The analysis of oil balance in crank bearing

Khachatur Kyureghyan[1], *Wiesław Piekarski*[2]

1) Department of Applied Mathematics and Computer Science, University of Life Sciences,
20-950 Lublin, Akademicka 13, e-mail: chaczik@wp.pl
2) Department of Energy and Transportation, University of Life Sciences,
20-959 Lublin, Poniatowskiego 1, e-mail: wieslaw.piekarski @ up.lublin.pl

Summary. The paper presents an analytical calculation of the balance of oil flowing through the dynamically loaded main and crank bearing. Theoretical considerations are based on solving the Reynolds equation (analytical distribution of oil pressure in the cross slide bearing) with boundary conditions that characterize the working conditions in the bearings used in motors s-4002/4003 (agricultural tractors).The quantitative and volumetric evaluations of lubricant fluid flowing through the bearing are presented adequately to the construction of a diagnostic signal, which allows for a dynamic comparative analysis carried out for the model bearings, the new ones as well as the ones with a particular classification of wear.

Key words: hydrodynamic lubrication, cross slide bearing, Reynolds equation, diagnostic analysis, diagnostic signal parameters.

- Physical quantities

h - thickness of the wedge [*mm*],
ε - relative bearing eccentricity [-],
c - bearing clearance [*mm*],
η - coefficient of dynamic oil viscosity [$Pa \cdot s$],
V - peripheral speed of crankshaft [*m/s*],
e - eccentricity [*mm*],
$P(x,z)$ - oil pressure inside bearing [*Pa*],
x,z - coordinate variables, $x \in \langle 0, 2\pi R \rangle, z \in \left\langle -\frac{L}{2}, \frac{L}{2} \right\rangle$ [*mm*],
L - the width of the pan [*mm*],
p_w, p_0 - supply pressure ambient pressure [*Pa*],
p_z - supply pressure [*Pa*],
a - cord diameter [*mm*],
Q - oil intensity passing through the crank bearing [dm^3/min],
Q_1, Q_2 - partial intensity[dm^3/min],
S_0, S_{gr} - values for Q_1, Q_2 from experiment[dm^3/min],
$h_0 = \min\limits_{x \in \langle 0, 2\pi R \rangle} h(x)$ - minimum thickness of oil wedge [*mm*],
$c_{dot} = c_0$ - clearance after getting proper association (at optimal operation after 100mth) [*mm*],
c_{gr} - critical value of clearance [*mm*],
D_p - coefficent of dinamic passing [-],
d_p - experimenthal value of D_p determined as a parametr of the diagnostic signal

- Mathematical solutions

$\Delta[f]_a^b = f(a) - f(b)$ - differential operator
$f_K = \sum_{k=1}^{K} f_k$ - summing operator, **N**,
$[f]$ - total part of expression *f*,
C_1, C_{1l}, C_{1K} - Reynolds equation constants determined by boundary conditions

INTRODUCTION

The parameters of diagnostic signals defining the course of change in the tightness of slide bearings are a very important factor in assessing the technical state of engines examined in this study on the example of farm tractors. It should be noted that in the lubrication subsystem the main slide bearings and the crank ones of the crankshaft determine the technical condition of the engine, and its ability to perform useful functions. Increased clearances in the bearings make the lubricant oil flow freely through the gaps between the pivots and the cups, which in turn is manifested by an increase in oil flow and pressure drop in the engine lubrication system.

In diagnostic considerations, the evaluation of work behavior of a dynamically loaded slide bearing is based on the simplified model of a standard cross slide bearing and is based mostly on the minimum thickness of oil gap and the value of the maximum pressure and maximum temperature of the oil film.

However, as noted by several authors [1,2,4,7,17], in diagnostic tests, particularly in predicting the technical state of emergency crank system operation, application of a single-parameter diagnostic signal based solely on the measurement of relative drop in oil pressure at selected points of the bearing, is not sufficient to assess the degree of bearing wear. [18,17] presented a multivariate statistical analysis (curvilinear regression model for the associated random variables) of the diagnostic signal based on measurements of four parameters, where the measurement of the relative pressure drop and loss of lubricant in the bearings were considered as associated variables. The purpose of this study is to analyze the theoretical value of the diagnostic signal parameter described by an analytical balance of oil in the cross slide bearing.

