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THE METHOD OF STANDING TREES ALLOCATION TO DIFFERENT BIOSOCIAL CLASSES

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Summary

The discriminant analysis method was used for the purpose of allocation trees to Kraft classes. The Scots pine trees were used to determine discriminant functions. Only traits which can be measured on the standing trees were taken into consideration. They included height, breast height diameter inside and without bark, basal area, double bark thickness, 5 and 10 years increment in breast, height diameter, tree slenderness. Some sets of uncorrelated traits were taken into account to select the better discriminant functions guided by the minimum of erroneous classification probability. The discriminant model based on the height, 10-years increment at the breast high diameter, basal area and slenderness, has proven to be the best for grouping the trees.

Key words and phrases: discriminant analysis, Kraft classes, Scots pine (*Pinus sylvestris* L.)

Classification AMS 2000: 62F25

1. Introduction

The trees from the same forest, despite equal age, are at a different condition. Some of them are very weak but others are strong and robust, thus for each tree the social class is determined. The natural classes of trees, which combined the social position of the tree in the stand with the degree of crown formation, were distinguished by Kraft in 1884 year (Assmann 1970). He recognized the following classes of trees (Figure 1): I – predominant trees with exceptionally well-developed crowns, II – dominant trees, forming the main stand as a rule with relatively well-developed crowns, III – low co-dominant trees; crown shape is still normal and hence the trees are similar to those in the second tree class in this respect, yet they are relatively weakly developed and restricted often already with the onset of degeneration, IV – dominated trees, with crowns more or less dying back, restricted on all sides or on two sides, or with one sided development. This class is divided into IVa – intermediate trees, essentially free of canopy cover with restricted lateral crown growth and IVb – partially overtopped crowns, the upper crown free, the lower crown under canopy cover. The last class is the V – entirely overtopped trees, which is divided into Va – with crowns capable of growth and Vb – with dead crowns.

The classes I-III are called the dominant stand and the classes IV-V are called the suppressed stand. Belonging to a given social class reflects a position of a tree in a stand, and through this, its growth potential.

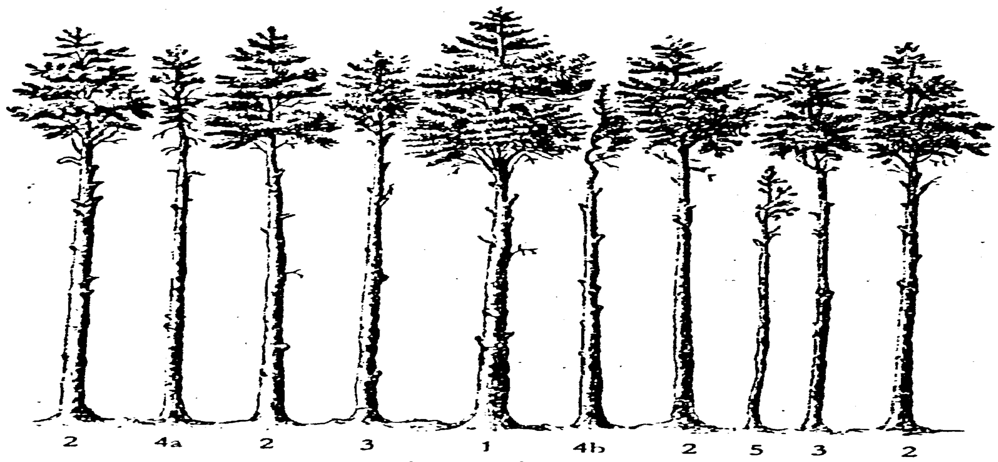


Fig. 1. Kraft's classes from 1884

The aim of this study was to designate one or more functions, by which, using some traits, the Kraft class of a given tree can be appointed. Moreover, only the traits that can be measured on standing trees were considered. The discriminant analysis was used for this purpose based on the known classification of trees made under visual assessment. Functions obtained in this way are used to classify trees from the different stands. Because for these trees the Kraft classes are known, thus the probability of misclassification can be calculated.

2. Experimental material

The experimental material included selected results of 302 pine-trees derived from 35−years−old pure Scots pine stand growing on fresh coniferous forest site in Zielonka. In the study measurement traits such as tree height (h) with a tolerance of up to 0.1 m; breast height diameter inside bark ($d_{1.3zk}$) in cm; tree basal area ($g_{1.3}$) in m^2 (the area of a circle with a diameter equal to breast height diameter); double bark thickness (K) – measured in cm at the height of 1.3 m from the base of the tree; 5-years and 10-years increment at breast height diameter (Zd_5,Zd_{10}) with a tolerance of up to 0.01 cm; tree slenderness (s) defined as the ratio of height in m to breast height diameter, in cm ($h/d_{1.3zk}$) were used. Moreover, the $d_{1.3bk}$ were calculated as $d_{1.3zk}$ -K. All of the analyzed traits were determined for standing trees. The measurement results of 33 and 44-year-old pines from other stand than the tested one were used as a test material.

3. Methods

The discriminant analysis method (for example Krzyśko 1990, 2000; Krzyśko et al. 2008; Kornacki and Ćwik 2005; Anderson 2003) was used to classify trees as regards the biosocial classes on the basis of the same measured traits which are determined on the standing trees.

