

## The size reduction theories of solid foods

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**Summary.** The paper presents the characteristics of grinding processes of solid foods. Especially the factors determining the size reduction process of solid foods were described and the most commonly used grinding theories were presented. The most important grinding laws were discussed. It can be concluded that the description of grinding process of solid foods is more complicated than of the comminuting process of other materials such as minerals and there are neither universal theories reported today nor any empirical comminution laws describing the process of solid foods size reduction over the entire range of particle sizes, especially in the field of ultrafine grinding.

**Key words:** grinding, food, grinding energy.

### INTRODUCTION

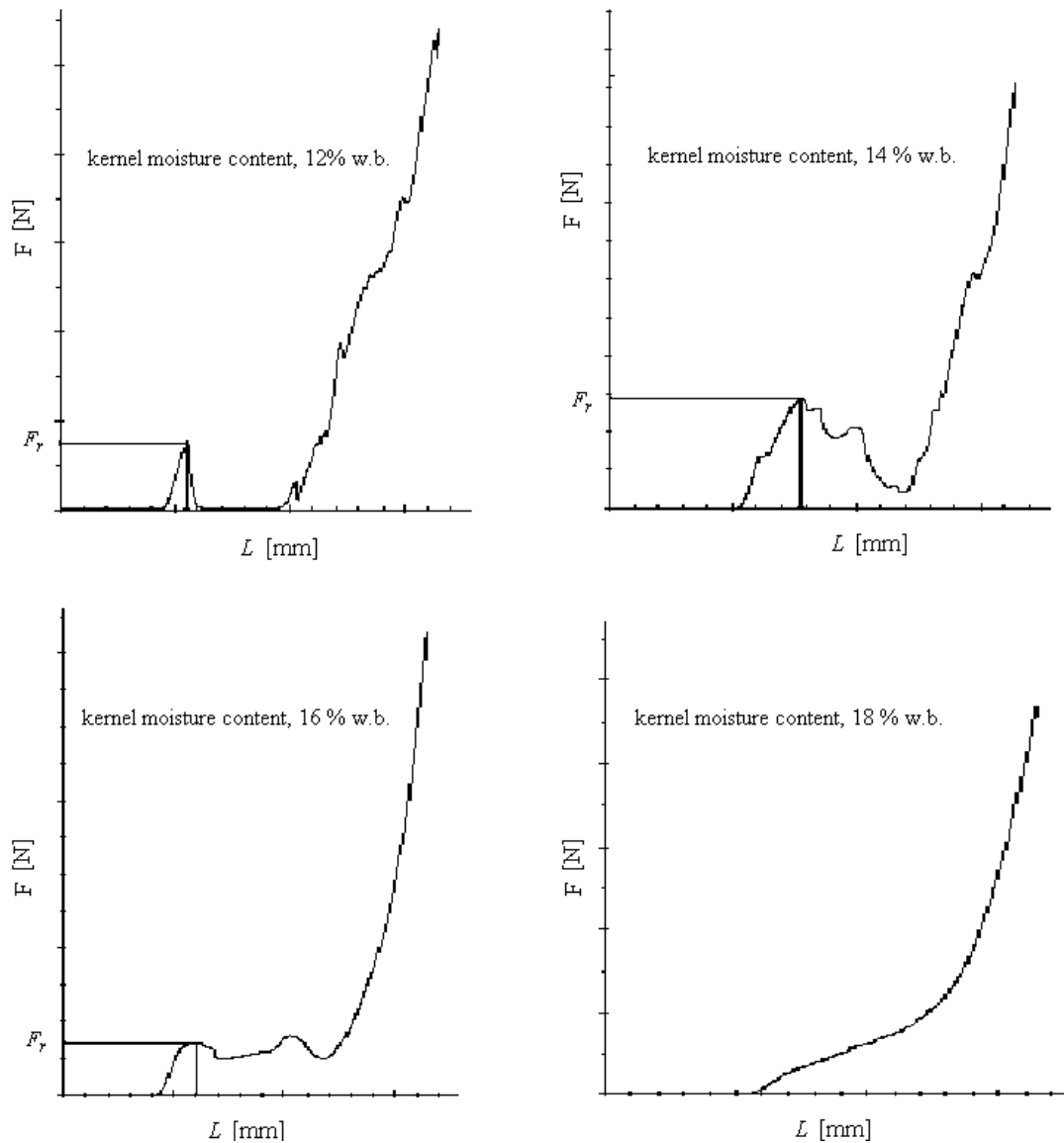
Size reduction has got many benefits in solid food processing. For example, dry milling yields flour and semolina, which can be used for making breakfast cereals, puffed snacks, pasta products and staple food items like cuscus. Dry milling has also been reported to help in redistribution of aflatoxins which got concentrated in the by-products during screening, whereas the main products contained only 12–30% of the level in the original grain [19]. This size reduction process is usually conducted by mechanical destruction of large fragments by impact or compressive action in devices of various engineering designs.

