are the sources of the signals are series of events that repeat periodically. That is why the characteristics of these signals are usually determined in the frequency domain.

Computers of high computational power and modern algorithms allow cheap and quick calculations of different, sometimes very complex frequency characteristics of the diagnostic signal through numerical methods. For the effective use of the numerical methods of signal processing one needs to know how the information encoded in the signal is generated during its processing and how the information is distorted and presented in the signal characteristics.

THE SUBJECT OF INVESTIGATION

The subject of the investigation is a mechanical device composed of elements joined in kinematic pairs. An operating device can be perceived as a controlled acting system, schematically presented in figure 1. The system is characterized by an open flow of the stream of mass, energy and information that is divided into two components. The first appears at the output in the form of a working process. The other is an uncontrolled stream of mass, energy and information that accompanies this process. In a correctly functioning device the amount of mass and energy in the second component is miniscule in comparison to the mass and energy of the first one. Yet, the flow of information related to the functioning principle is similar to the first component [10, 17].

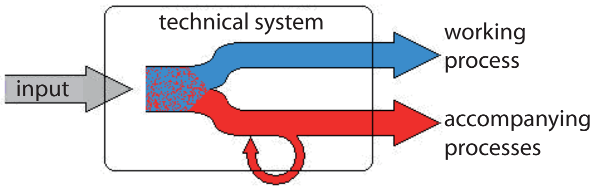

Fig. 1. Flow of stream of mass, energy and information

The second component of the flow stream appears in the form of processes that accompany the operation of the device. The most important are: deterioration, vibroacoustic and thermal processes. The cause-and-effect relation between the symptoms that characterize the conditions of the device and the processes that take place in this device as well as the relation between the processes emitted to the outside allows using 'reflection' for diagnostic purposes. The physical quantities that characterize these processes are used as signals carrying information on the device.

During the operation sources of wave distortions located in various kinematic pairs and parts of the mechanical device activate. Some of them have been shown in figure 2.

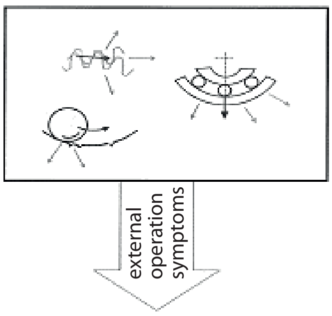

Fig. 2. Sources of wave distortions

The wave distortions generated by the sources, marked with $x_1(t)$,.....,$x_n(t)$ in figure 3 are subject to feedback through propagation. Between the sources and the output of the system representing the investigated device forms a channel of flow of information. At the output of the channel appears signal $\vec{y}(t)$ whose coordinates are the selected physical quantities [10, 11].

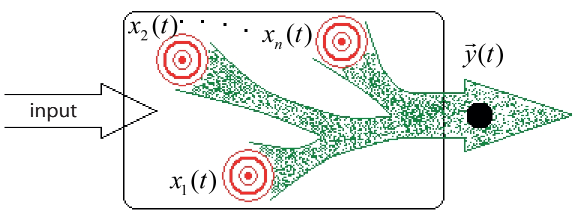

Fig. 3. The channel of the flow of information

In practice, individual time realizations of these quantities are used as diagnostic signals. The amount of information included in realization $f(t)$ depends on its versatility that shows through changes in time. Each change is one bit of information. The higher the frequency of these changes the more information in a time unit will be conveyed to the output of the flow channel. In this case the vibroacoustic signals (mechanical and acoustic vibrations) are the best choice out of all signal emissions by the operating device. An additional advantage of these signals is their accessibility outside of the device without distorting their operation [2,10, 17].

FOURIER TRANSFORM

In 1807, the baron Jean-Baptiste Joseph Fourier, a mathematician, engineer, member of the French Academy presented an essay treating on the fact that periodic, linear function $f(t)$ fulfilling the Dirichlet conditions can be expressed as a sum of series:

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2\pi \nu n t + b_n \sin 2\pi \nu n t\right) \qquad (1)$$

where: $a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$, $a_n = \frac{2}{T}\int_{=T/2}^{T/2} f(t)\cos 2\pi\nu n t dt$,

$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin 2\pi\nu n t dt$.

Coefficient a_0 is an average value, a_n, b_n are the coefficients of the harmonic distribution of function $f(t)$. Index $n = 1,2,...,\infty$ denotes the expressions of the series, T – period of function $f(t)$, $\nu = 1/T$ – frequency. Because: $\nu_n = \nu n = n/T$ and $\nu_{n+1} = \nu(n+1) = (n+1)/T$ the increment $\Delta\nu = \nu = 1/T = const$. Equation (1) presents function $f(t)$ as an infinite sum of cosine and sine functions of discrete frequencies growing by a step of $\Delta\nu$.