In diagnostic studies of changes in oil flow through the crankshaft bearing and at the constant supply pressure in static conditions, the oil flow rate depends mainly on the size of oil gap. And according to the relation:

$$h = L_t(1 - \varepsilon\cos\theta),$$

the gap value depends on oil bearing clearance (L_t) and relative bearing eccentricity (ε), in further considerations we use a simplified designation:

$$L_t = c.$$

Determination of this value in the experimental way is based on measurements of diagnostic parameters, the commonest of which are oil pressure or relative drop of oil pressure in the lubrication subsystem (bearing seal). By way of contrast, the theoretical determination of the oil gap is based on the equations of hydrodynamics for a viscous substance.

MATHEMATICAL MODEL OF BEARING AND ASSUMPTIONS

Assessment of work of dynamically loaded crank bearing is one of the most important tasks of the crank subsystem diagnostics in internal combustion engines. In this paper, theoretical considerations and the resulting mathematical calculations are shown on the basis of a cross slide bearing model, characterizing the operating parameters of crank bearing in S-4002/4003 type of internal combustion engines. In the process of exploitation, the growing bearing clearance values cause an increase in the impact of oil flowing from the bearing, causing a decrease in tightness of the bearings.

The balance of oil flowing through a dynamically loaded crank bearings can be determined analytically using a pressure distribution which is the solution of the Reynolds equation:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial z}\right) = 6V\frac{\partial h}{\partial x}. \tag{1}$$

After considering the constant coefficient of dynamic oil viscosity ($\eta = const$) as well as the following simplified expressions for the functions describing the thickness of the wedge and the bearing wear [5,9,14,20]:

$$h = h(x) = c\left(1 - \varepsilon\cos {}^{x}\!/\!_{R}\right), \tag{2}$$

$$\frac{3}{h(x)}\frac{\partial h(x)}{\partial x} = \frac{3e\sin {}^{x}\!/\!_{R}}{Rc\left(1-\varepsilon\cos {}^{x}\!/\!_{R}\right)}, \tag{3}$$

equation (1) can be written as:

$$P''_{xx}(x,z) + P''_{zz}(x,z) + \frac{3e\sin {}^{x}\!/\!_{R}}{Rc\left(1-\varepsilon\cos {}^{x}\!/\!_{R}\right)}P'_{x}(x,z) - \frac{6\eta e\sin {}^{x}\!/\!_{R}}{Rc^3\left(1-\varepsilon\cos {}^{x}\!/\!_{R}\right)^3}V = 0, \quad ,(4)$$

with the boundary conditions:

$$P = p_0 \text{ for } z = \pm\frac{L}{2}, \tag{5}$$

$$P = p_w(x) \text{ for } z = 0, \tag{6}$$

$$\left(\frac{\partial P(x,z)}{\partial z}\right)_W = -\frac{3\pi a^4 (p_z - p_w)}{4c^3\eta L\left(1-\varepsilon\cos {}^{x}\!/\!_{R}\right)^3} \text{ for } z = 0 \tag{7}$$

where:
p_0 - the ambient pressure [Pa],
p_z - supply pressure [Pa],
p_w - oil pressure at the inlet to the placenta [Pa],
a - cord diameter [mm],
L - width of the pan [mm].

According to [5,12,14], the analytical distribution in the bearing oil pressure as the solution of equation (4) can be written as follows:

$$P(x,z) = p_0 + C_1\left(z - \frac{L}{2}\right) + \left[\frac{1-\varepsilon\cos {}^{x}\!/\!_{R}}{1+\varepsilon}\right]^{-\frac{3e}{\varepsilon}} \cdot \sum_{K=1}^{\infty} C_{1K}\left(e^{Kz} - e^{K(L-z)}\right)\left[\Phi_{RK}\left({}^{x}\!/\!_{R}\right) + \Psi_{RKV}^{-2}\left({}^{x}\!/\!_{R}\right)\right], \tag{8}$$

where:

