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Deformation of rubber-metal vibration and seismic isolators

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S u m m a r y. The different going is considered near the decision of task about the tensely-deformed state of rubber-metal vibration isolators are considered The method of the solution of a task about deformation of constructions from nearly incompressible material on application moment scheme of finite elements is offered. For the decision of task of nonlinear viscoelasticity deformation of construction the newton-Kantorovich method is used.
K e y w o r d s : finite element method, vibration isolator, nearly incompressibility.

INTRODUCTION

The considerable part of the population of the globe lives in the seismoactive territory. Therefore protection of buildings, constructions, infrastructure objects against adverse effect of pulse and vibration loadings of a geological and technogenic origin is actual. Traditional systems of seismoprotection provide seismic stability at the expense of increase of bearing ability of designs and their connections that stimulates creation of stronger, rigid and monolithic constructions. Thus construction cost in seismic countries in comparison with not seismic increases by 4-12% depending on seismodanger size.

Use of nonconventional systems of vibroseismoprotection allows to provide safety of buildings and constructions at earthquakes and technogenic influences. Thus the budget cost of construction decreases by 3-6%, a material capacity of buildings and constructions for 5-10%, and also the scope of standard designs by control of areas with the increased seismicity extends, height of buildings increases when using the same designs.

Among nonconventional ways of vibration and seismic protection the most perspective is application of vibration and seismic isolators on the basis of rubber-metal designs. On many parameters – simplicity of production, reliability, dimensions, costs such designs surpass traditional schemes of the same appointment. They allow to find essentially new constructive solutions of responsible knots of protection of modern technical systems The analysis of world practice of vibration and seismic protection of cars, buildings and constructions shows that systems with use of rubber-metal blocks is the most perspective. Such systems allow to protect cars and buildings at seismic influences not only in the horizontal and vertical planes, but also from torsion. Besides, application of rubber-metal layered vibration isolators allows to protect buildings and the people who were in them from influences of the subway, motor and railway transport Besides, application of rubber-metal layered vibration isolators allows to protect buildings and the people who were in them from influences of the subway, auto and railway transport. The problem of calculation intense the deformed condition of polymeric elements of designs is rather actual [Lavendel E. E. 1976, Dymnikov S. I. 1968, Kirichevskiy V. V. 2005, Biderman V. L, Sukhova N. A. 1968, Malkov V. M 1998, Ray M. 2010, Dyrda V. I. 2010, Grigolyuk E. I., Kulikov G. M. 1988, Peng R. W. Landel R. F. 1975, Mooney M.A., 1940].

RESEARCH OBJECT

Rubber elements of vibration isolators have rather simple form. However conditions of loading are defined not only external loadings, but also a form of the metal elements of vibration isolators interfaced to them. Besides, rubber possesses weak compressibility. Calculation of parameters intense the deformed condition of such elements of designs probably various methods – experimental, empirical, approximate analytical, numerical. One of the main characteristics of a vibration and seismic isolators is rigidity on compression at vertical loading.

RESULTS OF RESEARCH

Let's consider the multilayered vibration isolators consisting of three metal plates of rather big thickness and two rubber blocks of a cylindrical form. For such vibration and seismic isolators during static tests rigidity on compression was defined at the various size of loading of rubber blocks: diameter 400mm, height 120mm. As a result of simple recalculations it is possible to define dependence between an deformation of a support and the enclosed loading:

$$\Delta = \frac{P}{C_{st}}, \tag{1}$$

where: Δ – seismic support contraction; P – compression loading; C_{st} – vertical rigidity of a seismic support.

On the other hand, for small deformations ($\varepsilon < 0.1$) Dyrda V.I. received analytical dependence between an contraction of a cylindrical rubber layer with free end faces and enclosed loading by the method of Ritz:

$$\Delta = \frac{P_0 h}{3\pi R^2 G}\left(1 - \frac{R}{h\sqrt{6}}\, th \frac{h\sqrt{6}}{R}\right) \quad , \tag{2}$$

where: P_0 – compression loading on rubber layer with free end faces; h – height of a rubber layer R – radius of a rubber layer; G – module of shift of rubber.