We can assume that for the non-periodic function $T\to\infty$, then $\Delta\nu\to d\nu$. Taking this assumption into account, after transforming series (1), we obtain a formula for the Fourier Transform [1, 3, 13]:

$$\mathbf{F}(\nu) = \int_{-\infty}^{\infty} f(t)\exp(-j2\pi\nu t)dt \qquad (2)$$

The integral (2) is a functional, in which the nucleus $\exp(-j2\pi\nu t)dt$, in function $f(t)$ seeks a harmonic wave of the frequency of $\nu\in[0,\infty)$ and describes it with a complex number, containing the time-averaged $t\in(-\infty,\infty)$ information on the amplitude and phase of this wave. In the complex coordinate system this number determines the point that is the end of a vector (in this paper the complex values will be typed in bold):

$$\mathbf{F}(\nu) = a(\nu) + jb(\nu) \qquad (3)$$

of the phase module and angle:

$$\left|\mathbf{F}(\nu)\right| = \sqrt{a^2(\nu) + b^2(\nu)} \qquad (4)$$

$\Psi(\nu) = arctg[b(\nu)/a(\nu)] + \pi m$, where $m = 0,\pm1,\pm2,...$.

Vector $\mathbf{F}(v)$ presented in figure 4 is a spectrum of the harmonic wave of frequency v_n contained in function $f(t)$. Because operation (2) of the transformation of function $f(t)$ is repeated for all frequencies from the range $[0,\infty)$, transform $\mathbf{F}(v)$ show the course of signal $f(t)$ in the frequency domain.

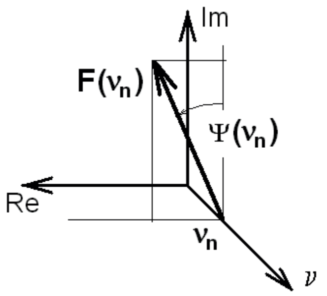

Fig. 4. The spectrum of the harmonic component of function $f(t)$

Using the phase module and angle from dependence (4), equation (3) can be notated according to the Euler theorem in the trigonometric form:

$$\mathbf{F}(v)=|\mathbf{F}(v)|\cos\psi(v)\pm j|\mathbf{F}(v)|\sin\psi(v)=|\mathbf{F}(v)|\exp[\pm j\psi(v)] \quad (5)$$

Hence:

$$a(v)=|\mathbf{F}(v)|\cos\psi(v),\ -b(v)|\mathbf{F}(v)|\sin\psi(v) \quad (6)$$

In the system of three coordinates: real part $a(v)$, imaginary unit $jb(v)$, frequency v - continuous course of the Fourier transform is a place of the geometrical end of vectors $\mathbf{F}(v)$ just like the ones in figure 4, distributed on the axis of frequency every $dv\to 0$. Taking the dependences (4) into account, we can present the transform in the form of two courses: module spectrum $|\mathbf{F}(v)|$ and spectrum of the phase angle $\psi(v)$.

Time realization $f(t)$ is always a real function of time. The results of the Fourier transformation of such a function will be a complex transform. Knowing the course of the cosine and sine functions we can prove that $a(v)=a(-v)$ and $-b(v)=b(-v)$. The Fourier transform of the real function fulfills the Hermite conditions. This means that the real part is even and the imaginary unit is odd. As a consequence the course of the module is an even function: $|\mathbf{F}(v)|=|\mathbf{F}(-v)|$ and the course of the phase angle is an odd function: $-\psi(v)=\psi(-v)$ [1, 3].

Time realization $f(t)$ and Fourier transform $\mathbf{F}(v)$ remain in the equivalence relation. The equivalent pair $f(t)\Leftrightarrow\mathbf{F}(v)$ fulfills the theorems of linearity and additiveness and mutual symmetry. The transform of the product of two functions $f_1(t)\cdot f_2(t)$ equals to the wreath product (convolution) of their transforms $\mathbf{F}_1(v)*\mathbf{F}_2(v)$; the reverse theorem is also true.

THE MEASUREMENT WINDOW

An infinite period of function $f(t)$ manifests through infinite boundaries of integration of transformation (2). In diagnostic research of mechanical devices we retrieve a fragment of a signal $f(t): t\in[-T/2, T/2]$. The finite retrieving time T becomes a basic period of function $f(t)$ and determined the frequency of the periodization of the retrieved fragment: $v_T=1/T$. This is contrary to the assumption that $T\to\infty$ for non-periodic function $f(t)$ that the basis for the transformation of series (1) to the form of an integral (2) [1, 6, 8, 10].

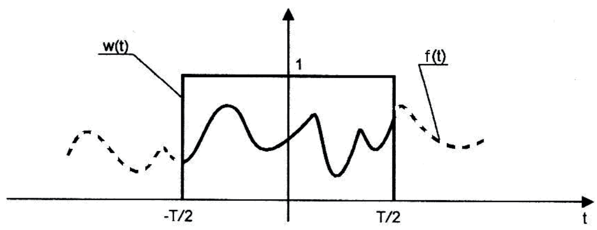

Fig. 5. Diagnostic signal in the measurement window

Range $[-T/2, T/2]$ determines a rectangular measurement window such that: $w(t)=1$ for $t\in[-T/2, T/2]$ and $w(t)=0$ for $t\notin[-T/2, T/2]$ presented in figure 5. The Fourier transformation is performed on signal $f(t): t\in[-T/2, T/2] = [f(t): t\in(-\infty,\infty)]\cdot w(t)$:

$$\int_{-\infty}^{\infty}[f(t)\cdot w(t)]\cdot\exp(-j2\pi vt)dt =$$

$$= \int_{-T/2}^{T/2} f(t)\cdot\exp(-j2\pi vt)dt = \mathbf{F}(v) * \mathbf{W}(v) \quad (7)$$

where the transform of the rectangular measurement window was $w(t)$:

$$\mathbf{W}(v)=\int_{-\infty}^{\infty} w(t)\exp(-j2\pi vt)dt=\int_{-T/2}^{T/2}\exp(-j2\pi vt)dt = T\frac{\sin\pi vT}{\pi vT} \quad (8)$$

