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An algorithm for approximate identification of relaxation modulus of viscoelastic materials from non-ideal ramp-test histories

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Summary. A new fast scheme for approximate identification of linear relaxation modulus of viscoelastic materials on the basis of the discrete stress data from non-ideal ramp-tests, where a time-variable strain rate is followed by a constant strain, is proposed. The approximations of the relaxation modulus in successive time instants are determined on the basis of the stress measurements in only three appropriately chosen sampling points. The numerical simulations are conducted for KWW relaxation modulus, which indicate that the presented approach is suitable for estimating the relaxation modulus. The approximation of relaxation modulus is more accurate than in Sorvari-Malinen method. However, the model errors are greater that in the case of Zapas-Phillips approach and the quality deterioration is acceptable. The noise robustness of the scheme must be noted, especially if compared with Sorvari-Malinen scheme.

Key words: relaxation test, non-ideal ramp-test, relaxation modulus, KWW model, identification method.

INTRODUCTION

Relaxation modulus is probably the most important mechanical characteristic in the framework of linear viscoelastic behavior [1,7,10,11]. The time-variable relaxation modulus $G(t)$, $t \geq 0$, is theoretically the stress that occurs in the material response to a unit step strain $\varepsilon(t)$. However, it is impossible to apply a step strain in experiments. Loading is never done infinitely fast [3,10,16]. In non-ideal stress relaxation tests the strain increases during the loading interval $[0, t_R]$ until a predetermined strain ε_0 is reached at ramp-time t_R, after which that strain ε_0 is maintained constant at that value. In ideal ramp-test [10] the strain increases along a constant strain rate path. However, usually the constant strain rate in the loading phase cannot be achieved experimentally [10,14,22]. Following Flory and McKenna [3], see also [16] and [22], we assume that the strain in non-ideal ramp-test is described by the function:

$$\varepsilon(t) = \begin{cases} 0 & for \;\; t < 0 \\ \frac{a}{3}\left(t - \frac{t_R}{2}\right)^3 + bt + c & for \;\; 0 \leq t < t_R, \\ \varepsilon_0 & for \;\; t \geq t_R \end{cases} \qquad (1)$$

where: the strain parameters are: $a = -\frac{3}{4}\varepsilon_0\left(\frac{2}{t_R}\right)^3$, $b = \frac{3}{2}\frac{\varepsilon_0}{t_R}$ and $c = -\frac{1}{4}\varepsilon_0$. The strain $\varepsilon(t)$ (1) is shown in Figure 1, where the ideal step-strain $\varepsilon_0(t)$ and the ideal ramp-test strain $\varepsilon_l(t)$ corresponding to linear loading phase strain, are also plotted.

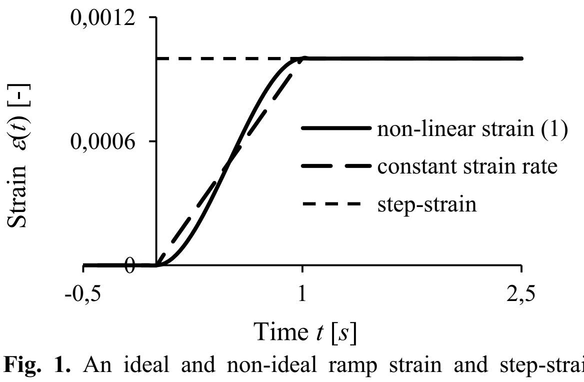

Fig. 1. An ideal and non-ideal ramp strain and step-strain; $t_R = 1$ [s], $\varepsilon_0 = 0.001$ [$-$]

Different methods [7,10,14-16,18,19,22,23] have been proposed during the last few decades for the relaxation modulus determination using the stress data histories from non-ideal relaxation tests. Most of them are addressed for the case of the linear loading phase strain. Only the classical Zapas-Phillips [23] method and the optimal relaxation modulus identification schemes presented in [15,22] have been designed for non-ideal stress relaxation test with a time variable loading phase strain rate. For detailed references and an overview, see [3] and the recent publication [22].

In the previous paper [20], based on the mathematical properties of the problem of relaxation modulus recovery from time-measurements of the stress $\sigma(t)$ in the non-ideal ramp-test (1), the analytical formulas to approximate the relaxation modulus for an arbitrary time instant have been derived. Under standard and mild assumption concerning the relaxation modulus of the viscoelastic material and the rheological experiment it has been proved, that for noise-free stress measurements the resulting relaxation modulus model is monotonically decreasing continuous function of time with at most one discontinuity point.