At axial compression for small deformations ($\varepsilon < 0.1$) dependence between an contraction of a rubber layer and enclosed loading is defined by a formula;

$$\Delta = \frac{P_0 h}{3\pi R^2 G}. \tag{3}$$

At calculation of seismic support it is necessary to consider that end faces of a rubber layer is vulcanized to metal plates. Then instead of loading P_0 it is necessary to insert the corrected value of real loading P which considers increase in rigidity at the expense of fixing of end faces into formulas (2) and (3):

$$P_0 = \frac{P}{\beta}, \tag{4}$$

where: $\beta = 1 + 0.413\rho^2$ [Payne A. R., 1959] or $\beta = 0.92 + 0.5\rho^2$ [Lavendel E. E., 1976].

V. I. Dyrda suggested to calculate coefficient β on a formula:

$$\beta = 1 + 0.83\rho^2, \tag{5}$$

where: $\rho = \frac{R}{h}$; β – coefficient of increase in rigidity at the expense of fixing of end faces.

Universal numerical method of calculation of rubber vibration and seismic isolators which allows to consider asymmetry of loadings and fixing, and also to receive a full picture intense the deformed condition is finite element method. Thus the traditional final element method doesn't allow take account for a weak compressibility of rubber. For constructions from elastomers the moment scheme of finite elements with use of threefold approximation a component of a vector of movement, a tensor of deformations and function of change of volume of rubber is used. Approximating functions are accepted in the form of square polynoms:

$$\begin{aligned}
u_i = {} & \omega_i^{(000)} + \omega_i^{(100)}\psi^{(100)} + \omega_i^{(010)}\psi^{(010)} + \\
& + \omega_i^{(001)}\psi^{(001)} + \omega_i^{(110)}\psi^{(110)} + \omega_i^{(101)}\psi^{(101)} + \\
& + \omega_i^{(011)}\psi^{(011)} + \omega_i^{(111)}\psi^{(111)} + \omega_i^{(200)}\psi^{(200)} + \\
& + \omega_i^{(020)}\psi^{(020)} + \omega_i^{(002)}\psi^{(002)} + \omega_i^{(210)}\psi^{(210)} + \\
& + \omega_i^{(201)}\psi^{(201)} + \omega_i^{(120)}\psi^{(120)} + \omega_i^{(021)}\psi^{(021)} + \\
& + \omega_i^{(102)}\psi^{(102)} + \omega_i^{(012)}\psi^{(012)} + \omega_i^{(211)}\psi^{(211)} + \\
& + \omega_i^{(121)}\psi^{(121)} + \omega_i^{(112)}\psi^{(112)},
\end{aligned}$$

$$\begin{aligned}
\varepsilon_{11} = {} & e_{11}^{(000)} + e_{11}^{(001)}\psi^{(001)} + e_{11}^{(010)}\psi^{(010)} + \\
& + e_{11}^{(011)}\psi^{(011)} + e_{11}^{(100)}\psi^{(100)} + e_{11}^{(110)}\psi^{(110)} + \\
& + e_{11}^{(101)}\psi^{(101)} + e_{11}^{(020)}\psi^{(020)} + e_{11}^{(102)}\psi^{(102)} + \\
& + e_{11}^{(111)}\psi^{(111)} + e_{11}^{(021)}\psi^{(021)} + e_{11}^{(012)}\psi^{(012)},
\end{aligned}$$

$$\begin{aligned}
\varepsilon_{22} = {} & e_{22}^{(000)} + e_{22}^{(001)}\psi^{(001)} + e_{22}^{(100)}\psi^{(010)} + \\
& + e_{22}^{(101)}\psi^{(101)} + e_{22}^{(001)}\psi^{(001)} + e_{22}^{(110)}\psi^{(110)} + \\
& + e_{22}^{(011)}\psi^{(011)} + e_{22}^{(002)}\psi^{(002)} + e_{22}^{(102)}\psi^{(102)} + \\
& + e_{22}^{(111)}\psi^{(111)} + e_{22}^{(201)}\psi^{(201)} + e_{22}^{(012)}\psi^{(012)},
\end{aligned}$$