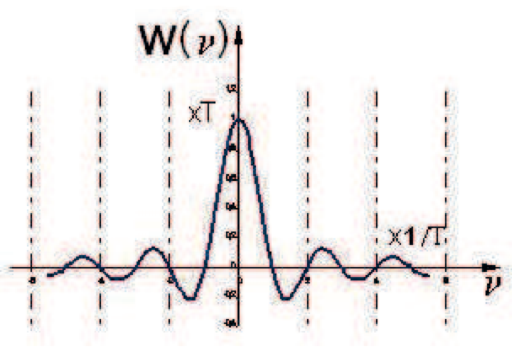

Fig. 6. Transform of the rectangular measurement window

The course of the transform $\mathbf{W}(v)$ for $v=0$, presented in Figure 6, is suspended on the abscissa of the coordinate system. If signal $f(t)$ contains harmonic component of this frequency and amplitude that equals one then this component should manifest in figure 6 in the form of a stripe of module spectrum $|\mathbf{F}(v)|$ on the ordinate. Because the signal was retrieved in finite time T, then as a result of the convolution (7) the stripe of the spectrum will be represented by local maximum of the course

W(ν) for $\nu=0$. Because for $\nu\to 0$ $\lim \frac{\sin \pi\nu T}{\pi\nu T}=1$ value T determines the scale that will adjust the height of this maximum against the length of the stripe.

In the spectrum of the module of the actual signal, the stripes denoting the amplitudes of the individual harmonic components occur for different frequency ν_n, where $n=1,2,3,\ldots$. The expression (8) will take the form:

$$\mathbf{W}(\nu)=T\frac{\sin \pi(\nu-\nu_n)T}{\pi(\nu-\nu_n)T} \tag{9}$$

The stripe of the module spectrum for $\nu=\nu_n$ will be distorted by the rectangular measurement window identically as the stripe for $\nu=0$.

The rectangular measurement window effect shows as a leakage and lateral waves of the Fourier transform of signal $f(t): t\in[-T/2, T/2]$. The leakage makes the differentiation of the neighboring stripes difficult and sometimes impossible and the lateral waves distort their value.

THE PROCESSING PRINCIPLES OF VIBROACOUSTIC SIGNALS

The time realization retrieved as a diagnostic signal is in fact an ergodic and stochastic process $f(\zeta,t)$, where: t – time, ζ – random variable. In a sufficiently long range function $f(t): t\in[-T/2, T/2]$ carries encoded information contained in process $f(\zeta,t)$. The processing that should enable reading of this information consists in determining of the non-random characteristics from function $f(t): t\in[-T/2, T/2]$. It is assumed that in range $[-T/2, T/2]$this function is linear, stationary and fulfills the Dirichlet conditions [1, 10].

The characteristics can describe signal $f(t): t\in[-T/2, T/2]$ in three domains: value, time and frequency. In the diagnostic of mechanical devices the most frequent sample is the one consisting in transformation of the signal to the frequency domain with the use of the Fourier Integral. This is particularly the case in vibroacoustic signals.

Equivalence $f(t)\Leftrightarrow \mathbf{F}(\nu)$ means that the following relation is true: $|f(t)|^2\Leftrightarrow|\mathbf{F}(\nu)|^2$. The squared course of the module spectrum is a frequency image of the energy contained in signal $f(t)$ and carries information of the intensity of the wave distortions that were activated while the mechanical device was in operation. This information, read and properly interpreted may form a message regarding the technical condition of the parts and kinematic pairs of the device where these sources are located [2, 10].

The sources of distortions are characterized by periodic repeatability of reversible events that manifest as wave phenomena. The values of frequencies determined based on the knowledge of the system composition and structure represented by the device can be deemed as common addresses of spectral components and sources that generated it. The addresses of a given pair source-component are usually different for each pair. The information resulting from relation $|f(t)|^2\Leftrightarrow|\mathbf{F}(\nu)|^2$ regarding the value of the module of the spectral component of the signal is at the same time information regarding the energy resulting from the intensity of functioning of a given source of distortion. The schematics of the conveyance channel presented in figure 3 shows that signal $f(t): t\in[-T/2, T/2]$ carries information regarding the energy of different sources that are mixed due to mutual feedbacks. Course $|\mathbf{F}(\nu)|$depicts the frequency decomposition of the signal that allows identifying and estimating the energy generated by given sources. That is why the Fourier transformation is an important tool in signal processing used in vibracoustic diagnostics of mechanical devices.

The computers allow calculations of the estimates of different characteristics that are difficult to determine through direct analogue measurement. The Fourier spectrum is a perfect example here. The database for the calculations of the spectrum is a series of values representing continuous time realizations of signal fragment $f(t): t\in[-T/2, T/2]$ [10, 11, 16].