To develop a fast algorithm for the relaxation modulus identification using discrete-time stress measurements obtained in the ramp-test $\varepsilon(t)$ (1), in which the relaxation modulus approximations in the successive time instants are calculated on the basis of at most three points of the stress data, is a basic concern. The numerical analysis is performed using the KWW material example both for noise free as well as noise corrupted stress measurements. Comparing the results obtained for the new and two other known algorithms with the true values of the relaxation modulus we will draw conclusions regarding the accuracy and applicability of the method proposed.

MODEL OF THE RELAXATION MODULUS

In the previous paper [20] under standard assumptions concerning the relaxation modulus $G(t)$ of the linear viscoelastic material [20; Assumption (4)] the following formula is derived for $0 < t \leq t_R/4$:

$$G^{(NM)}(t) = \frac{t_R{}^3}{12\varepsilon_0(t_R-t)t^2}\sigma(2t), \tag{2}$$

and the next rule is developed for $t > t_R/4$:

$$G^{(NM)}(t) = \frac{8}{3\varepsilon_0}\sigma\left(t+\frac{3t_R}{4}\right) - \frac{7}{3\varepsilon_0}\sigma(t+t_R) + + \frac{2}{3\varepsilon_0}\sigma\left(t+\frac{5t_R}{4}\right), \tag{3}$$

which approximate the modulus $G(t)$. In subsequent sections a complete algorithm for computing the relaxation modulus using discrete-time stress measurements from non-ideal ramp test (1) is presented and examined for simulated KWW model data.

IDENTIFICATION ALGORITHM

The computation of the relaxation modulus according to the above formulas involves the following steps.

1. Design the experiment – ramp-test (1) – with the predetermined constant strain level ε_0 and the ramp-time t_R, i.e., select the sampling instants t_i, $i = 1, \dots, N$, such that $t_1 = h$, $t_{i+1} - t_i = h$ and $t_R = 4i_0h$ for some integer $i_0 \geq 1$.
2. Perform the stress relaxation test (1), record and store the stress measurements $\bar{\sigma}(t_i) = \sigma(t_i) + z(t_i)$, corresponding to the chosen points $t_i, i = 1, \dots, N$, where $z(t_i)$ is additive measurement noise.
3. For $i = 1, \dots, i_0$ calculate the relaxation modulus $G^{(NM)}(t_i)$ according to formula:

$$G^{(NM)}(t_i) = \frac{t_R{}^3}{12\varepsilon_0(t_R-t_i)t_i{}^2}\bar{\sigma}(2t_i).$$

4. For $i = i_0 + 1, \dots, N - 5i_0$ determine the relaxation modulus $G^{(NM)}(t_i)$ using the rule:

$$G^{(NM)}(t_i) = \frac{8}{3\varepsilon_0}\bar{\sigma}\left(t_i+\frac{3t_R}{4}\right) - \frac{7}{3\varepsilon_0}\bar{\sigma}(t_i+t_R) + + \frac{2}{3\varepsilon_0}\bar{\sigma}\left(t_i+\frac{5t_R}{4}\right).$$

Remark. Note, that in view of (3), when the equidistant time sampling $t_{i+1} - t_i = h$ is applied, the existence of an integer $i_0 \geq 1$ such that $t_R = 4i_0h$ is obvious applicability condition of the scheme.

OTHER APPROXIMATE METHODS

Zapas and Phillips [23] developed a method, where the correction $t - \frac{t_R}{2}$ of the time is used as follows:

$$G^{(ZP)}\left(t_i - \frac{t_R}{2}\right) = \frac{1}{\varepsilon_0}\bar{\sigma}(t_i),$$

for an arbitrary $t_i \geq t_R$. Thus, the relaxation modulus approximation can be computed for $t_i \geq t_R/2$. For constant loading strain rate Sorvari and Malinen [14] proposed differential formula:

$$G^{(SM)}(t_i) = \frac{1}{\varepsilon_0}\bar{\sigma}(t_i+t_R) - \frac{t_R}{2\varepsilon_0}\dot{\bar{\sigma}}\,(t_i+t_R),$$

according to which the approximate value of the relaxation modulus for an arbitrary $t_i \geq 0$ is computed using the stress measurements and their derivatives for $t_i + t_R$. The above methods are used in numerical studies due to their approximate "nature".

