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EXTREMAL DEPENDENCIES ON COMMODITY FUTURES MARKETS

Key words: tail dependencies, Copula-ARMA-GARCH, commodity futures contracts

ABSTRACT. The aim of this study was to assess dependencies between extreme rates of return from commodity futures contracts on selected markets in the years 2000-2018. In periods of upheavals and turbulences, in markets for investors and portfolio management, it is crucial to estimate the probability of risk factors simultaneously taking extreme values. The analyses were conducted on dependencies between extreme rates of return (asymptotic dependencies) on markets of futures contracts for energy, metals and agricultural products in the years 2000-2018, applying the Copula-ARMA-GARCH models and tail dependence coefficients. Relatively strong and permanent asymptotic dependencies were found for pairs of futures contracts for crude oil and heating oil, while either no such dependencies were observed or only appeared during the subprime crisis and assumed very low values for other energy pairs of futures contracts and pairs of agricultural futures contracts, in which at least one of the contracts was concluded for soft commodities.

INTRODUCTION

Since the beginning of the 21st century we have been observing the dynamic development of derivative instruments in commodity exchange markets. The main causes for this phenomenon include an influx of capital from financial investors, the development of e-trade as well as the emergence of passively managed indexed funds and exchange traded funds (ETF) [Irwin, Sanders 2012]. The key reason for the decision to invest in commodities is connected both with the potential to attain comparable or even higher rates of return than those available on the markets of traditional financial assets and the potential to diversify portfolios [Gorton, Rouwenhorst 2004, Inamura et al. 2011]. In turn, Zvi Bodie and Victor Rosansky [1980] stated that a 40% share of commodity futures contracts considerably reduces the portfolio risk, while at the same time increases the expected return. Similar results were also reported by other researchers[1]. Another reason for investments in commodities results from their role as safeguards against inflation [Gorton, Rouwenhorst 2004], since commodity prices grow in periods of high inflation. Additionally, prices for many commodities increase as a result of shock triggered by catastrophic events (e.g. drought, hurricane, war), which threaten supplies of these commodities and exert an adverse effect on the markets of stocks and bonds [Krawiec 2016].

[1] A list of these studies was given e.g. by Adam Zaremba [2015] and Monika Krawiec [2016].

An increased number of participants involved in commodity transactions, commodity futures contracts and commodity indexes has resulted in the accelerated integration of certain commodity markets, as well as the integration of commodity and financial markets [see: Irwin, Sanders 2012, Tang, Xiong 2012, Attaf et al. 2015]. Apart from the structure of "averaged" dependencies in commodity and financial markets, in the aspect of risk management, investors need to focus on the analysis of dependencies between extreme values, called extremal dependencies or asymptotic dependencies. Such an analysis makes it possible to determine the probability of a simultaneous occurrence of extreme rates of return (extremely high or low) in various markets as a result of extreme events (e.g. natural disasters, wars, economic crises, speculation on these markets). Extreme events occur rarely, but when they do occur they lead to huge losses. The primary tool in the analysis of such dependencies is provided by copulas, which enable the modelling of the structure of these dependencies excluding marginal distributions. In the years 2000-2018, apart from dynamic growth on commodity futures contracts markets, it was possible to observe numerous upheavals connected with the occurrence of extreme events. Thus, the aim of this study was to assess dependencies between extreme rates of return from commodity futures contracts on selected markets (energy, metals, agricultural products), in the years 2000-2018, applying the Copula-ARMA-GARCH models. Although there are many studies on dependencies between commodities and macroeconomic variables, oil and other commodities [see Attaf et al. 2015], the literature on the dependencies between other commodities is sparse, especially studies on the extremal dependencies between commodities from one sector. Thus, this work significantly complements existing studies on extremal dependencies between commodity futures contracts.

MATERIAL AND METHODS

This study is based on a historical time series of daily closing quotes for commodity futures contracts from the period 2000-2018 [stooq.pl], included in the Thomson Reuters Equal Weight Commodity Index. Contracts were divided into three groups: energy (heating oil HO.F, natural gas NG.F, crude oil CL.F), metals (gold GC.F, silver SI.F, platinum PL.F, copper HG.F) and agricultural products (corn ZC.F, wheat ZW.F, soybean ZS.F, soybean oil ZL.F, cotton CT.F, sugar SB.F, coffee KC.F, cocoa CC.F). Analyses were conducted on percentage daily logarithmic rates of return from futures contracts:

$$r_{i,t} = 100 \ln\left(P_{i,t} / P_{i,t-1}\right), \quad (i = 1,\dots,N,\ t = 1,\dots,T) \tag{1}$$

where $P_{i,t}$ denotes the closing quote of the i-th futures contract in period t.