MODEL OF A DISCRETE SIGNAL

The retrieved time realizations $f(t): t\in[-T/2, T/2]$ are continuous. A series of values from which the computer will calculate the Fourier transform is generated in two phases by the A/C converter from the retrieved signal:

1. sampling that consists in discretization of the function argument: $f(t): t\in[-T/2, T/2]$ in moments: $t_0,t_1,\ldots,t_k,\ldots t_{N-1}$ $\in[-T/2, T/2]$. The difference: $\Delta t=t_{k+1}-t_k=T_e$ determines the constant period of sampling and the frequency of sampling: $\nu_e=1/T_e$. Values T and T_e, determine the number of samples: $N=T/T_e=$int. that will fit into the measurement window,
2. quantization of realization $f(t): t\in[-T/2, T/2]$ in moments: $t_0,t_1,\ldots,t_k,\ldots t_{N-1}$ that determines the value of signal $f(kT_e)$ for: $k=0,1,2,\ldots,N-1$.

The discrete function: $f(kT_e)$ represents a series of subsequent events each of which has an assigned number value. Separately, none of them carries any important information, but a series of N – events in timely order, does. In the database obtained during the A/C conversion information is stored regarding the signal in the measurement window in the moment of sampling. Other information that can exist in the signal is lost.

In order to analyze the process of generating the information during numerical processing we can use a model that reflects the A/C conversion and the periodization of the fragment of the retrieved signal in a closed time frame [10, 12,15]. For the creation of such a model we can use the Dirac comb:

$$Ш(t)=\bigcup_{\alpha=-\infty}^{\infty}\delta(t-\alpha T_e),\ \text{where}\ \alpha=-\infty,\ldots,-1,0,1,\ldots,\infty,$$

shown in figure 7a. It is a series of impulses evenly distributed on the time axis maintaining a distance equal to signal sampling period $\Delta t=T_e$=const. The multiplication by the function of window $w(t)$ takes into account the retrieval of the signal of the period of $[-T/2, T/2]$ and limits the number of samples determined by the Dirac impulses to the value N:

$$f^{\bullet}(kT_e) = \left(f(t) : t \in (-\infty, \infty)\right) \cdot \\ \cdot w(t) \cdot \bigcup_{\alpha=-\infty}^{\infty} \delta(t - \alpha T_e) \qquad (10)$$

The periodization of signal (10) we can reflect through convolution of signal $f^{\bullet}(kT_e)$ through Dirac comb Ш'$(t) = \bigcup_{\eta=-\infty}^{\infty} \delta(t-\eta T)$ in which the impulses are distributed on the time axis every $\Delta t = T$=const. and are numbered with an index: $\eta = -\infty,...,-1,0,1,2,...,\infty$:

$$f(kT_e) = \left(f(t) : t \in (-\infty, \infty)\right) \cdot \\ \cdot w(t) \cdot \bigcup_{\alpha=-\infty}^{\infty} \delta(t - \alpha T_e) * \bigcup_{\eta=-\infty}^{\infty} \delta(t - \eta T) \qquad (11)$$

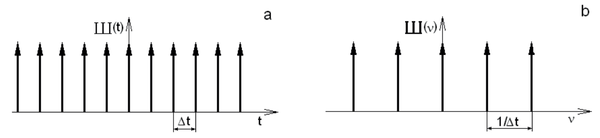

Fig. 7. Dirac comb in the time and frequency domain

Signal $f(kT_e)$ for $k=0,1,2,...,N-1$ is a series of events that is why operations (10) and (11) marked $\bigcup$, are not algebraic addition but a summing of events.

DISCRETE FOURIER TRANSFORM

Taking into account the theorem on the transform of the algebraic product and convolution of two functions the Fourier transformation of signal $f(kT_e)$ can be notated as follows:

$$\mathbf{F}(\nu) = [\int_{-T/2}^{T/2} f(t)\exp(-j2\pi\nu t)dt \cdot \\ \cdot \int_{-\infty}^{\infty} [\text{Ш}'(t)] \cdot \exp(-j2\pi\nu t)dt * \int_{-\infty}^{\infty} [\text{Ш}(t)] \cdot \exp(-j2\pi\nu t)dt \qquad (12)$$

The Fourier integral from $-T/2$ to $T/2$ takes into account the algebraic multiplication through window $w(t)$. The result of the integration is the convolution of the function $f(t) \in (-\infty,\infty)$ transform and the transform of the rectangular measurement window occurring on the right side in formula (7).

The Fourier transform of the Dirac comb that is determined in the time domain remains the Dirac comb in the frequency domain [15]. It has a form of distribution of a period of $1/\Delta t$, shown in figure 7b. The transforms Ш(t) and Ш'(t) will be respectively:

$$\mathbf{Ш}(\nu) = \nu_e \bigcup_{\alpha=-\infty}^{\infty} (\delta - \alpha\nu_e),\ \mathbf{Ш}'(\nu) = \nu_T \bigcup_{\eta=-\infty}^{\infty} (\delta - \eta\nu_T),$$

where: $\nu_e = 1/\Delta t = 1/T_e$, $\nu_T = 1/\Delta t = 1/T$ and $\alpha, \eta = -\infty,...,-1,0,1,...,\infty$.

Algebraic multiplication through comb **Ш'**(t) will result in a discretization of the spectrum to the form of a series of vectors referred to as stripes. Value $\nu_T = 1/T$ determines a constant distance between the stripes and determines the discrete spectrum resolution. The values $\eta\nu_T$ determine frequencies for which these stripes can appear. The rectangular measurement window limits the number of stripes to value N.

