

Jan Święch

## APPLICATION OF MULTIDIMENSIONAL ANALYTIC GEOMETRY TO THE ESTIMATION OF THE EFFECT OF JOINT ACTION OF INSECTICIDES IN MIXTURES

### I. INTRODUCTION

The effect of the action of insecticide on the organism of an insect ( $R =$  reaction) is a function of the applied dose of insecticide ( $D =$  dose).  $R = f(D)$ . We expect that after mixing insecticides the effect of the action of the mixture ( $R_m$ ) will be a function of the sum of doses of insecticides ( $D_m$ ):  $R_m = f(D_m)$  (= addition).

However, the effect of the action of mixtures — as the result of joint action of insecticides in the process of intoxication of the organism of an insect — can be higher than the expected effect:  $R_m > f(D_m)$  (= synergism) or lower:  $R_m < f(D_m)$  (= antagonism).

In view of the practical aspect of synergism there are used various methods in studies on mixtures of insecticides, tending towards quantitative assesment of the effect of joint action of the components of these mixtures in the shape of so-called coefficient of synergism (Bakuniak, 1973; Cole and Clark, 1962; Gowing, 1960; Sun and Johnson, 1960; Tammes, 1964).

These methods are generally limited to studies on two-component mixtures. The autor proposes the application of multimentional analytic geometry to calculate the effect of joint action of any number of insecticides in mixture.

### II. METHOD

To evalutate the interaction of insecticides in  $n$ -component mixtures we can apply the method of multidimensional analytic geometry. We consider the  $n$ -component mixture of insecticides as a set of systems ( $LD_1, LD_2, LD_3, \dots, LD_n$ ) in  $n$ -dimentional Cartesian space  $C_n$ , where:  $LD_i$  — 'do-

sis letalis' of this i-of insecticide stated for instance by Finney (1952) or Litchfield and Wilcoxon (1949; see: Bojanowska, 1961).

And thus, the individual toxicity of the particular insecticides create in space  $C_n$  following sets:

$$\begin{aligned} I_1 & (LD_1, 0, 0, \dots, 0) \\ I_2 & (0, LD_2, 0, \dots, 0) \\ I_3 & (0, 0, LD_3, \dots, 0) \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \dots, \quad \cdot \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \dots, \quad \cdot \\ & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \dots, \quad \cdot \\ I_n & (0, 0, 0, \dots, LD_n), \end{aligned}$$

and the toxicity of these insecticides in the mixture create in  $C_n$  the set:

$$M (LD_{m_1}, LD_{m_2}, LD_{m_3}, \dots, LD_{m_n}).$$

The sets  $I_1, I_2, I_3, \dots, I_n$  delimit in the space  $C_n$  the  $(n-1)$ -dimensional hyperspace which designates the expected effect of the joint action of  $n$  insecticides in mixture (= addition). This hyperspace is described by the equation (1):

$$(1) \quad 0 = \begin{vmatrix} LD_{m_1} - LD_1 & LD_{m_2} & LD_{m_3} & \dots & LD_{m_n} \\ -LD_1 & LD_2 & 0 & \dots & 0 \\ -LD_1 & 0 & LD_3 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ -LD_1 & 0 & 0 & \dots & LD_n \end{vmatrix} \leftrightarrow$$

$$\leftrightarrow LD_{m_1} LD_2 LD_3 \cdot \dots \cdot LD_n + LD_{m_2} LD_1 LD_3 \cdot \dots \cdot LD_n + LD_{m_3} LD_1 LD_2 \cdot \dots \cdot LD_n + LD_{m_n} LD_1 LD_2 \cdot \dots \cdot LD_{n-1} - LD_1 LD_2 LD_3 \cdot \dots \cdot LD_n = 0.$$

Denoting  $LD_{m_i}$  by  $x_i$  and the succeeding products  $LD_i$  with  $LD_{m_i}$  by  $A_1, A_2, A_3, \dots, A_n$  and the product  $LD_1 LD_2 LD_3 \cdot \dots \cdot LD_n$  by  $A_0$  we obtain the equation of hyperspace  $H_{n-1}$  in the general form (2) (Borsuk, 1976):

$$(2) \quad A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots + A_n x_n - A_0 = 0.$$

In the instance of addition, after substitution for the equation of the hyperspace  $H_{n-1}$  of the adequate values  $LD_i$  and  $LD_{m_i}$ , it satisfies the equation (2). Taking the value of addition as the unity (= 1) the coefficient of the joint action of insecticides in the mixture equals (3):

$$(3) \quad WW = \frac{A_0}{A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots + A_n x_n},$$

when:  $WW = 1$  -addition,  
 $WW > 1$  -synergism,  
 $WW < 1$  -antagonism.