The dynamics of dependencies between rates of return from prices of commodity futures contracts were described applying the Copula-ARMA-GARCH models, while the strength of an asymptotic dependence was measured using dynamic tail dependency coefficients.

The application of the conditional copula enables the modelling of joint distributions of the N-dimensional vector $\mathbf{y}_t = (y_{1,t},\dots,y_{N,t})$ $(t = 1,\dots,T)$, conditional in relation to a set of information $\mathcal{F}_{t-1}$ available by time point $t-1$. The general model of a conditional copula takes the form [Patton 2007]:

$$y_{1,t}|\mathcal{F}_{t-1} \sim F_{1,t}(\cdot|\mathcal{F}_{t-1}),..., y_{N,t}|\mathcal{F}_{t-1} \sim F_{N,t}(\cdot|\mathcal{F}_{t-1}) \tag{2}$$

$$\mathbf{y}_t|\mathcal{F}_{t-1} \sim F_t(\cdot|\mathcal{F}_{t-1}) \tag{3}$$

$$F_t(\mathbf{y}_t|\mathcal{F}_{t-1}) = C_t(F_{1,t}(y_{1,t}|\mathcal{F}_{t-1}),..., F_{N,t}(y_{N,t}|\mathcal{F}_{t-1})|\mathcal{F}_{t-1}) \tag{4}$$

where: C_t is the copula, while F_t and $F_{i,t}$ are the function of the joint distribution $\mathbf{y}_t$ and the marginal distribution function $y_{i,t}$ at time point t.

It was assumed that the rates of return $r_{i,t}$ $(i = 1,..., N, t = 1,...,T)$:

$$r_{i,t} = \mu_{i,t} + y_{i,t} \tag{5}$$

$$\mu_{i,t} = E(r_{i,t}|\mathcal{F}_{t-1}) \tag{6}$$

$$y_{i,t} = \sqrt{h_{i,t}}\varepsilon_{i,t}, \ \varepsilon_{i,t} \sim iid(0;1) \tag{7}$$

$$h_{i,t} = Var(r_{i,t}|\mathcal{F}_{t-1}) \tag{8}$$

are described using the ARMA-GARCH model[2]. Empirical analysis was conducted using the GARCH models [Bollerslev 1986] and asymmetric GARCH models, i.e. EGARCH [Nelson 1991], GJR-GARCH [Glosten et al. 1993] and APARCH [Ding et al. 1993] with various innovation distributions. In the Copula-ARMA-GARCH model, it is assumed that the conditional joint distribution of the *N*-dimensional vector $\boldsymbol{\varepsilon}_t$ is modelled applying a conditional copula with conditional correlations $\mathbf{R}_t$. In turn, the matrix of conditional correlations is determined from the model of dynamic conditional correlations (DCC) [Engle 2002]:

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t \tag{9}$$

$$\mathbf{D}_t = diag\left(\sqrt{h_{1,t}},...,\sqrt{h_{N,t}}\right) \tag{10}$$

$$\mathbf{R}_t = (diag(\mathbf{Q}_t))^{-1/2}\mathbf{Q}_t(diag(\mathbf{Q}_t))^{-1/2} \tag{11}$$

$$\mathbf{Q}_t = \left(1 - \sum_{k=1}^{K} a_k - \sum_{l=1}^{L} b_l\right)\overline{\mathbf{Q}} + \sum_{k=1}^{K} a_k \mathbf{u}_{t-k}\mathbf{u}'_{t-k} + \sum_{l=1}^{L} b_l \mathbf{Q}_{t-l} \tag{12}$$

where: conditional variance $h_{i,t}$ is modelled using the GARCH model, $\overline{\mathbf{Q}}$ is an unconditional covariance matrix of variables $\mathbf{u}_t$, where $u_{i,t} = y_{i,t}/\sqrt{h_{i,t}}$, and parameters a_k, b_l meet conditions $a_k \geq 0$, $b_l \geq 0$, $\sum_{k=1}^{K} a_k + \sum_{l=1}^{L} b_l < 1$.

When a_k and b_l equal 0, the DCC model is reduced to the constant conditional correlation (CCC) model [Bollerslev 1990]. In the empirical study 2-dimensional Copula-ARMA-GARCH models were estimated applying the maximum likelihood method and the semiparametric transformation method for innovations from the GARCH models. Thus, Gaussian and Student *t* copulas were investigated.