$$\Phi_{RK}\left({}^{x}\!/\!_{R}\right) = -\frac{\sin^2 {}^{x}\!/\!_{R}}{4}\left(\frac{1-\cos {}^{x}\!/\!_{R}}{2}\right)^{-\left(R^2K^2+1\right)}, \tag{9}$$

$$\Psi_{RKV}\left({}^{x}\!/\!_{R}\right) = \left(\frac{2\pi RK}{nV}\right)^2 \sum_{i=1}^{\left[R^2K^2\right]}\left[\left(\frac{R^2K^2V}{i}\right)\frac{1}{i}\left(\frac{1-\cos {}^{x}\!/\!_{R}}{2}\right) - \ln\left(\frac{1-\cos {}^{x}\!/\!_{R}}{1+\cos {}^{x}\!/\!_{R}}\right)^i - \left(\frac{1-\varepsilon\sqrt{V}\cos {}^{x}\!/\!_{R}}{\pi\left(1+e\sqrt{V}\right)}\right)^{-\frac{3e}{\varepsilon}}\right], \tag{10}$$

$$C_1 = \frac{\Delta_L}{1+\frac{L}{2}\Delta_L}\left(p_z - p_0\right), \tag{11}$$

$$C_{11} = \frac{3\Delta_L \Gamma_L\left(p_z - p_0\right)}{\left(1+\varepsilon\right)^{\frac{3e}{\varepsilon}}\left[1+\frac{1+\varepsilon}{1-\varepsilon}\left(\Phi(0)+\Psi^{-2}(0)\right)-\left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{\frac{3e}{\varepsilon}}\left(\Phi\left(\pi/2\right)+\Psi^{-2}\left(\pi/2\right)\right)\right]}, \tag{12}$$

$$\Delta_L = \frac{15\pi a^4}{2c^3\eta L}, \tag{13}$$

$$\Gamma_L = \frac{1-\varepsilon^2}{1+3\varepsilon}\Delta_L\left(e^L - 1\right). \tag{14}$$

ANALYTICAL BALANCE OF OIL

Analytical determination of the amount of oil flowing through bearing is based on the solution of equation (4), i.e. function P (x, z) describing the distribution of pressure in the crank bearing [9,20]. The value of the intensity of Q passing through the crank bearing can be represented as two partial streams Q_1,Q_2, where $\mathbf{Q_1}$ is part of the stream flow caused by the rotation of the crankshaft, while Q_2 is part of a stream of pressurized forced power Q_2. By virtue of the above considerations, the dependencies representing the partial streams of flowing oil can be written as follows:

$$Q_1 = Q_{1,x=0} - Q_{1,x=R\pi} = \frac{1}{6\eta}\int_{-L/2}^{L/2} h^3 \Delta\left[\frac{\partial P}{\partial x}\right]_0^{R\pi} dz =$$
$$= \frac{1}{6\eta}\int_{-L/2}^{L/2} h^3\left[\frac{\partial P}{\partial x}\right]_{x=0} dz - \frac{1}{6\eta}\int_{-L/2}^{L/2} h^3\left[\frac{\partial P}{\partial x}\right]_{x=R\pi} dz\ , \tag{15}$$

$$Q_2 = Q_{2,z=0} - Q_{2,z=\pm\frac{L}{2}} = \frac{1}{6\eta}\int_0^{2\pi} h^3 \Delta\left[\frac{\partial P}{\partial z}\right]_0^{\pm\frac{L}{2}} dx =$$
$$= \frac{1}{6\eta}\int_0^{2\pi} h^3\left[\frac{\partial P}{\partial z}\right]_{z=0} dx - \frac{1}{6\eta}\int_0^{2\pi} h^3\left[\frac{\partial P}{\partial z}\right]_{z=\pm\frac{L}{2}} dx\ , \tag{16}$$

where:

$$Q_{1,x=0} = \frac{c^3 C_{1K}\left(1-\exp\{KL\}\right)}{6\eta RK}$$
$$\left(1-\varepsilon\right)^3\left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{\frac{3e}{\varepsilon}}, \tag{17}$$

$$Q_{1,x=R\pi} = \frac{c^3 C_{1K}\left(1-\exp\{KL\}\right)}{48\eta RK}\left(1+\varepsilon\right)^3, \tag{18}$$