NUMERICAL ANALYSIS

Example 1 – noise-free measurements. Consider viscoelastic material whose relaxation modulus is described by KWW (Kohlrausch- Williams-Watts) model of the form [3, 16]:

$$G(t) = G_0 e^{-(t/\tau)^\beta}, \tag{4}$$

where: $G_0 = 10^9\ [Pa]$, the parameter $\beta = 0.5\ [-]$, and the relaxation times are $\tau = 3\ [s]$ (material A) and $\tau = 100\ [s]$ (material B). The strain $\varepsilon_0 = 0.001\ [-]$ and the time-horizon $T = 20\ [s]$ are assumed for experiment. The test data was equally spaced in the time interval $[0, T]$. The ramp times: $t_R = 1$ $[s]$, $t_R = 2$ $[s]$ and $t_R = 5$ $[s]$ have been taken in experiment studies. Thus, the condition $t_R \leq 18\tau$ [20, Remark 1] which is sufficient for the monotonicity of the relaxation modulus model determined according to the proposed method is satisfied for material (4) in every case examined. The Zapas-Phillips and Sorvari-Malinen rules and the new method are studied. For any method the number N of sampling points t_i are chosen according to the applicability conditions of the respective scheme. Noise free measurements $\bar{\sigma}(t_i) = \sigma(t_i)$, $i = 1, \dots, N$, are assumed. To estimate the approximation error of the relaxation modulus (4) for the new method the following mean square relative index is taken:

$$ERRG^{(NM)} = \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \left[\frac{G^{(NM)}(t_i) - G(t_i)}{G(t_i)}\right]^2 100\%,$$

where: $\bar{N} = N - 5i_0$ is the corrected number of sampling instants t_i for which the approximate value of the relaxation modulus $G^{(NM)}(t_i)$ can be determined using the scheme proposed. The indices $ERRG^{(ZP)}$ and $ERRG^{(SM)}$ for Zapas-Phillips and Sorvari-Malinen methods are defined by analogy. The indices $ERRG^{(NM)}$, $ERRG^{(ZP)}$ and $ERRG^{(SM)}$ for noise-free stress measurements are given in Table 1 for material A. For material B the courses of this indices as a function of N are shifted in Figure 2 using the logarithmic axis to obtain the best clearness if this graph.

Table 1. Mean relative errors of the relaxation modulus approximation for the new method and Zapas-Phillips and Sorvari-Malinen rules; material A, noise-free case

Mean relative error	Noise-free case, $t_R = 1$ $[s]$					
	$N = 80$	$N = 160$	$N = 240$	$N = 320$	$N = 400$	$N = 480$
$ERRG^{(NM)}$ [%]	4,32E-3	2,46E-3	8,202E-4	4,158E-4	3,038E-4	2,843E-4
$ERRG^{(ZP)}$ [%]	6,39E-4	5,02E-4	4,61E-4	4,415E-4	4,302E-4	4,228E-4
$ERRG^{(SM)}$ [%]	0,273	0,101	0,061	0,044	0,035	0,03
	Noise-free case, $t_R = 2$ $[s]$					
	$N = 40$	$N = 80$	$N = 120$	$N = 160$	$N = 200$	$N = 240$
$ERRG^{(NM)}$ [%]	2,77E-3	0,011	3,997E-3	2,204E-3	1,637E-3	1,467E-3
$ERRG^{(ZP)}$ [%]	3,45E-3	2,74E-3	2,533E-3	2,432E-3	2,373E-3	2,335E-3
$ERRG^{(SM)}$ [%]	1,02	0,399	0,247	0,182	0,148	0,127
	Noise-free case, $t_R = 5$ $[s]$					
	$N = 32$	$N = 80$	$N = 112$	$N = 160$	$N = 208$	$N = 240$
$ERRG^{(NM)}$ [%]	0,103	0,017	0,013	0,014	0,014	0,015
$ERRG^{(ZP)}$ [%]	0,031	0,027	0,026	0,026	0,026	0,025
$ERRG^{(SM)}$ [%]	2,562	1,057	0,831	0,675	0,597	0,563