The convolution by **Ш**(ν) results in a periodization of the discrete spectrum. It is shown as a reproduction of the spectrum every ν_e value for an infinite number of times because $\alpha=-\infty,...,-1,0,1,...,\infty$. Frequencies ν_T and ν_e, occurring before the sums of events in the transforms of the Dirac combs **Ш'**(ν) and **Ш**(ν) determine the scale of the multiplication operation and do not influence the way the discrete spectrum is formed.

Figures 8a, b and c show the process of the formation of the discrete spectrum of the module of the tested signal represented by the real function of time.

According to the Hermit conditions, the spectrum of the module of the real function is even. In figure 8a spectrum $|\mathbf{F}(\nu)|$ and its even reflection $|\mathbf{F}(-\nu)|$ are symmetrical and form a unity that as a result of periodization is 'suspended' on the abscissa determined by stripes $0 \cdot \nu_e$ and $1 \cdot \nu_e$. The reproduction of the spectrum results in that the information in all ranges $[\alpha \cdot \nu_e, (\alpha+1) \cdot \nu_e)$, where $\alpha=-\infty,...,-1,0,1,...,\infty$ will be the same. That is why for the considerations only one range needs to be included: $[0, \nu_e)$. In this range $N=\nu_e/\nu_T$, stripes will fit that are numbered with index $n=0,1,...,N-1$.

In figure 8b spectrum $|\mathbf{F}(\nu)|$, suspended on abscissa $\nu_e = 0 \cdot \nu_e$ and its left hand reflection $|\mathbf{F}(-\nu)|$ suspended on abscissa $\nu_e = 1 \cdot \nu_e$ are subject to superposition in frequency range $[0, \nu_e)$. The determined spectrum is a sum of superimposed courses: $|\mathbf{F}(\nu)|$ and $|\mathbf{F}(-\nu)|$. This phenomenon is called aliasing.

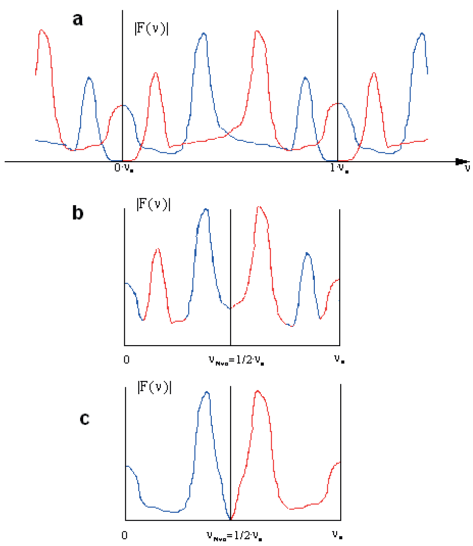

Fig. 8. Periodization, aliasing and filtering of the discrete spectrum

The middle of range $[0,v_e)$ is determined by the Nyquist frequency: $v_{Nyq}=0{,}5v_e$. If in spectrum $|\mathbf{F}(v)|$ that is suspended on stripe $0\cdot v_e$ occurs a local maximum for frequency $v>v_{Nyq}$ then, due to symmetry, the same maximum will occur in spectrum $|\mathbf{F}(-v)|$, suspended on stripe $1\cdot v_e$, for $v<v_{Nyq}$. As a result of aliasing in frequency range $[0,v_{Nyq})$ spectral information will be conveyed that is not contained in the signal. If we want to avoid that, we should use a low-pass filter of the signal that will damp the harmonic components of frequencies equal or greater than the Nyquist frequencies as shown in figure 8c.

If signal $f(kT_e)$ determined by formula (11) is subjected to low-pass filtering then in frequency range $[0,v_{Nyq})$ then at the most $N/2$ of the stripes of the discrete spectrum will fit that are numbered with an index: $n=0,1,2,...N/2–1$.

Replacing the integration with addition after $k=0,1,2,...N/2–1$ and substituting time and frequency in the discrete form from dependence (12) we can derive a formula for the discrete Fourier transform:

$$\mathbf{F}(n\nu_T)=\frac{1}{T}\sum_{k=0}^{N-1}f(kT_e)\cdot\exp(-j2\pi n\nu_T kT_e) \tag{13}$$

In the dimensionless domain of indexes k and n, transformation (13) takes the form of Discrete Fourier Transform (DFT):

$$\mathbf{F}(n)=\frac{1}{N}\sum_{k=0}^{N-1}f(k)\cdot\exp(-j2\pi nk/N) \tag{14}$$

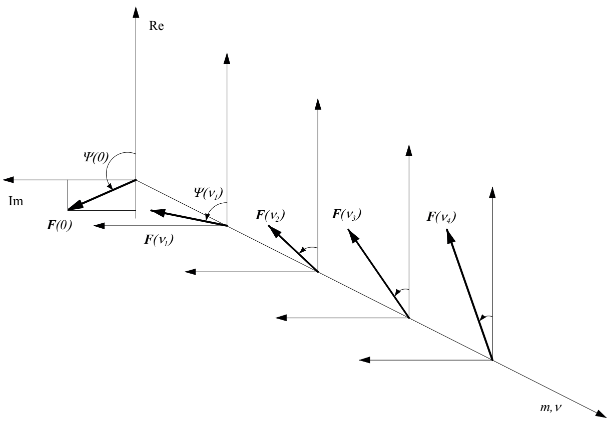

Fig 9. Signal discrete spectrum

Dependence (14) shows a system of equations for $n=0,1,2,...N/2–1$. The results of the solution of each equation is transform $\mathbf{F}(n)$ in the form of a complex number that determines the module and phase angle of a single stripe of a spectrum for frequency $n\nu_T$. The whole spectrum is a sum of harmonic components of a signal occurring on the frequency axis for values $n\nu_T$:

$$\mathbf{F}(\nu_n)=\bigcup_{n=0}^{N/2-1}\mathbf{F}(n) \tag{15}$$

Figure 9 presents the signal discrete spectrum calculated from the DFT dependence.