$$Q_{2,z=0} = \frac{c^3 K\pi\left(1+\exp\{KL\}\right)}{6\eta}\left[2C_1 + \frac{C_{1K}\left(6e+\varepsilon\right)}{3e+2\varepsilon}\left(1-\varepsilon\right)^{\frac{3e}{\varepsilon}}\right], \tag{19}$$

$$Q_{2,z=\pm\frac{L}{2}} = \frac{c^3 K\pi\exp\{KL\}}{3\eta}\left\{2C_1 + \frac{C_{1K}\left(6e+\varepsilon\right)}{3e+2\varepsilon}\left(1-\varepsilon\right)^{\frac{3e}{\varepsilon}}\right\}. \tag{20}$$

The relationships (17) - (20) omitted elements of the order o(ε^3) and used the following indications:

$$C_1 = -\frac{\Delta_L}{1+\frac{L}{2}\Delta_L}\left(p_z - p_0\right), \tag{21}$$

$$C_{1K} = \sum_{k=1}^{K} C_{1k} = \sum_{k=1}^{K}\left(p_0 - C_1\frac{L}{2} + \left[\frac{1-\varepsilon}{1+\varepsilon}\right]^{-\frac{3e}{\varepsilon}}\frac{\Delta_L}{\pi C_1\left(1-\frac{KL}{4}\right)}\right), \tag{22}$$

where:

$$K = 1,2,\ldots,\left[\left(\frac{R}{e}\right)^2 - 1\right]. \tag{23}$$

Due to (17) - (20), oil streams Q_1, Q_2 can be written as follows:

$$Q_1 = \frac{c^3 C_{1K}\left(1-\exp\{KL\}\right)}{6\eta RK}\left[(1-\varepsilon)^3\left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{\frac{3e}{\varepsilon}} - \left(\frac{1+\varepsilon}{2}\right)^3\right], \tag{24}$$

$$Q_2 = \frac{K\pi\left(1-\exp\{KL\}\right)}{6\eta}\left[2C_1 + \frac{C_{1K}\left(6e+\varepsilon\right)}{3e+2\varepsilon}\left(1-\varepsilon\right)^{\frac{3e}{\varepsilon}}\right]. \tag{25}$$

Finally, by equation (15), (16), the balance of oil flowing through the crank bearing can be written as the following relationship:

$$Q = Q_1 + Q_2 = \frac{c^3\left(1-\exp\{KL\}\right)}{6\eta}\left[2C_1 + C_{1K}\left(\frac{\Omega_1}{RK} + K\pi\Omega_2\right)\right], \tag{26}$$

where:

$$\Omega_1 = (1-\varepsilon)^3\left(\frac{1+\varepsilon}{1-\varepsilon}\right)^{\frac{3e}{\varepsilon}} - \left(\frac{1+\varepsilon}{2}\right)^3, \tag{27}$$

$$\Omega_2 = \frac{6e+\varepsilon}{3e+2\varepsilon}\left(1-\varepsilon\right)^{\frac{3e}{\varepsilon}}. \tag{28}$$

PERFORMANCE ANALYSIS OF DIAGNOSTIC SIGNAL

In the crank bearings of the engine type S-4002/4003 (agricultural tractors C-355, 360), the maximum value of bearing clearance is characterized by accelerated wear of crank mechanism (the boundary condition of the bearing crank is 0.2 mm), which can be derived from the average dependence given by [10,20,21]:

$$C_{gr} = \frac{C_{dot}^2}{4h_0}, \tag{29}$$

using the classical formula Vogelpohl:

$$C_{dot} = 0.92 \cdot 10^{-3} \sqrt[4]{V}, \tag{30}$$

where:

C_{dot} - clearance after getting proper association (at optimal operation after 100mth),

h_0 - the minimum thickness of oil wedge,

V - the peripheral speed of crankshaft.