[2] ARMA-GARCH models are presented e.g. in [Tsay 2005].

Tail dependence coefficients (TDC) constitute basic measures of dependencies between extreme values of random variables X_1 and X_2. These coefficients estimate the conditional probability of a simultaneous occurrence of extreme values for both variables (extremely high (low) values of both variables). If variables X_1 and X_2 have distribution functions F_1 and F_2 and are connected with copula C, then dependence coefficients in the upper tail λ^U and lower tail λ^L are defined by formulas (13) and (14) [M. Doman, R. Doman 2014]:

$$\lambda^U = \lim_{\alpha \to 1^-} P\left(X_2 > F_2^{-1}(\alpha) \middle| X_1 > F_1^{-1}(\alpha)\right) = \lim_{\alpha \to 0^+} \frac{\hat{C}(\alpha,\alpha)}{\alpha} \tag{13}$$

$$\lambda^L = \lim_{\alpha \to 0^+} P\left(X_2 \le F_2^{-1}(\alpha) \middle| X_1 \le F_1^{-1}(\alpha)\right) = \lim_{\alpha \to 0^+} \frac{C(\alpha,\alpha)}{\alpha} \tag{14}$$

where $\hat{C}(u_1,u_2) = u_1 + u_2 - 1 + C(1-u_1, 1-u_2)$.

For the Gaussian copula $\lambda^U = \lambda^L = 0$. In turn, for the Student *t* copula with ν degrees of freedom and the correlation coefficient $\rho > -1$:

$$\lambda^U = \lambda^L = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}}\right) \tag{15}$$

RESULTS

The estimation of the Copula-ARMA-GARCH models was conducted in two stages. In the first stage, the ARMA-GARCH models were fitted to one-dimensional series of rates of return from commodity futures contracts. These models were selected based on information criteria (Akaike information criterion, AIC and Bayesian information criterion, BIC) and properties of residuals. Types of fitted ARMA-GARCH models are presented in Table 1. Over the analysed period numerous upheavals related to crises were

Table 1. Types of fitted ARMA-GARCH models for investigated series of rates of return from commodity futures contracts

Contract	Model	Contract	Model	Contract	Model
HO.F	GARCH(1,1) std	NG.F	ARMA(0,1)-GARCH(1,1) sstd	CL.F	ARMA(0,1)-EGARCH(1,1) sstd
GC.F	GARCH(1,1) sstd	HG.F	ARMA(1,0)-EGARCH(1,1) sstd	PL.F	ARMA(1,2)-GARCH(1,1) sstd
SI.F	APARCH(1,1) sstd	ZC.F	GARCH(1,1) sstd	ZW.F	APARCH(1,1) sstd
ZS.F	APARCH(1,1) sstd	ZL.F	GARCH(1,1) sstd	CT.F	GARCH(1,1) std
SB.F	GARCH(1,1) sstd	CC.F	GARCH(1,1) sstd	KC.F	EGARCH(1,1) sstd

HO.F – heating oil, NG.F – natural gas, CL.F – crude oil, GC.F – gold, SI.F – silver, PL.F – platinum, HG.F – copper, ZC.F – corn, ZW.F – wheat, ZS.F – soybean, ZL.F – soybean oil, CT.F – cotton, SB.F – sugar, KC.F – coffee, CC.F – cocoa

Std – Student *t* distribution for innovation, sstd – skewed Student *t* distribution for innovation

Source: own adjustment based on data from financial service [stooq.pl]

observed in the investigated markets. Thus, the models describing volatility are either GARCH models or asymmetric GARCH models with the skewed Student *t* distribution as the innovation distribution.

In the second stage of the study two-dimensional Student *t* or Gaussian models of conditional copulas were fitted with DCC dynamics for futures contracts for energy, metals and agricultural products. An appropriate model was selected based on information criteria (AIC and BIC). Fitted types of copula models are presented in Table 2. For most pairs of analysed series of rates of return, a conditional Student *t* copula model (21 pairs) and a conditional Gaussian copula model (15 pairs) were fitted with DCC dynamics, for one pair it was a constant conditional Student *t* copula model.