SPECTRUM OBTAINED THROUGH THE DFT METHOD

While generating spectrum $\mathbf{F}(n)$ of signal $f(k)$ the discretization with period T_e results in a periodization of the spectrum with period $v_e=1/T_e$. Sampling frequency $v_e=1/T_e$ determines band $[0,v_{Nyq}=1/2v_e$ where the stripes of the discrete spectrum are contained.

The periodization of the signal $f(k)$ with period T results in a discretization of the spectrum with resolution $v_T=1/T$. The DFT procedure generates stripes only for admissible frequencies nv_T of values equal to the total multiplicity of the inverse of signal retrieval time. The strips carrying original information on the harmonic components of the signal, are numbered with index $n=0,1,2,...,N/2–1$.

The frequency structure of the signals generated by the sources of wave distortions that function in mechanical devices is unknown. The frequencies of the harmonic components that form the signal depend on the physical nature of the sources and they are independent from value T and $v_e=1/T_e$. That is why we should expect that the frequencies of many components (possibly all) of the retrieved signal would not belong to the admissible set [7, 10, 11].

Figure 10a shows the signal in the form of a harmonic wave that was subjected to a discrete Fourier transformation. Because the retrieval time $T=n\cdot T_{f.h.}$ where $n=2$ and $T_{f.h.}$ – the period, the frequency of the wave belongs to the admissible set. The spectrum of the module, presents the stripe for $n=2$ shown in figure 10b. The reduction of time T results in that $T\neq n\cdot T_{f.h.}$ and frequency $v_T=1/T$ of the wave does not belong to the admissible set. In order to present this wave the DFT procedure generates a substitute spectrum in the form of a set of waves of admissible frequencies that do not exist in the signal. The local maximum of the substitute spectrum shown in figure 10c occurs for admissible frequencies close to the value of $1/T_{f.h.}$. We can expect that the result of the superposition of waves of the substitute spectrum will be the approximate course of the signal.

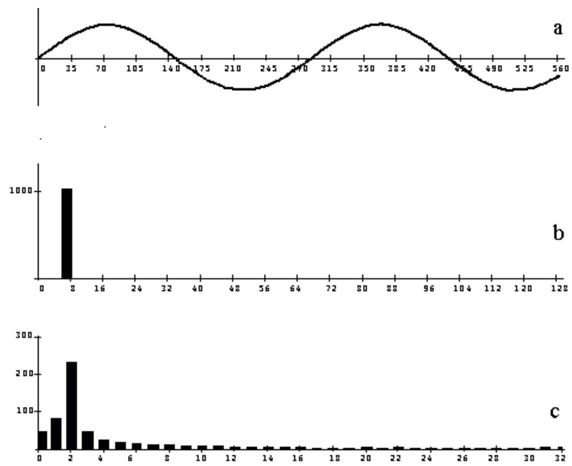

Fig. 10. The signal, its real and substitute spectrum

The analysis of the results presented in Figure 10 gives grounds for a formulation of the following hypothesis: the waves of the discrete substitute spectrum are determined by the DFT procedure according to the Fourier series for a preset and limited set of admissible frequencies $n\nu_T$. The trueness of the hypothesis can be verified through comparing the spectrum of the signal obtained from the development into a Fourier series with the spectrum of this signal calculated with the DFT method.

After a transformation of the sum of the cosine and sine function in formula (1) the Fourier series can be notated in the form of a sine function taking into account the phase angle:

$$f(k) = \sum_{n=0}^{N/2-1} A_n \sin\left(\frac{2\pi nk}{N} + \psi_n\right) \tag{16}$$

In series (16) function $f(k)$ where $k=0.1,2,...,N-1$ represents a discrete form of the investigated wave $f(t)=A\sin 2\pi\nu t$ retrieved in finite time T. The argument of the sine function determines the dimensionless domain of admissible frequencies that belong to range $[0,\nu_{Nyq})$ marked with index: $n=0,1,2,...,N/2-1$. In comparison to series (1), in formula (16) the summing was limited to $N/2$ of the components. Symbol ψ_n denotes the phase angle of the n–th component of function $f(k)$.

Because in the experiment $T=const$ was assumed, the set of admissible frequencies preset for the development of function $f(k)$ will be constant. In order to obtain information that could confirm (or reject) the assumed hypothesis one should investigate different substitute spectrums. To this end we need to ensure the possibility of modification of the wave frequency without changing the retrieval time.