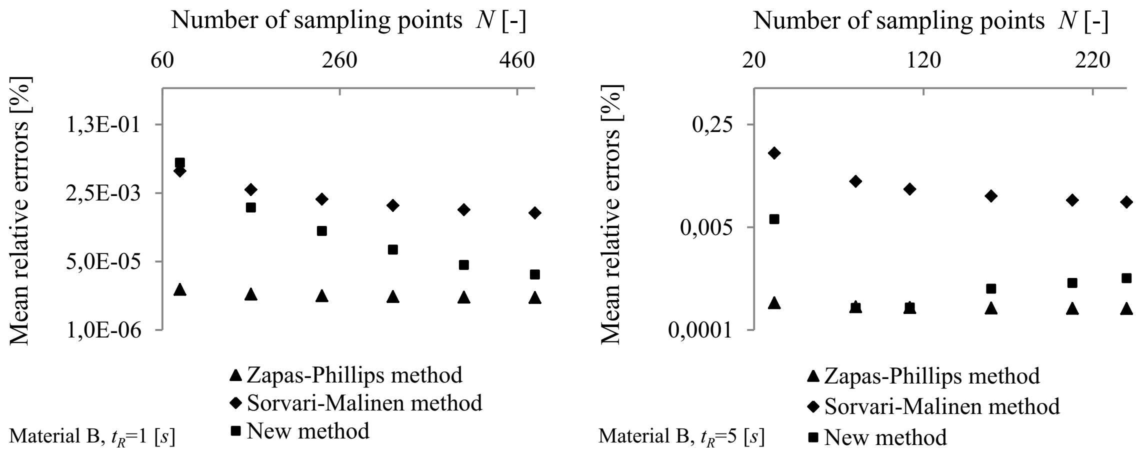

Fig. 2. Mean relative errors of the relaxation modulus approximation for the new method and Zapas-Phillips and Sorvari-Malinen rules; material B, $t_R = 1$ $[s]$ and $t_R = 5$ $[s]$, noise-free case

It can be seen from the above simulation results that for noise-free stress measurements the new algorithm ensures very good accuracy of the relaxation modulus approximation, which is comparable with the Zapas-Phillips rule and better than for the Sorvari-Malinen method, whenever the number of the measurements is appropriately chosen in accordance with the ramp-time t_R and the relaxation time of the material. The relaxation modulus $G^{(NM)}(t)$, $G^{(ZP)}(t)$ and $G^{(SM)}(t)$ calculated according to the considered methods for time interval $[0; 3{,}75]$ seconds are plotted for $t_R = 2\,[s]$ and $N = 160$ in Figure 3 (material A) and in Figure 4 (material B). The relaxation modulus $G(t)$ (4) is also marked in both figures.

Example 2 – noise robustness. We consider again the materials A and B described by the KWW relaxation modulus (4). In order to model the noise produced by a test machine the noises $z(t_i)$ have been generated independently by random choice with normal distribution with zero mean value and variance equal to 100% and 200% of the mean integral value $\frac{1}{T}\int_0^T \sigma(\lambda)d\lambda$ of the signal $\sigma(t)$ in the time interval $[0, T]$. Such measurement noises are even strongest than the true disturbances recorded for the plant materials (see [17; Chapter 5.5.4]). The experiment and next the computations of the approximate relaxation modulus model have been repeated n=100 times. The mean values $MERRG^{(ZP)}$, $MERRG^{(SM)}$ and $MERRG^{(NM)}$ of the indices $ERRG^{(ZP)}$, $ERRG^{(SM)}$ and $ERRG^{(NM)}$, respectively, obtained for material A are given in Table 2 for the case of weak 100% noises and in Table 3 for 200% strong noises. The Zapas-Phillips relaxation modulus model $G^{(ZP)}(t)$ does not depend essentially on the measurement noises, while

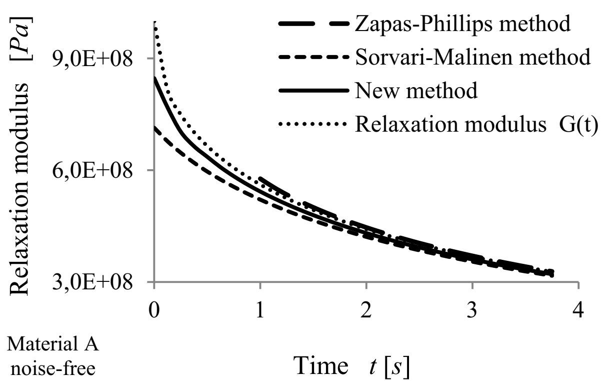

Fig. 3. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material A, noise-free case, $t_R = 2\,[s]$

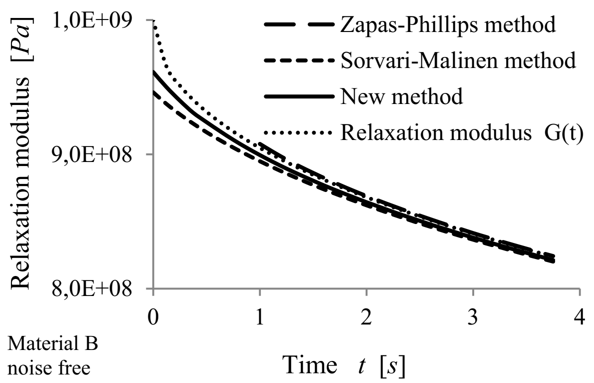

Fig. 4. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material B, noise-free case, $t_R = 2\,[s]$