Table 2. Types of fitted copula models for investigated pairs of series of rates of return from commodity futures contracts

Model	Contract–contract
Conditional Student *t* copula model with DCC dynamics	HO.F-CL.F; GC.F-SI.F; GC.F-PL.F; GC.F-HG.F; SI.F-PL.F; SI.F-HG.F; PL.F-HG.F; ZC.F-ZW.F; ZC.F-ZS.F; ZW.F-ZS.F; SB.F-KC.F; CC.F-KC.F; ZC.F-ZL.F; ZC.F-CT.F; ZW.F-ZL.F; ZW.F-CT.F; ZW.F-CC.F; ZS.F-ZL.F; ZS.F-CT.F; ZS.F-KC.F; ZL.F-CT.F
Conditional Gaussian copula model with DCC dynamics	HO.F-NG.F; NG.F-CL.F; CT.F-SB.F; CT.F-CC.F; CT.F-KC.F; SB.F-CC.F; ZC.F-SB.F; ZC.F-CC.F; ZC.F-KC.F; ZW.F-SB.F; ZW.F-KC.F; ZS.F-SB.F; ZL.F-SB.F; ZL.F-CC.F; ZL.F-KC.F
Constant conditional Student *t* copula model	ZS.F-CC.F

HO.F – heating oil, NG.F – natural gas, CL.F – crude oil, GC.F – gold, SI.F – silver, PL.F – platinum, HG.F – copper, ZC.F – corn, ZW.F – wheat, ZS.F – soybean, ZL.F – soybean oil, CT.F – cotton, SB.F – sugar, KC.F – coffee, CC.F – cocoa
Source: own adjustment based on data from [stooq.pl]

Asymptotic dependencies between pairs of series of rates of return from commodity futures contracts in the analysed markets were measured applying dynamic dependence coefficients in the upper and lower tails in the period 2000-2018. These coefficients are presented in Figures 1-3.

In the case of futures contracts for energy for rates of return from contracts for heating oil and natural gas (HO.F-NG.F) and for natural gas and crude oil (NG.F-CL.F), two-dimensional models were estimated for the conditional Gaussian copula with the DCC dynamics (Table 2). This means that tail dependence coefficients equal 0. In turn, tail dependencies in the joint conditional distribution for rates of return from futures contracts for heating oil and crude oil (HO.F-CL.F) were relatively strong (0.1-0.7) and permanent in the analysed period (Figure 1). Thus, it may be stated that the probability of transfer of extreme events between the markets for these contracts is high. This phenomenon is observed, because heating oil is obtained from the distillation of crude oil.

For asymptotic dependencies between pairs of rates of return from commodity futures contracts for metals the situation is very different. In the case of a pair of futures contracts for gold and silver (GC.F-SI.F), a relatively high level of dependencies was observed in

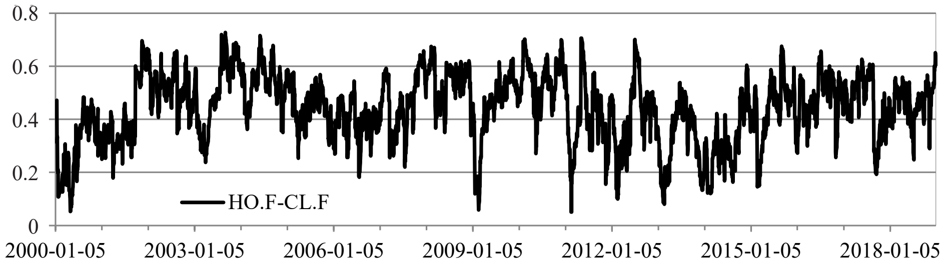

Figure 1. Tail dependence coefficients for a pair of futures contracts for energy
Source: own adjustment based on data from financial service [stooq.pl]

tails from October 2004 to the end of 2018 (0.2-0.44). Also, in the case of pairs of futures contracts for gold and platinum (GC.F-PL.F), as well as silver and platinum (SI.F-PL.F) we may see a higher level of asymptotic dependencies from 2006 than in the previous years (2000-2005). The lowest level of asymptotic dependencies was recorded for futures contracts for copper in pairs with other contracts (GC.F-HG.F, SI.F-HG.F, PL.F-HG.F). The probability of the joint occurrence for the extreme rates of return in these cases increased in the period 2006-2014 (although it did not exceed 0.15) and obviously was related to turbulences in the markets for futures contracts observed during crises (Figure 2).

The obtained results indicate a diverse dynamic of dependencies in tails of joint conditional distributions for pairs of rates of return from agricultural futures contracts.