Let wave frequency $\nu=(1+\varepsilon)/T$. Then for $\varepsilon=0$ – a single wave period equals T and its frequency $\nu\equiv\nu_T=1/T$. For each value $\varepsilon\in(0,1)$ maintaining a constant retrieval time the frequency of the wave will not belong to the admissible set. After substituting the dependence: $\nu=(1+\varepsilon)/T$, $T=NT_e$ and $t=kT_e$, the argument of the sine function in formula (16), assumes the form $2\pi(1+\varepsilon)k/N$. Then:

$$f(k) = A\sin 2\pi(1+\varepsilon)\frac{k}{N} \tag{17}$$

Wave $f(k)$ has been determined for $A=1$ and $N=16$ samples numbered with index: $k=0,1,2,...,15$. The modified wave presented in figure 11 was generated for: $\varepsilon=2/3$. Taking the Nyquist criterion into account the spectrum was determined for $N/2-1$ of the frequencies marked with index $n=0,1,...,7$.

Substitute spectrums of the module of the investigated wave calculated from the Fourier series and through the DFT method have been presented in figure 12. The stripes superimposed on the admissible frequency mesh show a spectrum obtained from formula (16) and the stripes in the immediate vicinity through the DFT method. The height of the stripes obtained with two methods is identical, which is confirmed by the adopted hypothesis.

DISCRETE SPECTRUM DISTORTION

The average value of the original wave of $\varepsilon=0$, represented by the component of the spectrum for $n=0$, equals zero. The performed modification of the frequency of this wave results in an unintended and incidental change of the average value of the retrieved fragment of the signal. That is why in the substitute spectrum in figure 12 the stripes for $n=0$ are non-zero.

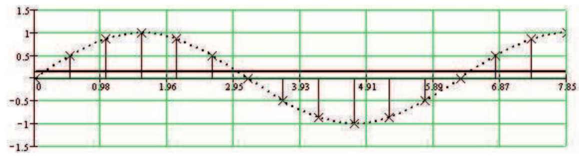

Fig. 11. Wave for $\varepsilon=2/3$

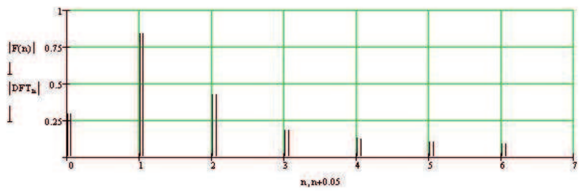

Fig. 12. Discrete wave spectrums for $\varepsilon=2/3$

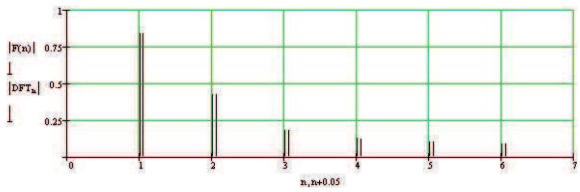

Fig. 13. Discrete wave spectrums for $\varepsilon=2/3$ after resetting of the average value

In figure 11 the axis of abscissa has been shifted so that the average value of the fragment of the wave for $\varepsilon=2/3$ equaled zero. Figure 13 presents the courses of the discrete spectrum of the module of this fragment of the wave after shifting the axis of abscissa. The component of the spectrum for $n=0$ equals zero and the stripes representing the outstanding components are the same as in figure 12 before the shift has been carried out.

From the course of transform $\mathbf{W}(\nu)$ in figure 6 and dependence (8) results that for the admissible frequencies $n/T=n\nu_T$ when $n=1,2,...=\text{int}$, complex integer $\mathbf{F}(\nu)^*\mathbf{W}(\nu)=0$ because $\mathbf{W}(\nu)=0$. For $n=0$, this product is other than zero and the transform of the window influences the stripe of the spectrum through change of the scale. Formula (9) confirms that this statement is correct for each stripe of the spectrum irrespective of value $n\nu_T$ for which it occurred on the condition that $n=\text{int}$.

Because in the discrete spectrum the total multiplicity of the inverse of the retrieval time determines the set of admissible frequencies, the discretization of the spectrum eliminates the distortions triggered by the rectangular measurement window in the form of a leakage and lateral waves. The filtering characteristics of periodical distribution **Ш'**(t) starts manifesting itself. Because dependence (7) remains true we can assume that the distortions of the discrete spectrums from the rectangular measurement window would display themselves in a different form than it directly results from this dependence.

In order to estimate the transfer of the information the original harmonic waves has been reproduced from the substitute spectrum. Figure 14 presents the comparison of the course of the wave generated for $\varepsilon=2/3$ and the course of the wave that was a result of a superposition of the harmonic waves recovered from the stripes of the substitute spectrum. Figure 15 shows the residuum determined by the difference between the recovered and the original wave.

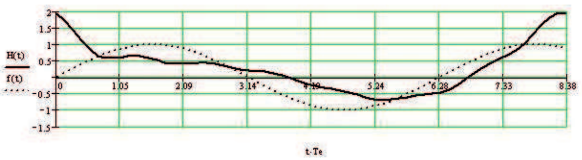

Fig. 14. The original and recovered waves for $\varepsilon=2/3$

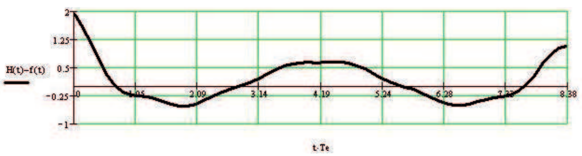

Fig. 15. Residuum of the original and recovered waves for $\varepsilon=2/3$

CONCLUSIONS

The discrete spectrums generated and emitted by mechanical devices in operation are a result of a superposition of waves representing the harmonic components of the signal of frequencies that belong to the admissible set as well as waves of frequencies that do not belong to this set and form substitute spectrums. Thus resulting distortions of the discrete spectrum are consequence of the existence of a rectangular measurement window of retrieval time of *T*.