However, this process is one of the most energy consuming ones in food processing. For example, grinding consumes a majority of the total power during the wheat flour milling and during feed production. Among the physical properties of solid foods, the mechanical properties have the greatest influence on grinding energy. The amount of energy needed to fracture food is determined by its tendency to crack (its friability), which in turn depends on the structure of the food. Harder foods absorb higher energy and consequently require a greater energy input to create fractures [6]. The method of grinding

depends on the properties of raw material. For example, compression forces are used to fracture friable foods; combined impact and shearing forces are necessary for fibrous materials, and shearing forces are used for fine grinding of softer foods. It is thought that foods fracture at lower stress levels if force is applied for longer times. The extent of size reduction, energy expended and the amount of heat generated in food, therefore, depend on both the size of the forces that are applied and the time in which the food is subjected to the forces [7].

Moisture content greatly affects the mechanical properties of foods. Figure 1 presents an example of compression curves obtained for barley kernels with different moisture contents. Thus, the water content significantly affects both the degree of fineness and the mechanism of breakdown in foods. Dry solid foods are brittle and easy to grind, they also need less energy for grinding [12]. An increase in water content causes an increase in food plasticity, especially when higher energy is required for grinding. During the comminution of solid foods, also the raw material temperature has a significant influence on the grinding results. Low temperature causes change of mechanical properties of food. The materials become brittle, they crumble easily, permitting grinding to a finer and more consistent size. An especially considerably smaller size can be obtained under cryogenic conditions [9]. Apart from this grinding in inert atmosphere of liquid nitrogen, gas reduces the risk of fire hazards and dust explosion [13].

Three types of force are used to reduce the size of foods: compression forces, impact forces and shearing (or attrition) forces. In most size reduction equipment all the three forces are present, but often one is more important than the others. For example, when hammer mill is used for pulverizing, the impact forces are more important than shearing forces, and compression forces are the least important [5].



**Fig. 1.** Exemplary curves obtained during compression of barley kernel with different moisture contents,  $F$  – loading force,  $L$  – displacement of measuring head,  $F_r$  – rapture force

Grinding energy is one of the most common measurement parameter. To calculate the needed grinding energy against the grain size reduction many of size reduction theories were proposed. The aim of the work was to work out the characteristics and comparison of the grinding laws.

#### THE GRINDING LAWS

The amount of energy required for size reduction of solid foods can be theoretically calculated based on the following equation:

$$dE = -K \frac{dx}{x^n}, \quad (1)$$

where:  $dE$  is the energy required in breaking a unit mass of diameter  $x$  about size  $dx$ ,  $K$  and  $n$  are constants depending on the ground material and grinding methods.

The relation (1) has been classically interpreted in many ways, referred to as Rittinger, Bond, Kick, Sokolowski and other grinding theories. The equation (1) has many solutions depending on  $n$  value. After the integration of equation (1) in the range from  $D$  to  $d$  we obtain:

$$E = \int_D^d -Kx^{-n} dx, \quad (2)$$

where:  $D$  and  $d$  represent the particle size before and after grinding, respectively. When we assume, that  $n > 1$ , the solution of equation (2) is the following:

$$E = \left( \frac{K}{n-1} \right) \left( \frac{1}{d^{n-1}} - \frac{1}{D^{n-1}} \right). \quad (3)$$

When  $n = 1$ , integration of the basic equation gives Kick's law [11]:

$$E_K = K_K (\ln D - \ln d). \quad (4)$$

The Kick's law states that the energy required to reduce the size of particles is proportional to the ratio of the initial size of a typical dimension (for example the diameter of the particles) to the final size of that dimension. This relation is derived directly from the elasticity theory of ideal brittle solids. In practice it has been found that Kick's law gives reasonably good results for coarse grinding in which there is a relatively small increase in surface area per unit mass [8].