Table 2. Mean relative errors of the relaxation modulus approximation for the new method and Zapas-Phillips and Sorvari-Malinen rules; material A, weak noises 100%

Mean relative errors	Weak noises 100%, $t_R = 1\,[s]$			
	$N = 80$	$N = 160$	$N = 240$	$N = 320$
$MERRG^{(NM)}$ [%]	0,0474	0,04749	0,04618	0,04602
$MERRG^{(ZP)}$ [%]	0,004516	0,004493	0,004324	0,004399
$MERRG^{(SM)}$ [%]	0,3195	0,2425	0,3595	0,5662

	Weak noises 100%, $t_R = 2\,[s]$					
	$N = 40$	$N = 80$	$N = 120$	$N = 160$	$N = 200$	$N = 240$
$MERRG^{(NM)}$ [%]	0,04374	0,04892	0,04264	0,04122	0,04084	0,04357
$MERRG^{(ZP)}$ [%]	0,007173	0,006491	0,006279	0,006276	0,006211	0,006055
$MERRG^{(SM)}$ [%]	1,057	0,526	0,5102	0,6218	0,83	1,103
	Weak noises 100%, $t_R = 5\,[s]$					
	$N = 32$	$N = 80$	$N = 112$	$N = 160$	$N = 208$	$N = 240$
$MERRG^{(NM)}$ [%]	0,1263	0,04797	0,04636	0,0611	0,07399	0,1013
$MERRG^{(ZP)}$ [%]	0,03425	0,03039	0,02964	0,0292	0,02892	0,0289
$MERRG^{(SM)}$ [%]	2,647	1,526	1,75	2,499	3,65	4,625

the mean value of the noise is zero. Both for the new and Sorvari-Malinen methods the accuracy of the relaxation modulus approximation are dependent on the noises, however, for Sorvari-Malinen method the mean approximation errors are multiple greater than in the case of the new method. Similar simulation results are obtained for material B. An example of the models $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ and the "true" relaxation modulus of material A are illustrated in Figure 5 for weak 100% noises and in Figure 6 for strong 200% noises; the ramp-time $t_R = 2\ [s]$ and the number of measurements $N = 160$ are applied here. The respective characteristics for material B are given in Figures 7 (weak noises) and 8 (strong noises). The models $G^{(ZP)}(t)$ are generally fairly smooth, however the models $G^{(NM)}(t)$ and especially $G^{(SM)}(t)$ are not.

In turn, the relaxation modulus $G(t)$ (4) of material B and the corresponding models for ramp-times $t_R = 1\ [s]$ and $t_R = 5\ [s]$ are recorded in Figures 9 and 10, respectively, for the case of weak noises; as previously, the number of measurements $N = 160$ is taken. Through a comparison of the Figures 7,9 and 10 we can realize how the accuracy of the relaxation modulus approximation strongly depend on the respective choice of such experiment parameters as the ramp-time t_R and the sampling period h.

Example 3 – Young modulus identification. The identifiability of the constant uniaxial Young modulus $G(0)$ has been also examined. The accuracy of the initial value $G(0)$ identification in the case of the new method has been measured by relative means error $ERR0^{(NM)}$ defined as a mean value of the square relative errors

Table 3. Mean relative errors of the relaxation modulus approximation for the for the new method and Zapas-Phillips and Sorvari-Malinen rules; material A, strong noises 200%

Mean relative error	Strong noises 200%, $t_R = 1\ [s]$					
	$N = 80$	$N = 160$	$N = 240$	$N = 320$		
$MERRG^{(NM)}$ [%]	0,097	0,092	0,092	0,09		
$MERRG^{(ZP)}$ [%]	8,527E-3	8,283E-3	8,186E-3	8,362E-3		
$MERRG^{(SM)}$ [%]	0,363	0,388	0,659	1,053		
	Strong noises 200%, $t_R = 2\ [s]$					
	$N = 40$	$N = 80$	$N = 120$	$N = 160$	$N = 200$	$N = 240$
$MERRG^{(NM)}$ [%]	0,082	0,086	0,081	0,078	0,08	0,083
$MERRG^{(ZP)}$ [%]	0,011	0,01	9,96E-3	9,96E-3	9,841E-3	9,797E-3
$MERRG^{(SM)}$ [%]	1,098	0,649	0,794	1,103	1,553	2,077
	Strong noises 200%, $t_R = 5\ [s]$					
	$N = 32$	$N = 80$	$N = 112$	$N = 160$	$N = 208$	$N = 240$
$MERRG^{(NM)}$ [%]	0,15	0,07	0,074	0,108	0,147	0,181
$MERRG^{(ZP)}$ [%]	0,038	0,034	0,033	0,033	0,033	0,032
$MERRG^{(SM)}$ [%]	2,733	2,002	2,717	4,293	6,713	8,776