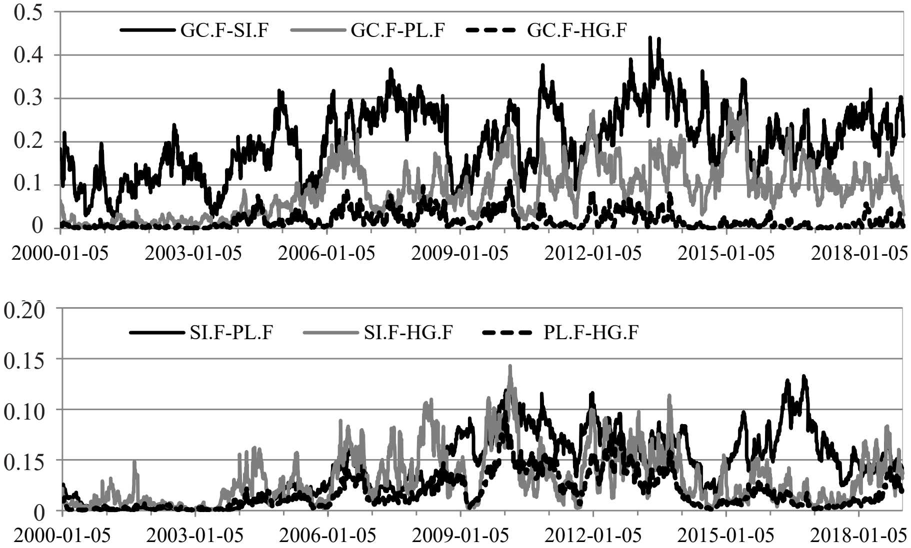

Figure. 2. Tail dependence coefficients for pairs of futures contracts for metals
Source: own adjustment based on data from financial service [stooq.pl]

An asymptotic dependence is a typical property in the case of tails in two-dimensional distributions for futures contracts for agricultural products, which are connected with fundamental factors. This indicates a simultaneous occurrence of extreme rates of return for pairs of futures contracts for corn and wheat (ZC.F-ZW.F) in mid-2005, from the 4th quarter of 2008 to the end of the 2nd quarter of 2013, from the 3rd quarter of 2015 to the end of 2018; for corn and soybean (ZC.F-ZS.F) in the years 2000–2018; for soybean and soybean oil (ZS.F-ZL.F) from March to mid-July 2004, from April to October 2005, from