In the boundary conditions of dynamically loaded crank bearing's operation, the diagnostic signal parameters describing the relative decline in the oil pressure inside the bearing and its flow through the bearing score significantly higher values than when working under optimal conditions, which points to accelerated wear of the crank. Piekarski [17] applied the model to assess the value of diagnostic signal parameters based on measurements of the relative pressure drop and oil flow dynamics at the measurement narrowing. As an indicator of the dynamics, the following value describing oil flow through the bearing was accepted:

$$d_p = \frac{S_{gr} - S_0}{S_0}, \tag{31}$$

Analytical interpretation of the above relation, using the average value of clearance limit (at the speed of the shaft 1200 min^{-1}):

$$C_{gr} = 2,3 \cdot 10^{-3} \sqrt{V} = 1,42 \cdot C_0$$

can be obtained by equation (26) as follows:

$$D_p == \left(\frac{c_{gr}}{c_0}\right)^3 \cdot \frac{2C_1 + C_{1K}\left(\frac{\Omega_{1gr}}{RK} + K\pi\Omega_{2gr}\right)}{2C_1 + C_{1K}\left(\frac{\Omega_{10}}{RK} + K\pi\Omega_{20}\right)} - 1 = 2,86 \cdot \frac{\Phi_{gr}}{\Phi_0} - 1, \tag{32}$$

where:

Ω_{i0} - quantity calculated from solutions (27),(28) for $c = c_0$, $i=1;2$,

Ω_{igr} - quantity calculated from solutions (27),(28) for $c = c_{gr}$, $i=1;2$,

$$\Phi_0 = 2C_{10} + C_{1K}\left(\frac{\Omega_{10}}{RK} + K\pi\Omega_{20}\right),$$

$$\Phi_{gr} = 2C_{1gr} + C_{1K}\left(\frac{\Omega_{1gr}}{RK} + K\pi\Omega_{2gr}\right),$$

$$K = \begin{cases} 1 & for\ c = c_0 \\ 2 & for\ c \in \left(c_0, c_{gr}\right) \\ 3 & for\ c = c_{gr} \end{cases} .$$

In diagnostic tests, the dynamic of determined signal changes should be as high as possible. It is assumed that the determined change induced by an increase in consumption occurring in the crank subsystem is the relative increase in oil pressure drop, which is treated as a diagnostic signal.

CONCLUSIONS

Knowledge of the dynamics of steam friction (pivot - cup) in a crank system, expressed by escalation of clearance between its elements, allows to determine the probability of reliable operation of the considered friction pair. A proper technical maintenance of the engine operation is necessary, provided the information is available on its properties.

This information can be known only by the changes of bearing clearance and course changes in the dynamics of the diagnostic signal.

Terms of co-operation of the functional subsystem crank pivot - cup decide on the reliable operation of the engine. The worsening conditions for cooperation of these subsystems as a result of processes of consumption leads to premature engine wear, and even more to significant increase in fuel and oil consumption and increased difficulty in starting.

Requirements for operational progress are becoming more frequently recognized and formulated. It was noted that the effectiveness of the management of technical objects in many cases reduces the high operating expenses, which may even get higher than expenses associated with designing and manufacturing. The high operating expenses can be reduced by improving the quality of technical objects, as well as conditions of their use and handling. For this purpose, the pursuit is necessary of rational, science-based exploitation of technical objects.

REFERENCES

1. **Bałuciński J., Stępiński J., 1993:** Wykorzystanie metod oceny stanu technicznego silników przy kwalifikowaniu ich do naprawy. Techn. Moto., 11, 48-52.
2. **Będkowski L., 1981:** Elementy ogólnej teorii diagnostyki technicznej. Biul. WAT, 3.
3. **Cislikowski B., Pedryc N., 2009:** Koncepcja nadzoru nad maszynami przez computer pokładowy ciągnika w czasie rzeczywistym z wykorzystaniem magistrali informatycznej LIN. Inżynieria Rolnicza 9(118), 29-33
4. **Das S., Guha S.K., Chattopadhyay A.K., 2005:** Linear stability analisis od hydrodynamic journal bearings under micropolar lubrication. Trybology International,38, 31-35
5. **Hebda M., Wachal A., 1980:** Trybologia, WNT, Warszawa.
6. **Mienann G., 1980:** Mashinenelemente. Springer Verlag, Berlin.
7. **Niewczas A., 1991:** Zastosowanie pomiarów przedmuchów spalin do prognozowania trwałości silników spalinowych. Krajowa Konf. Naukowo – Techniczna Kiekrz, 67, 53 – 62.
8. **Koniuszy A., 2008:** A new metod of usability evaluation of agricultural tractors based on optimum working index. TEKA commission of motorization and power industry in agriculture VIIIa, 93-99.