Rittinger based on empirical findings assumed that  $n = 2$  and the integration of equation (2) give the following solution:

$$E_R = K_R \left( \frac{1}{d} - \frac{1}{D} \right). \quad (5)$$

Rittinger's law [5] states that the energy required for size reduction is proportional to the change in surface area of the particles. Rittinger's law gives better results with fine grinding where there is a much larger increase in surface area.

According to Rittinger [5], the  $D$  and  $d$  should be calculated as follows:

$$D = \frac{1}{\sum_j \frac{G_j}{D_j}}, \quad (6)$$

$$d = \frac{1}{\sum_i \frac{g_i}{d_i}}, \quad (7)$$

where:  $G_j$  and  $g_i$  represent the mass fractions of particles  $D_j$  and  $d_i$ , respectively.

Bond [2] assumed that  $n = 1.5$  and the integration of equation (2) give the following solution:

$$E_B = K_B \left( \frac{1}{\sqrt{d_{80}}} - \frac{1}{\sqrt{D_{80}}} \right), \quad (8)$$

where:  $K_B$  is the Bond's constant and expresses the energy needed to reduce the unit of mass theoretically from infinity to such a size for which 80% of particles is sieved to values lower than 100  $\mu\text{m}$  [15].

Bond expressed  $K_B$  as function of  $W_i$ :

$$E_B = 10W_i \left( \frac{1}{\sqrt{d_{80}}} - \frac{1}{\sqrt{D_{80}}} \right), \quad (9)$$

where:  $W_i$  is defined as work index and characterizes the resistance of material to grinding. This index can be

also defined as comminution energy of the unit of mass from infinity size up to 100  $\mu\text{m}$  [4]:

$$E(\infty \rightarrow 100) = 10W_i \left( \frac{1}{100} - \frac{1}{\sqrt{\infty}} \right) = W_i. \quad (10)$$

Using the laboratory methods, Bond determined, the values of  $W_i$  for different materials. Therefore, his grinding theory is very often used in practice. Bond's law combines the Kick's and Rittinger's laws by means of fitting factors which take into account the mechanical properties of the materials subjected to size reduction and their destruction conditions.

However, it is worth noting that the values of  $D$  and  $d$  in the equations (4) and (5) had different meanings in comparison to equation (9), although many authors have not differentiated between them [4, 20]. Sokołowski [17] has found out that the parameters  $D$  and  $d$  should be calculated as follows:

$$D_w = \left( \frac{1}{\sum_j \frac{G_j}{D_j^w}} \right)^{\frac{1}{w}}, \quad (11)$$

$$d_w = \left( \frac{1}{\sum_j \frac{g_j}{d_i^w}} \right)^{\frac{1}{w}}, \quad (12)$$

where:  $G_j$  and  $g_i$  have the same meaning as in the equations (6) and (7), respectively.

Sokołowski [17] showed that the value of exponent ( $w$ ) changed from 0.25 to 0.85 for comminuted materials and when we assume the average value of  $w = 0.5$ , the error of estimation is low, but the solutions of equations (11) and (12) are simple, and we obtain:

$$D_{0.5} = \left( \frac{1}{\sum_j \frac{G_j}{\sqrt{D_j}}} \right)^2, \quad (13)$$

$$d_{0.5} = \left( \frac{1}{\sum_j \frac{g_j}{\sqrt{d_i}}} \right)^2, \quad (14)$$

then, the solution of equation (3) can be calculated as follows:

$$E_S = K_S \left( \frac{1}{\sqrt{d_{0.5}}} - \frac{1}{\sqrt{D_{0.5}}} \right), \quad (15)$$

where:  $K_S$  is the Sokołowski's constant and  $d_{0.5}$  and  $D_{0.5}$  represents the particle size before and after grinding, respectively

It is worth noting that equations (4), (5), (8) and (15) were developed mainly from studies of hard mineral materials such as coal and limestone, but many authors have used these laws for description of comminution process of solid foods. Djantou et al. [4] studied the effect of pre-treatment on the grinding ability of dried mango for powder production. They observed that values of constants  $K_K$  and  $K_R$  differed significantly. Walde et al. [20] studied the grinding characteristics of microwave dried wheat. They found that the values of  $K_B$  were almost two times higher than values of  $K_K$ . Pujol et al [16] showed that Sokołowski's constant changed from 22  $\text{kJkg}^{-1}\text{mm}^{0.5}$  for soft wheat to 54  $\text{kJkg}^{-1}\text{mm}^{0.5}$  for *durum* wheat. Dziki [5] found that this constant also depended on the method of grinding.