For instance, the formula for calculation of the index  $WW$  in the mixture of three insecticides has the form (4):

$$(4) \quad WW = \frac{LD_1 LD_2 LD_3}{LD_{m_1} LD_2 LD_3 + LD_{m_2} LD_1 LD_3 + LD_{m_3} LD_1 LD_2}$$

and in the instance of the mixture of two insecticides (5):

$$(5) \quad WW = \frac{LD_1 LD_2}{LD_{m_1} LD_2 + LD_{m_2} LD_1}$$

or in the instance of the mixture of one insecticide with a synergist which itself does not show any insecticidal activity (6):

$$(6) \quad WW = \frac{LD_1}{LD_{m_1}}.$$

The significance of appearance of synergism in the mixture of insecticides we can test at the accepted level of probability the significance of the average result  $|WW-1|$ , for instance out of five repetitions. The Dunnet's two-sided test (Dunnet, 1955) is the most applicable. The autor proposes the program 'MIX' (to the microcomputer ZX Spectrum Sinclair) to calculate the effect of joint action of insecticides in mixtures.

### III. EXAMPLE

The three component mixture containing 10% of Insecticide No. 1 + 50% of Insecticide No. 2 + 40% of Insecticide No. 3. The calculation of results of the investigation on synergism of the mixture are presented in Table 1.

Program 'MIX' to calculate the effect of joint action of insecticides in mixture to the microcomputer ZX SPECTRUM:

```

10 REM ,MIX': Joint action of insecticides
20 REM ZX SPECTRUM 48 K
30 CLS: INPUT "Number of insecticides" ' "in mixtures?";
  n' "Number of repetitions?"; r: DIM a(r): DIM b(r, n): DIM c(r, n):
  DIM d(n) : DIM m(r) : DIM p(n) : DIM x(r) : DIM z(r)
40 LET s1 = 0: FOR j = 1 TO n: INPUT ("Percentage ";j; "-insecticide") ; "in mixture?" ; p(j) : LET s1 = s1 + p (j) : IF s1 = 100 THEN GO TO 60
50 BEEP .2,17.6: BEEP .3,13.4: PRINT FLASH 1; AT 12,12; "Sum{ } 100!": GO TO 40

```

Table 1

The results of the investigation on synergism of the mixture of insecticides:  
 Ins. No. 1 — 10%, Ins. No. 2 — 50% and Ins. No. 3 — 40%.  
 LD<sub>50</sub> microgram per insect

Insecticide	LD <sub>1</sub>	Repetitions				
		1	2	3	4	5
Ins. No. 1:	LD <sub>1</sub>	0,21	0,19	0,24	0,19	0,22
Ins. No. 2:	LD <sub>2</sub>	0,67	0,54	0,60	0,72	0,66
Ins. No. 3:	LD <sub>3</sub>	1,60	1,58	1,46	1,60	1,63
Mixture 1:5:4	LD <sub>m</sub>	0,27	0,25	0,30	0,22	0,24
LD <sub>m</sub> . 0,1	LD <sub>m</sub> <sub>1</sub>	0,027	0,025	0,030	0,022	0,024
LD <sub>m</sub> . 0,5	LD <sub>m</sub> <sub>2</sub>	0,135	0,125	0,150	0,110	0,120
LD <sub>m</sub> . 0,4	LD <sub>m</sub> <sub>3</sub>	0,108	0,100	0,120	0,088	0,096
Index	WW	2,52	2,35	2,19	3,03	2,86

WW = 2.59; s = 0.351; S<sub>ww</sub> = 0.157;

Confidence interval — p = 0.95 : 0.157 · 3.88 = 0.610;

— p = 0.99 : 0.157 · 5.73 = 0.900;

WW-1 = 2.59-1 = 1.59 > 0.900.