$$\begin{aligned}\varepsilon_{33} &= e_{33}^{(000)} + e_{33}^{(100)}\psi^{(100)} + e_{33}^{(010)}\psi^{(010)} + \\ &+ e_{33}^{(110)}\psi^{(110)} + e_{33}^{(001)}\psi^{(001)} + e_{33}^{(200)}\psi^{(200)} + \\ &+ e_{33}^{(020)}\psi^{(020)} + e_{33}^{(101)}\psi^{(101)} + e_{33}^{(011)}\psi^{(011)} + \\ &+ e_{33}^{(210)}\psi^{(210)} + e_{33}^{(120)}\psi^{(120)} + e_{33}^{(111)}\psi^{(111)}, \\ \varepsilon_{12} &= e_{12}^{(000)} + e_{12}^{(001)}\psi^{(001)} + e_{12}^{(100)}\psi^{(100)} + \\ &+ e_{12}^{(110)}\psi^{(110)} + e_{12}^{(100)}\psi^{(100)} + e_{12}^{(101)}\psi^{(101)} + \\ &+ e_{12}^{(011)}\psi^{(011)} + e_{12}^{(002)}\psi^{(002)}, \\ \varepsilon_{13} &= e_{13}^{(000)} + e_{13}^{(100)}\psi^{(100)} + e_{13}^{(010)}\psi^{(010)} + \\ &+ e_{13}^{(001)}\psi^{(001)} + e_{13}^{(110)}\psi^{(110)} + e_{13}^{(020)}\psi^{(020)} + \\ &+ e_{13}^{(011)}\psi^{(011)} + e_{11}^{(020)}\psi^{(020)}, \\ \varepsilon_{23} &= e_{23}^{(000)} + e_{23}^{(100)}\psi^{(100)} + e_{23}^{(010)}\psi^{(010)} + \\ &+ e_{23}^{(001)}\psi^{(001)} + e_{23}^{(001)}\psi^{(001)} + e_{11}^{(200)}\psi^{(200)} + \\ &+ e_{11}^{(110)}\psi^{(110)} + e_{11}^{(101)}\psi^{(101)}, \\ \theta &= \xi^{(000)} + \xi^{(100)}\psi^{(100)} + \xi^{(010)}\psi^{(010)} + \\ &+ \xi^{(001)}\psi^{(001)} + \xi^{(101)}\psi^{(101)} + \xi^{(110)}\psi^{(110)} + \\ &+ \xi e^{(011)}\psi^{(011)} + \xi^{(111)}\psi^{(111)},\end{aligned} \tag{6}$$

where: u_i - components of a vector of movements; ε_{ij} – components of a tensor of deformations; θ - function of change of volume; $\omega_i^{(pqr)}$ – decomposition components; $\psi^{(pqr)}$ – a set of sedate coordinate functions of a look

$$\psi^{(pqr)} = \frac{(x^1)^p}{p!} \cdot \frac{(x^2)^q}{q!} \cdot \frac{(x^3)^r}{r!}, \; (p,\ q,\ r = 0,\ 1,\ 2). \tag{7}$$

Let's find to a contraction for a two-layer seismic support under the influence of loading $P = 50\kappa N$, (the module of shift of rubber $G = 0.63 MPa$). Dyrda V. I. Lisitsa N.I., etc. received the solution of a nonlinear task a precipitation of the continuous cylinder taking into account toughening at end faces by means of the accuracy Runge-Kutt method of the fourth order. he received value a seismic insulator precipitation ($\Delta = 0.0127m$) rather well coincides with experimental data.

The analysis of behavior of vibration and seismic isolator at imposition of cyclic or impulsive loading requires the account of viscoelastic properties of rubber elements. A viscoelasticity determines it dumping properties. For description of viscoelasticity deformation it is possible to take advantage of Volterra's equalizations

$$\sigma = E_0\left[\varepsilon - \int_0^t R(t-\tau)\varepsilon(\tau)d\tau\right]. \tag{8}$$

where: $R(t-\tau)$ – kernel of relaxation.