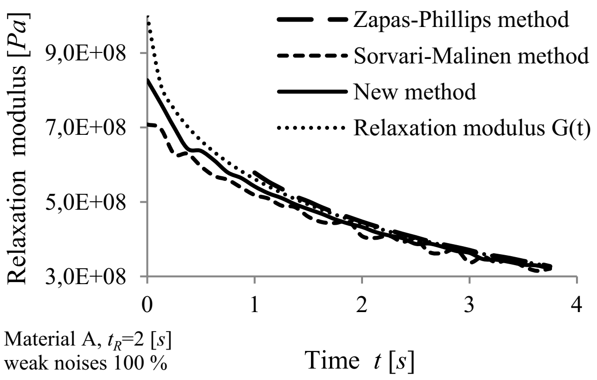

Fig. 5. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material A, weak noises 100%, $t_R = 2\ [s]$

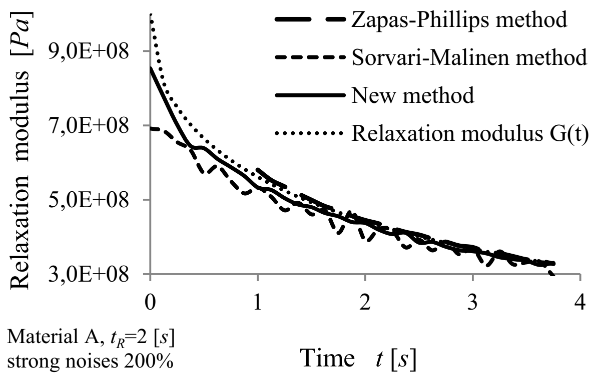

Fig. 6. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material A, strong noises 200%, $t_R = 2\ [s]$

$\left[\frac{G^{(NM)}(t_1)-G(0)}{G(0)}\right]^2$ for 100 repetitions of the numerically simulated stress relaxation test (1) in time interval $[0,T]$. The respective indices for the two other methods are defined by analogy. In the case of Zapas-Phillips method the value of $G^{(ZP)}(t_R/2)$ is chosen as a $G(0)$ approximation. The numerical studies results obtained for material A are summarized in Table 4 for weak and in Table 5 for strong noises. Clearly, the Young modulus approximation errors in the case of Zapas-Phillips method is nearly independent of the number of sampling points and the noises intensity, however the errors increases with the ramp-time t_R. For Sorvari-Malinen method the errors decreases with the increasing number of sampling points, but still are unacceptable big. The smallest errors of $G(0)$ approximation are guaranteed by the new method. By respective choice of the experiment parameters t_R and N, the errors can be reduced below 1,5%.

CONCLUSIONS

1. A new fast scheme for approximate identification of linear relaxation modulus of viscoelastic materials using discrete time-measurements of the stress from non-ideal ramp-tests is proposed and analyzed. The approximations of the relaxation modulus in successive time instants are determined on the basis on the stress measurements in only three appropriately chosen sampling points. Numerical results are provided to compare the presented method with two other alternative procedures known in the literature.
2. The model obtained by using Zapas-Phillips method exactly describes the true KWW relaxation modulus, both for noise-free, see Figure 3 and 4, as well as for noise corrupted stress measurements, see Figures 5-10. However, the Zapas-Phillips method fails at the estimating of the initial value of $G(0)$. The approximation accuracy obtained by the new method is worse than that guaranteed by Zapas-Phillips rule, but the applicability for short time interval is an excellent advantage of the new scheme.
3. In Sorvari-Malinen method the relaxation modulus is poorly estimated. The new method provides better approximation accuracy than Sorvari-Malinen method, the more so that the simulation computations indicate that the approximation errors for Sorvari-Malinen method are extremely sensitive to the noises (see Figures 5-10). It seems also reasonable to use the new method to produce a good fit of the constant Young modulus.
4. Both theoretical analysis of the scheme conducted in [20] as well as the results of the numerical studies presented above, especially acceptable noise robustness, suggest