Result: Significance of synergism at p = 0.99.

```

60 CLS: FOR i = 1 TO r: PRINT AT 21, 0; i; "-repetition:": IN-
  PUT "LD50 of mixture?"; m(i): FOR j = 1 TO n: INPUT ("LD50
  of"; j; "-insecticide?"); b(i, j): NEXT j: NEXT i: CLS: FOR i = 1
  TO r: LET a(i) = 1: FOR j = 1 TO n: LET a(i) = a(i) * b(i, j): LET
  c(i, j) = m(i) * p(j)/100: NEXT j: NEXT i: FOR i = 1 TO r: LET
  z(i) = 0: FOR j = 1 TO n: LET d(j) = 1: LET e = b(i, j): LET
  b(i, j) = c(i, j): FOR k = 1 TO n: LET d(j) = d(j) * b(i, k): NEXT
  k: LET b(i, j) = e: LET z(i) = z(i) + d(j): NEXT j: NEXT i: LET
  s1 = 0: LET s2 = 0: FOR i = 1 TO r: LET x(i) = a(i)/z(i): LET
  s1 = s1 + x(i): LET s2 = s2 + x(i) * x(i): NEXT i: IF r = 1 THEN GO
  TO 80
70 LET s = SQR ((s2 - s1 * s1/r)/(r-1)): GO TO 90
80 LET s = 0
90 LET SWW = s/SQR r: LET WW = s1/r: CLS: PRINT TAB 5;
  "INDEX OF JOINT ACTION:": FOR i = 1 TO r: PRINT AT
  i+1, 8; "WW("; i; ") = "; x(i): NEXT i: PRINT "TAB 8; "MEAN:
  "; WW: INPUT #0; "Dunnet's two-sides test?" "to.05 = "; d1, "to.
  01 = "; d2: LET C1 = SWW * d1: LET C2 = SWW * d2: PRINT
  "TAB 7; "CONFIDENCE INTERVAL": "TAB 6; "at p = 0.95:
  "; C1 "TAB 6; "at p = 0.99: "; C2: LET a$ = "ANTAGONISM":
  LET s$ = "SYNERGISM": LET c$ = "Significance of": LET
  p$ = "at p = 0.95": LET q$ = "at p = 0.99": IF C1 > ABS
  (WW-1) THEN PRINT FLASH 1; "ADDITION": GO TO 150
  
```

```

100 IF C2 < ABS (WW-1) THEN GO TO 130
110 IF WW < 1 THEN PRINT FLASH 1'c$ + a$p$: GO TO 150
120 PRINT FLASH 1'c$ + s$p$: GO TO 150
130 IF WW < 1 THEN PRINT FLASH 1'c$ + a'q$: GO TO 150
140 PRINT FLASH 1'c$ + s'q$: GO TO 150
150 PRINT # 0; "START-press 's'", "END-press 'e'": PAUSE 0: IF
    INKEY$ = "s" OR INKEY$ = "S" THEN RUN
160 IF INKEY$ = "e" OR INKEY$ = "E" THEN NEW
170 GO TO 150

```

## REFERENCES

1. Bakuniak E. — 1973 — The synergism evaluating of two component insecticidal mixtures. *Pol. Pismo Entom.* 43, 395—414.
2. Bojanowska A. — 1961 — Statystyczna interpretacja biologicznych badań insektycydów. „Materiały do metodyki badań biologicznych oceny środków ochrony roślin”. Instytut Ochrony Roślin, Poznań. 53—94.
3. Borsuk K. — 1976 — Geometria analityczna wielowymiarowa. PWN Warszawa.
4. Cole M. M., Clark P. M. — 1962 — Toxicity of various carbamates and synergists to several strains of bode lice. *J. Econ. Entom.* 55. 98—102.
5. Dunnet C. W. — 1955 — A multiple comparison procedure for comparing several treatments with a control. *J. American Statist. Assoc.* 50. 1096—1121.
6. Finney D. J. — 1952 — Probit Analysis. University Press. Cambridge.
7. Gowing D. P. — 1960 — Comments of tests of herbicide combination. *Weeds*, 8. 379—381.
8. Sun Y. P., Johnson E. R. — 1960 — Analysis of joint action of insecticides against house flies. *J. Econ. Entom.* 53. 887—892.
9. Tammes P. M. — 1964 — Isoboles, a graphic presentation of synergism in pesticides. *Neth. Jour. Plant. Path.* 70. 73—80.