In addition, for description of behavior of nearly incompressible material different nonlinear laws are used. For example, Peng-Landel's law [Peng R. W., Landel R. F., 1975.]

$$\sigma^{ij} = I_3^{1/2}\left[\mu\left(\left(-I_3^{-1/3} + \frac{2}{9}(2I_1 - 3)(I_3 - 1)\right)G^{ij} + I_3^{-4/3}g^{ij}\right) + \frac{B}{2}(I_3 - 1)G^{ij}\right], \tag{9}$$

where: I_1, I_3 are invariants of tensor of deformations; μ, B are constant of material, G^{ij} is a metrical tensor.

We replace resilient permanent is the module of compression B and module of shear μ we get the Volterra's operators

$$\begin{aligned}\sigma^{ij} = I_3^{1/2}\Bigg[&\mu\left(\left(-I_3^{-1/3} + \frac{2}{9}(2I_1 - 3)(I_3 - 1)\right)G^{ij} + \right. \\ &\left. + I_3^{-4/3}g^{ij}\right) + \frac{B}{2}(I_3 - 1)G^{ij} - \\ &- \mu\int_{-\infty}^{t} R_\mu(t-\tau)\left(-I_3^{-1/3} + \frac{2}{9}(2I_1 - 3)(I_3 - 1)\right)G^{ij}d\tau - \\ &- \mu\int_{-\infty}^{t} R_\mu(t-\tau)I_3^{-4/3}g^{ij}G^{ij}d\tau \\ &- \frac{B}{2}\int_{-\infty}^{t} R_b(t-\tau)(I_3 - 1)G^{ij}d\tau\Bigg].\end{aligned} \tag{10}$$

The tensor of deformations can be presented as a sum linear and nonlinear constituents

$$\begin{aligned}\varepsilon_{ij} &= \varepsilon_{ij}^{l} + \varepsilon_{ij}^{n}, \\ \varepsilon_{ij}^{l} &= \frac{1}{2}\left(c_j^k u_{k,i} + c_i^k u_{k,j}\right), \\ \varepsilon_{ij}^{n} &= \frac{1}{2}u_{k,i}u_{,j}^{k}.\end{aligned} \tag{11}$$

Then the invariants of Cauchy- Green's tensor of deformations also can be presented as a sum linear and nonlinear parts

$$\begin{aligned}I_1 &= I_1^l + I_1^n, \\ I_1^l &= \varepsilon_{11}^l + \varepsilon_{22}^l + \varepsilon_{33}^l, \\ I_1^n &= \varepsilon_{11}^n + \varepsilon_{22}^n + \varepsilon_{33}^n.\end{aligned} \tag{12}$$

We put (12) in (10) and we lay out I_3 in the Taylor series about with $I_3 = 1$ as the center of the circle of convergence. Then we cast aside by virtue of weak compressibility of material members of decomposition the second order of trifle and we get the linearized correlation. For the decision of task of nonlinear deformation of constructions different methods are used [Dymnikov S.I., 1968, Lavendel E.E., 1980, Kirichevskiy V.V., 2005]. Most effective among them is the modified Newton-

Kantorovich method. At the use of this going near the decision of task on every step on loading get the specified linearized equalization

$$Ku^{i+1} = -N(u^i) - P_k\,, \qquad (13)$$

where: K is matrix of inflexibility of construction; $N(u^i)$ is a vector of the nonlinear additions, conditioned by physical and geometrical non-linearity; P_k is vector of key forces; u is a vector of the key moving. On the basis of this approach the task of determination is decided seismic support.