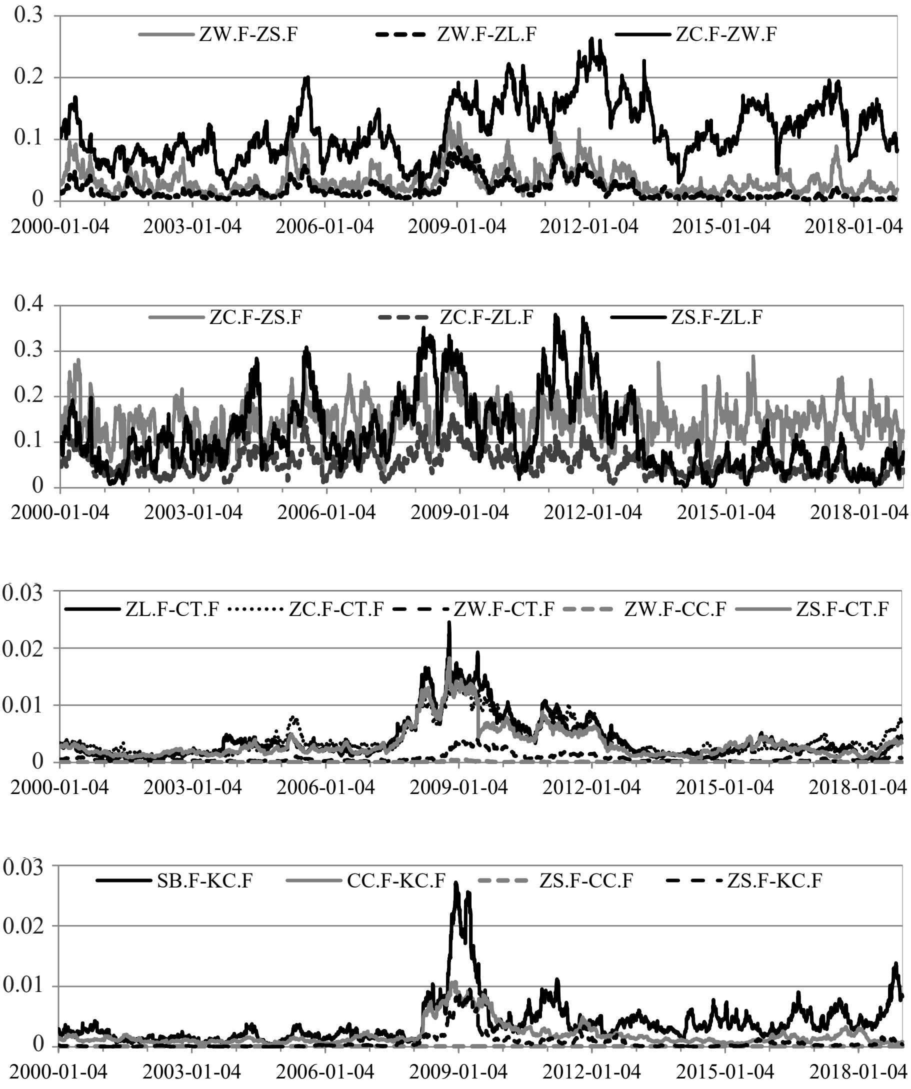

Figure 3. Tail dependence coefficients for pairs of futures contracts for agricultural products

Source: own adjustment based on data from financial service [stooq.pl]

the 2nd half of 2007 to the end of 2012. In the case of futures contracts for soft commodities, generally zero probability was found for the transfer of extreme events. An exception in this case was observed for pairs of futures contracts for sugar and coffee (SB.F-KC.F) and for cocoa and coffee (CC.F-KC.F), as the probability of transfer of extreme events in both cases increased during the subprime crisis and did not exceed 0.03. In the period of the subprime crisis a similar dynamic for tail dependence coefficients was also recorded for futures contracts for cotton in pairs with contracts for corn, soybean and soybean oil (ZC.F-CT.F, ZS.F-CT.F, ZL.F-CT.F) (Figure 3). Thus, it may be concluded that tail dependencies of joint conditional distributions in the case of agricultural futures contracts, if they ever occur, are weak and do not lead to any considerable increase in the risk for any portfolio composed of these contracts in the periods of market upheavals. An exception in this respect is provided by three pairs of futures contracts for grains and oilseeds (ZC.F-ZW.F, ZC.F-ZS.F, ZS.F-ZL.F).

CONCLUDING REMARKS

The identification of dependencies between extreme rates of return from commodity futures contracts is important from the point of view of risk management by market players. These dependencies on commodity futures contract markets, in the years 2000-2018, were investigated applying the Copula-ARMA-GARCH models, followed by the use of tail dependence coefficients in two-dimensional distributions for rates of return from futures contracts. The obtained results indicate a relatively permanent and high level of dependencies between extreme rates of return from futures contracts for crude oil and heating oil, as well as futures contracts for gold and silver. Asymptotic dependencies were found between all pairs of futures contracts only in the case of markets for metals. For pairs of agricultural futures contracts, in which at least one contract was concluded for soft commodities, the asymptotic dependencies were either absent or assumed very low values in the period of subprime crisis. A limitation in this study was connected to a lack of distinction for dependencies between extreme rates of return coming from left and right tails.

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ZALEŻNOŚCI EKSTREMALNE NA RYNKACH TOWAROWYCH KONTRAKTÓW FUTURES

Słowa kluczowe: zależności ogonowe, Copula-ARMA-GARCH, kontrakty futures na towary

ABSTRAKT

Celem pracy jest ocena zależności między ekstremalnymi stopami zwrotu z towarowych kontraktów futures na wybranych rynkach w latach 2000-2018. W okresach niepokojów i zawirowań na rynkach dla inwestorów i zarządzających portfelami ważne jest szacowanie prawdopodobieństwa tego, że czynniki ryzyka jednocześnie będą przyjmować wartości ekstremalne. Analizowano zależności pomiędzy ekstremalnymi stopami zwrotu (zależności asymptotyczne) na rynku kontraktów futures na: energię, metale i towary rolne w latach 2000-2018, przy użyciu modeli Copula-ARMA-GARCH i współczynników zależności w ogonach. Stosunkowo silne i trwałe zależności asymptotyczne występowały dla pary kontraktów futures na ropę naftową i olej opałowy, z kolei nie występowały tego typu zależności lub pojawiły się w okresie kryzysu subprime i przyjmowały niewielkie wartości dla pozostałych par energetycznych kontraktów futures i par rolniczych kontraktów futures, w których co najmniej jeden z kontraktów wystawiony był na towary miękkie.

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