Charels [3] extended existing theories of comminution and proposed the equation to calculate the comminution energy ( $E$ ) necessary to obtain the particle size  $y$  from the material with the initial size  $x_{max}$ :

$$E = \int_0^y \int_{x_{max}}^x (-Kx^{-n} dx) dM, \quad (16)$$

where:  $dM$  represents the mass of particles in the range of sizes from  $x$  to  $x+dx$ . According to Stambolidis [18] the mass of particles with sizes lower than  $x$  can be expressed as:

$$M_x = W_o \left( \frac{x}{y} \right)^\alpha, \quad (17)$$

where:  $W_o$  is the mass of particles taken for comminution and  $\alpha$  is the coefficient of particle size distribution. The derivative of equation (17) is as follows:

$$dM_x = \alpha W_o \frac{x^{\alpha-1}}{y^\alpha} dx. \quad (18)$$

The solution of equation (16), after dividing at both sides of equation by  $W_o$  can be expressed as:

$$E_{Ch} = \frac{K_{Ch}\alpha}{(n-1)(\alpha-n+1)} y^{1-n}, \quad (19)$$

where:  $K_{Ch}$  is a constant dependent on the properties of ground material.

The detailed way of determining equation (19) and coefficients  $\alpha$  i  $n$  was described by Stamboliadis [18]. He found out that for most materials the expression  $(\alpha-n+1)$  is equal to zero, thus the equation (19) cannot be used to determine the energy of comminution and he proposed the formula:

$$E_s = \frac{C}{n-1} \frac{\ln y^\alpha}{y^\alpha}. \quad (20)$$

Hukki [10] assumed that in the equation (19) exponent  $(1-n)$  is not constant, but depends on the size of comminuted material and degree of fineness. For large particles (order of magnitude 0,01 m) and when the degree of fineness is low, the grinding energy is mainly derived from the volume of material and Kick's theory of grinding is adequate. For smaller grinding (order of magnitude of ground particles 0,01 m) the Bond's grinding theory should be used and for the finest grinding (orders of magnitude 0,001), the grinding energy is proportional to the area of comminuted particles and thus the Rittinger's grinding theory can be used. It is caused by the fact that the small particles need much more stresses to comminution [1]. Similar conclusions were obtained by Morrell [14] which modified the Bond's theory and proposed the following equation:

$$E_r = M_i \cdot K (d_{80}^{f(d)} - D_{80}^{f(D)}), \quad (21)$$

where:  $M_i$  represents the index depending on the method of grinding,  $K$  is the grinding constant, and  $d_{80}$  and  $D_{80}$  have the same meaning as in the equation (8). For particles with size  $x$ , the function describing the changing of exponent can be calculated as follows [14]:

$$f(x) = -(a + x^b), \quad (23)$$

where:  $a$  and  $b$  are constants, and  $x$  is such a size of the screen diameter for which 80% of particles are sieved.

Beside the described grinding theories there are many others, but they are seldom used in practice.

## CONCLUSIONS

1. The description of grinding of solid foods is more complicated than of the comminuting processes of minerals or metals. It is due to the fact that the results of grinding depend strongly on food moisture content and temperature.
2. Some foods are sensitive to increases in temperature or oxidation during comminution, and mills are therefore cooled by chilled water, liquid nitrogen or carbon dioxide.
3. There are neither theories reported today nor any empirical comminution laws describing the process of solid foods comminution in the field of fine and ultrafine grinding.
4. None of the common mathematic approximations of the grinding behavior of solid foods is equally accurate over the entire range of particle size.

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#### TEORIE ROZDRABNIANIA ŻYWNOSCI O POSTACI STAŁEJ

Streszczenie. W pracy przedstawiono charakterystykę procesu rozdrabniania żywności o postaci stałej. W szczególności omówiono czynniki determinujące proces rozdrabniania żywności oraz skupiono się na najistotniejszych teoriach procesu rozdrabniania. Wykazano, że opis procesu rozdrabniania żywności jest znacznie bardziej skomplikowany niż dekohezja innych materiałów, takich jak np. materiały mineralne i nie ma uniwersalnej teorii rozdrabniania opisującej proces redukcji wymiarów żywności w całym przedziale wielkości cząstek, a w szczególności podczas rozdrabniania bardzo drobnego.

Słowa kluczowe: rozdrabnianie, żywność, energia rozdrabniania.