The problem is also solved on the basis of the moment scheme of finite elements on the basis of the obtaining complex «MIRELA+». In fig. 1-4 are presented finite element model and distribution of movements and tension in the radial section of a rubber element

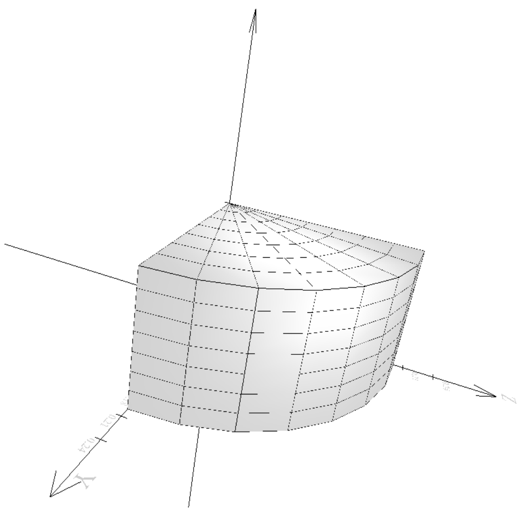

Fig. 1. Finite element model

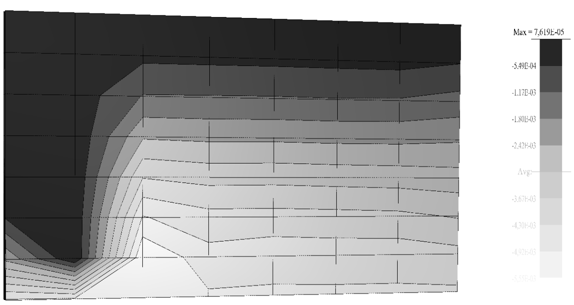

Fig. 2. Axial movement

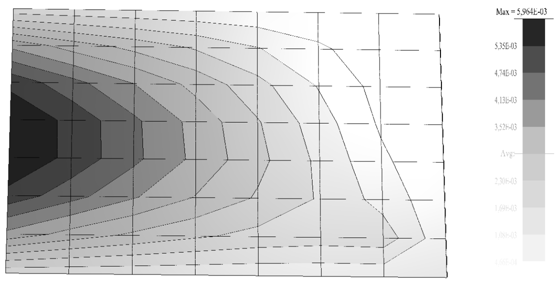

Fig. 3. Radial movement

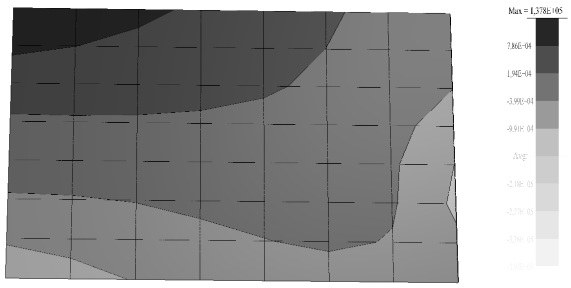

Fig. 4. Stress σ_{12}

As comparison we will give a calculation example a precipitation of a rubber layer of a seismic support on formulas (2) and (3). Results of calculations are given in table.

Table. Seismic support contraction

Indicator	Experiment	Formula		FEM
		(2)	(3)	
Δ, m	0.012	0.0084	0.0128	0.0122

CONCLUSIONS

The method of calculation of vibration and seismic isolators is developed. The analysis of the received results shows that use of a finite element method allows to receive a complete picture of distribution of tension and movements on the volume of an element of a design taking into account weak compressibility of a material.

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ДЕФОРМИРОВАНИЕ РЕЗИНОМЕТАЛЛИЧЕСКИХ ВИБРОСЕЙСМОИЗОЛЯТОРОВ

Юрий Козуб

А н н о т а ц и я. Рассмотрены различные подходы к решению задачи о напряженно-деформированном состоянии резинометаллических виброизоляторов. Предложен метод решения задачи о деформировании элементов конструкций из слабосжимаемого материала основанный на применении моментной схемы конечных элементов. Для решения задачи нелинейного вязкоупругого деформирования конструкции используется метод Ньютона-Канторовича.

К л ю ч е в ы е   с л о в а. Метод конечных элементов, виброизолятор, слабая сжимаемость.