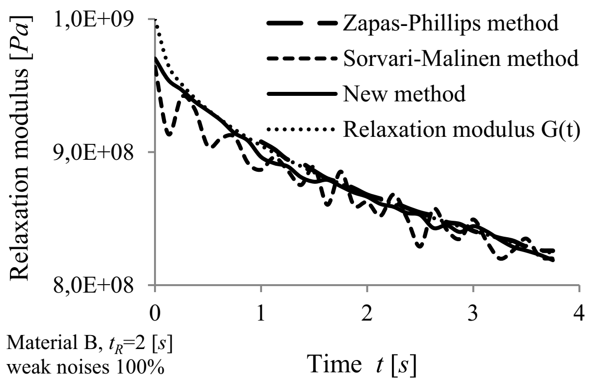

Fig. 7. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material B, weak noises 100%, $t_R = 2$ [s]

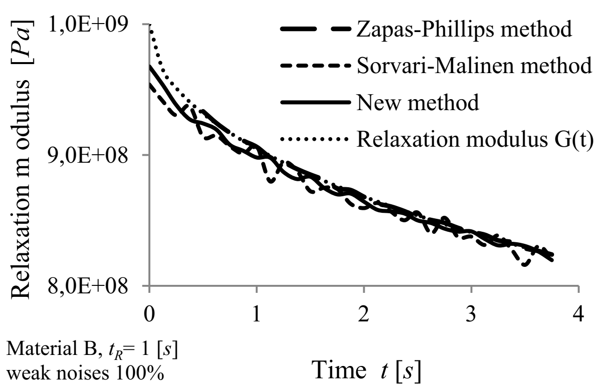

Fig. 9. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material B, weak noises 100%, $t_R = 1$ [s]

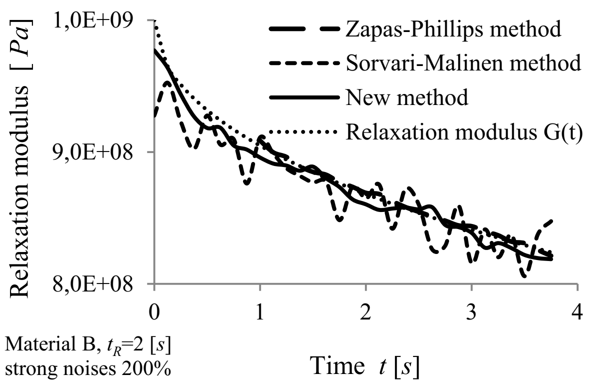

Fig. 8. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material B, strong noises 200%, $t_R = 2$ [s]

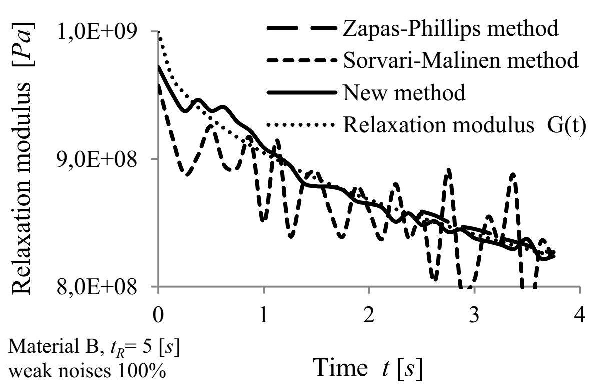

Fig. 10. Approximations $G^{(NM)}(t)$, $G^{(ZP)}(t)$, $G^{(SM)}(t)$ of the relaxation modulus $G(t)$; material B, weak noises 100%, $t_R = 5$ [s]

Table 4. Mean relative errors of $G(0)$ approximation for the for the new method and Zapas-Phillips and Sorvari-Malinen rules; material A, weak noises 100%

Mean relative errors	Weak noises 100%, $t_R = 1\ [s]$			
	$N = 80$	$N = 160$	$N = 240$	$N = 320$
$ERR0^{(NM)}$ [%]	8,568	2,858	1,689	1,238
$ERR0^{(ZP)}$ [%]	10,458	10,474	10,444	10,467
$ERR0^{(SM)}$ [%]	8,568	5,85	4,938	4,362

	Weak noises 100%, $t_R = 2\ [s]$					
	$N = 40$	$N = 80$	$N = 120$	$N = 160$	$N = 200$	$N = 240$
$ERR0^{(NM)}$ [%]	12,65	4,906	2,989	2,114	1,742	1,731
$ERR0^{(ZP)}$ [%]	17.892	17.888	17,9	17,88	17,86	17,87
$ERR0^{(SM)}$ [%]	15,36	10,81	9,027	8,239	7,715	7,452