Jan Święch

## ZASTOSOWANIE GEOMETRII ANALITYCZNEJ WIELOWYMIAROWEJ DO OCENY EFEKTU WSPÓŁDZIAŁANIA INSEKTYCYDÓW W MIESZANINACH

### STRESZCZENIE

Do obliczeń efektu interakcji insektycydów w mieszaninach (addycja, synergizm, antagonizm) zaproponowano metodę geometrii analitycznej wielowymiarowej. Mieszanina n-składnikowa traktowana jest jako zbiór

układów ( $LD_1, LD_2, LD_3, \dots, LD_n$ ) w  $n$ -wymiarowej przestrzeni kartezjańskiej  $C_n$ . Toksyczności indywidualne poszczególnych insektycydów tworzą w  $C_n$  układy  $I_1 (LD_1, 0, \dots, 0)$ ,  $I_2 (0, LD_2, 0, \dots, 0)$ ,  $I_3 (0, 0, LD_3, \dots, 0)$ , ...,  $I_n (0, 0, 0, \dots, LD_n)$ , zaś toksyczności ich w mieszaninie tworzą w  $C_n$  układ  $M (LD_{m_1}, LD_{m_2}, LD_{m_3}, \dots, LD_{m_n})$ .

Układy  $I_i$  wyznaczają w  $C_n$  hiperpłaszczyznę  $(n-1)$ -wymiarową określającą oczekiwany efekt współdziałania  $n$  insektycydów w mieszaninie (addycja). Hiperpłaszczyzna  $H_{n-1}$  opisana jest równaniem ogólnym:  $A_1x_1 + A_2x_2 + A_3x_3 + \dots + A_nx_n = 0$ . Przyjmując indeks addycji  $WW = 1$  otrzymamy:  $WW = A_0 / (A_1x_1 + A_2x_2 + A_3x_3 + \dots + A_nx_n)$ . Gdy  $WW > 1$  — synergizm, gdy  $WW < 1$  — antagonizm.

Istotność synergizmu bada się testując istotność różnicy  $|WW-1|$  według Dunnetta (1955).

Podano wzory na obliczanie wskaźników  $WW$  mieszaniny dwu- i trójskładnikowej oraz dla przypadku mieszaniny insektycydu z synergetykiem. Załączono listing programu dla mikrokomputera ZX Spectrum Sinclair obliczającego wartość wskaźnika  $WW$  i testującego istotność synergizmu  $n$ -składnikowej mieszaniny insektycydów.

Я Н Сьвенх

## ПРИМЕНЕНИЕ МНОГОРАЗМЕРНОЙ АНАЛИТИЧЕСКОЙ ГЕОМЕТРИИ ДЛЯ ОЦЕНКИ ЭФФЕКТА ВЗАИМОДЕЙСТВИЯ ИНСЕКТИЦИДОВ В СМЕСЯХ

### РЕЗЮМЕ

Для вычисления эффекта интеракции инсектицидов в смесях (аддичия, synergizm, antagonizm) предложено метод многомерной аналитической геометрии. Смесь  $n$ -элементов относится к множеству систем  $(LD_1, LD_2, LD_3, \dots, LD_n)$  в  $n$ -размерном картезианском пространстве  $C_n$ . Личные токсичности отдельных инсектицидов образуют в  $C_n$  системы  $I_1 (LD_1, 0, \dots, 0)$ ,  $I_2 (0, LD_2, 0, \dots, 0)$ ,  $I_3 (0, 0, LD_3, \dots, 0)$ , ...,  $I_n (0, 0, 0, \dots, LD_n)$ , зато их токсичности в смеси образуют в  $C_n$  систему  $M (LD_{m_1}, LD_{m_2}, LD_{m_3}, \dots, LD_{m_n})$ .

Системы  $I_n$  определяют в  $C_n$  гиперплоскость  $(n-1)$  — размерную определяющую ожидаемый эффект взаимодействия  $n$  инсектицидов в смеси аддичия. Гиперплоскость  $X_{n-1}$  описана общим уравнением:

$A_1x_1 + A_2x_2 + A_3x_3 + \dots + A_nx_n = 0$ . Принимая индекс аддиции  $WW = 1$  получим:  $WW = A_0/A_1x_1 + A_2x_2 + A_3x_3 + \dots + A_nx_n$ . Когда  $WW > 1$  — синергизм, когда  $WW < 1$  — антагонизм.

Существенность синергизма исследуется тестируя существенность разницы  $(WW - 1)$  по Дуннету (1955).

Дано формулы на вычисление показателей  $WW$  смеси дву- и три-элементной а также в случае смеси инсектицида с синергетиком. Приложено листинг программы для микрокомпьютера ZX Spectrum Sinclair вычисляющего величину показателя  $WW$  и тестирующего существенность синергизма  $n$ -элементной смеси инсектицидов.