The DFT procedure generates a substitute spectrum according to the Fourier series irrespective of the researcher's intention. The stripes of the spectrum are determined for admissible frequencies that belong to a limited range. The values of these frequencies and the boundary frequencies of this range are preset and determined by values *T* selected by the researcher. The substitute spectrum transfers approximately true frequency information in the form of a sum of untrue information that is not in the signal. The amount of information is limited by the width of the range $[0,v_{Nyq})$.

The superposition of the waves of the substitute spectrum, taking the phase shifts into account, recovers the course of the harmonic wave. The courses of the residuum determined from the comparison of the original wave and the same wave obtained from the substitute spectrum shows that the recovery is inaccurate and incomplete. This mainly results from the fact that the recovered wave is a result of limited number *N/2* – waves of the substitute spectrum contained in frequency range $[0,v_{Nyq})$ not a infinite number of waves as the Fourier series requires.

The finite retrieval time, selected by the researcher, results in the fact that the average value of the processed fragment of the signal is mostly non-zero. The information transferred by the stripe of the discrete spectrum for $n=0$ is incidental and thus unreliable. The distortion of the spectrum from the constant component can be eliminated by assuming this value to be zero by definition. From the courses of the residuum we can see that the wave recovered from the components of the substitute spectrum was more accurate as compared to the original wave. Such an action does not trigger a change of the components for $n>0$ and does not generate additional spectrum distortions.

The existence of the measurement window, which is an inevitable consequence of the limited retrieval time, as well as discretization of the signal during the A/C conversion are in opposition to the assumptions based on which from series (1) Fourier integral was derived (2). That is why the described distortions of the discrete spectrum are inevitable and with today's level of knowledge cannot be eliminated nor reduced through adjustment windows used when determining the spectrums with analogue methods (spectrometers).

Each adjustment window is a function of weight. The signal multiplied by this function assumes a zero value at the beginning and at the end of the retrieval. This multiplication results in deletion of some information contained therein and addition of other, carried by the window function not contained in the signal. Product (7) assumes a form $\mathbf{F}(v)*\mathbf{W}_k(v)$ and transform of the adjustment window is $\mathbf{W}_k(v)\neq 1$. That is why the discrete Fourier transform of the fragment of the signal improved by the adjustment window, beside the harmonic components of the signal and substitute spectrums also includes the components contained in the window function. This results in an additional inaccuracy of the information contained in the discrete spectrum.

The distortions of the discrete Fourier spectrum of the retrieved fragment of the signal that come from the substitute spectrums can be reduced by extending of the retrieval time. It is known that for constant bandwidth $[0,v_{Nyq})$ as the retrieval time *T* grows the size of the set of admissible frequencies increases and the distance between them decreases. The number of the signal components that are probable to find their place in the set of admissible frequencies grows. As a result, the amount of information that can be accurately conveyed in the discrete spectrum increases. Also increases the number of the components of the substitute spectrums recovering the components of the signal of frequencies that do not belong to this set, which results in a better accuracy of the recovery.

For $T\to\infty$ frequency $v_T\to 0$ the admissible frequency set becomes infinitely large and the discrete spectrum approaches a continuous form. A practical realization of such a case is impossible due to an infinite length of the retrieval time. Even if that were possible, the determination of a discrete spectrum of an infinitely dense set of admissible frequencies would not eliminate the distortions.

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TRANSFORMACJA FOURIERA – WAŻNE NARZĘDZIE W DIAGNOSTYCE WIBROAKUSTYCZNEJ

Streszczenie. Mierzalne wielkości fizyczne, charakteryzujące procesy towarzyszące funkcjonowaniu urządzenia mechanicznego, mogą być wykorzystane jako sygnały niosące zakodowane informacje. Aby je odczytać sygnał należy pobrać i obrobić. Obróbka polega najczęściej na transformacji sygnału do dziedziny częstotliwości metodą DFT. Wykorzystanie takiej obróbki wymaga wiedzy jak informacja diagnostyczna jest wytwarzana w dyskretnej charakterystyce sygnału. W tym celu należy badać model, który odzwierciedla pobranie sygnału w skończonym przedziale czasu, jego przetwarzanie A/C i periodyzację. Badania wykazują, że procedura DFT wytwarza prążki reprezentujące składowe harmoniczne sygnału tylko dla częstotliwości $n\nu_T$, które są całkowitą krotnością odwrotności czasu pobrania. Składowe o innych częstotliwościach są przedstawiane w postaci widm zastępczych. W konsekwencji, widma dyskretne są rezultatem superpozycji fal o częstotliwościach należących do zbioru dopuszczalnych i widma zastępczych reprezentujących fale, które do tego zbioru nie należą.

Słowa kluczowe: diagnostyka wibroakustyczna, numeryczna obróbka sygnałów transformacja Fouriera.