9. **Kozłowiecki H., 1974:** Łożyska tłokowych silników spalinowych. WKiL, Warszawa.
10. **Kozłowiecki H., Przustek J., 1975:** Współczesne metody obliczania parametrów pracy łożysk mechanizmu korbowego silników spalinowych. Silniki Spalinowe, 2, 62-69.
11. **Krasowski P., 2006:** Zmiana ciśnienia w poprzecznym łożysku ślizgowym przy laminarnym niestacjonarnym smarowaniu. Zeszyty Naukowe Akademii Morskiej w Szczecinie,10(82): 287-298.
12. **Kurowski W., 1978:** Systemy diagnostyczne urządzeń mechanicznych. DUM, Ossolineum.
13. **Kyureghyan Kh., 2002:** Theoretical calculation of size on an oil slit in a transverse sliping bering, Mathematical Methods in Agriculture,11, 388-393
14. **Kyureghyan Kh., Kornacki A., Piekarski W., 2006:** Analityczna zależność do wyznaczania ciśnienia w hydrostatycznym łożysku poprzecznym ślizgowym, Inżynieria Rolnicza, 6(98).
15. **Kyureghyan Kh., Piekarski W., 2008:** Analisis of detetermining pressure distribution in crank bearing, Eksploatacja i Niezawodność – Maintenance and Reliability, 4(40),19-24.
16. **Miroshnikov V. i inni, 2010:** Calculation of quantity characteristics of the created by local devices. TEKA commission of motorization and power industry in agriculture Xc, 190-197.
17. **Piekarski W., 1993:** Prognozowanie trwałości par trących się na przykładzie łożysk wału korbowego silnika S – 312C. Zeszyty Naukowe AR Szczecin,85, 372-386.
18. **Piekarski W. i inni., 1988:** Diagnostyka pojazdów rolniczych Wyd. AR, Lublin, 77.
19. **Remen V. i inni, 2010:** Mathematical model of valce-amplifierrs for pneumatic drives of mechanical systems. TEKA commission of motorization and power industry in agriculture Xc, 255-260.
20. **Thum H.:** Über den Verlauf des Schmierfilmdruckes in dynamisch beanspruchten Gleitlagern. Wiss. Z. Techn. Hochsch. Magdeburg, 5.99.
21. **Thum H., 1985:** Zur Bestimmung der Verschleissbedingungen Lebensdauer von Wälzlagern bei verschiedenen Überlebenwahrscheinlichkeiten. Schimrungstechnik,16, 101-112.
22. **Vogelpohl G., 1969:** Über die Ursachen der Unzureichenden Bewertung von Schemierungsfragen im Vorigen fahrundert. Schmiertechnik, 5.

ANALIZA BILANSU OLEJU W ŁOŻYSKU KORBOWYM

Streszczenie. Praca przedstawia analityczną kalkulację bilansu oleju przepływającego przez dynamicznie obciążone łożysko główne i korbowe. Teoretyczne rozważania bazują na rozwiązaniu równania Reynoldsa (analityczny rozkład ciśnienia oleju w łożysku poprzecznym ślizgowym) o warunkach brzegowych charakteryzujących warunki pracy w/w łożysk zastosowanych w silnikach S-4002/4003 (ciągniki rolnicze). Ilościowo-objętościową ocenę cieczy smarującej przepływającej przez łożysko przedstawiono adekwatnie do teoretycznej wartości sygnału diagnostycznego co pozwala na dynamiczną analizę porównawczą przeprowadzoną dla wzorcowych, nowych, jak również dla łożysk o ustalonej klasyfikacji stopnia zużycia.

Słowa kluczowe: smarowanie hydrodynamiczne, łożysko poprzeczne ślizgowe, równanie Reynoldsa, analiza diagnostyczna, parametry sygnału diagnostycznego.