	Weak noises 100%, $t_R = 5\ [s]$					
	$N = 32$	$N = 80$	$N = 112$	$N = 160$	$N = 208$	$N = 240$
$ERR0^{(NM)}$ [%]	9,625	3,909	3,101	4,112	10,79	12,42
$ERR0^{(ZP)}$ [%]	33.263	33,26	33,25	33,24	33,26	33,24
$ERR0^{(SM)}$ [%]	22,64	16,73	15,49	14,9	14,41	13,91

Table 5. Mean relative errors of $G(0)$ approximation for the for the new method and Zapas-Phillips and Sorvari-Malinen rules; material A, strong noises 200%

Mean relative error	Strong noises 200%, $t_R = 1\ [s]$			
	$N = 80$	$N = 160$	$N = 240$	$N = 320$
$ERR0^{(NM)}$ [%]	8,594	3,17	1,926	1,42
$ERR0^{(ZP)}$ [%]	10,46	10,46	10,46	10,46
$ERR0^{(SM)}$ [%]	8,674	5,907	4,804	4,405

	Strong noises 200%, $t_R = 2\ [s]$					
	$N = 40$	$N = 80$	$N = 120$	$N = 160$	$N = 200$	$N = 240$
$ERR0^{(NM)}$ [%]	12,6	5,138	3,301	2,36	1,74	2,183
$ERR0^{(ZP)}$ [%]	17,87	17,88	17,88	17,89	17,89	17,88
$ERR0^{(SM)}$ [%]	15,33	10,85	9,128	8,399	7,632	7,403

	Strong noises 200%, $t_R = 5\ [s]$					
	$N = 32$	$N = 80$	$N = 112$	$N = 160$	$N = 208$	$N = 240$
$ERR0^{(NM)}$ [%]	10,19	4,413	3,544	4,391	9,023	15,64
$ERR0^{(ZP)}$ [%]	33.23	33,26	33,25	33,26	33,24	33,23
$ERR0^{(SM)}$ [%]	22,54	16,78	15,35	14,64	14,42	13,97

that the proposed scheme can be used successfully within a satisfactory range of viscoelastic materials. Moreover, the practical point of view has been specially emphasized while the scheme derivation and because it does not require any other experimental technique more sophisticated than the equidistant sampling of time instants during rheological experiment, the presented algorithm is easy to implement and fast to use since only three measured stress data points are used to evaluate the relaxation modulus at any time instant. Thus the scheme can be easily implemented in an arbitrary computational environment supporting the rheological experiment.

5. The problems of viscoelastic model determination, in which the relaxation test stress history may provide experimental

data for identification, such as that considered in [1,6,9,13] for polymeric liquids and solids, in [12] for metals and their alloys or in [2, 5,8,17,21] for foods and biological materials, constitute the area of applicability of the scheme.

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ALGORYTM IDENTYFIKACJI MODUŁU RELAKSACJI MATERIAŁÓW LEPKOSPRĘŻYSTYCH NA PODSTAWIE NIEIDEALNEGO TESTU RELAKSACJI NAPRĘŻEŃ O NIELINIOWYM ODKSZTAŁCENIU WSTĘPNYM

Streszczenie. Celem pracy było opracowanie szybkiego algorytmu identyfikacji modułu relaksacji materiałów o własnościach liniowo lepkosprężystych na podstawie dyskretnych pomiarów naprężenia zgromadzonych w rzeczywistym teście relaksacji naprężeń o zmiennej w czasie prędkości odkształcania wstępnego. Zaproponowano prosty schemat identyfikacji, w którym przybliżenie modułu w wybranej chwili czasu wyznaczane jest na podstawie co najwyżej trzech pomiarów naprężenia. Przeprowadzone badania numeryczne wskazują, że opracowana metoda zapewnia lepsze przybliżenie modułu relaksacji niż metoda Sorvari-Malinena oraz akceptowalne pogorszenie jakości jego przybliżenia w porównaniu z regułą Zapasa-Phillipsa. Przewagę nowej metody nad regułą Sorvari-Malinena stanowi także większa odporność na zakłócenia pomiarowe, jej zaletą w porównaniu z regułą Zapasa-Phillipsa jest rozszerzenie zakresu stosowalności o początkowy odcinek czasu. Metoda zapewnia bardzo dobre przybliżenie wartości początkowej modułu relaksacji, a jej prosty algorytm umożliwia zastosowanie w trybie on-line w czasie eksperymentu reologicznego.

Słowa kluczowe: test relaksacji naprężeń, moduł relaksacji, model KWW, algorytm identyfikacji.