Let $\mathbf{X} = [x_{ij}]$ be a random sample from normal distributed population, x_{ij} is the observation of i^{th} ($i = 1,...,n$) object and j^{th} ($j = 1,...,m$) variables (trait). The discriminant variables are present as linear combination of standardized input variables form $\mathbf{F} = \mathbf{A'Z}$, where $\mathbf{F}$ is the $(t \times n)$ matrix of discriminant variables (t – is the number of discriminant variables), $\mathbf{A'}$ is the $(t \times m)$ matrix of coefficients of discriminant variables and $\mathbf{Z}$ is the $(m \times n)$ matrix of standardized observations.

Two matrices of variance are calculated namely inter classes $\mathbf{M} = \sum_{r=1}^{c} n_r (\bar{z}_r - \bar{z})(\bar{z}_r - \bar{z})'$ and inside classes

$\mathbf{W} = \sum_{r=1}^{c}\sum_{i=1}^{n_r}(z_{ri} - \bar{z}_r)(z_{ri} - \bar{z}_r)'$, where $\bar{z}_{ri} = [z_{rij}]$ is the m-vector of standardized variables of i^{th} object in the r^{th} group ($r = 1,...,c$), $\bar{z}_r = [\bar{z}_{rj}]$ is the m-vector of means $\bar{z}_{rj} = n_r^{-1}\sum_{i=1}^{n_r} z_{rij}$, $\bar{z} = [\bar{z}_j]$ is the m dimensional vector of means $\bar{z}_j = \sum_{r=1}^{c}\sum_{i=1}^{n_r} z_{rij}$. In this issue we are looking for coefficients of discriminant function which minimize the intragroup variability and maximize intergroup variability, the same maximize ratio of the intergroup variability for the intragroup variability. This approach leads to a linear equations system $(\mathbf{M} + \lambda_l \mathbf{W})\mathbf{a}_l = 0$, where $\mathbf{a}_l$ is the normalized eigenvector corresponding to the l^{th} eigenvalue ($l = 1,...,t$). The solution of equations is t non-negative eigenvalues λ_l ($\lambda_1 \geq \lambda_2 \geq ....\lambda_t \geq 0$) which correspond to normalized eigenvectors $\mathbf{a}_l$. The first discriminant function f_l is related to the largest (first) eigenvalue, the second is related to second eigenvalue etc. The absolute values of the coefficients $\mathbf{a}_l$, which were calculated from standardized input data, determine the discriminative power of prior variables.

The usefulness of discriminant function is determined by testing of hypothesis that not all eigenvalues are equal zero. If this hypothesis is rejected, the hypothesis that $t-1$ eigenvalues are equal zero (ignored the largest eigenvalue λ_1) is tested etc. This process is continued until the hypothesis is rejected since from this value the remaining eigenvalues are equal zero. These hypotheses are verified using the Lambda Wilks' statistics $\Lambda_k = \prod_{l=k+1}^{t} 1/(1+\lambda_l)$ ($k = 0,...,t-1$, $t = \min(m, c-1)$), where $-[n-(t+c)/2-1]\ln\Lambda_k$ has asymptotic χ^2 distribution with $(m-k)(z-k-1)$ degree of freedom.

To evaluate of the input variable the partial lambda Wilks' statistics, $\Lambda_+ = \Lambda_k / \Lambda_k^{(-j)}$ where Λ_k is calculated after being introduced into the model j^{th} variable and $\Lambda_k^{(-j)}$ is calculated before being introduced into the model j^{th} variable, can be used. Next the $F_j = [(n-c-m+1)(c-1)^{-1}](1-\Lambda_+)/\Lambda_+$ statistics, which has $c-1$ and $n-c-m+1$ degree of freedom for the numerator and denominator respectively, is designated.

If linear classifier is used, observation is classified to class r when the value of function $d_r(c) = \sum_{l=1}^{t}[a_l(x-\bar{x}_r)]^2$.

Determination of the discriminant function was carried out in several stages in this study. In the first step the variables were selected to models. It would be desirable that the variables included in the model were not too strongly correlated as such variable do not bring anything to the model. Moreover, this could hinder distinguishing groups (Joilliffe 2002, p.206). Due to the above, the correlation between individual pairs of traits were calculated. Moreover, the correlations between variables were presented in the PCA diagram (using the covariance matrix). In this diagram the standard deviations of variables were isolable as the length of the vectors (the variables presented by the longest vectors should be included to the model).

In the second step the discriminant power of individual variables (included to the models) was evaluated on the basis of partial lambda Wilks' statistics using the SAS *DISCRIM* procedure (Statistical Analysis System, version 9.3).

In the third step the number of discriminant functions was determined and cross validation method was used to evaluate proposed models. For this purpose the cross validation method proposed in the *DISCRIM* procedure of SAS system was used. In this way each training data point (tree) was classified as it was new and the discriminant function used in each case was constructed by taking that observation out of the data set. Moreover, the cross validation method was used, then all data (trees) was random by divided into two parts, proportionally to the Kraft classes, and the first half of the data was used to designate discriminant functions and the second one was used as new data (SAS *SURVEYSELECT* procedure). This division was repeatedly and the means of a posteriori probabilities of misclassification were calculated. These two methods of cross validation were used when a priori probabilities of misclassifications for a given Kraft cases are equal as well as proportional to the number in the class.

In the fourth step the models with the smallest a posteriori probabilities of misclassifications were chosen. In the last step the models were tested using the other data (33 and 44 old tress).

4. Results

In the first step of analysis, the variables should be selected to models. The correlation between individual pairs of traits was calculated and results were placed in Table 1. The very strong correlations (>95%) occur between $d_{1.3zk}$ with $d_{1.3bk}$ and $g_{1.3}$, next $g_{1.3}$ with $d_{1.3bk}$. Thus, when selecting variables to the model, the above pairs should be avoided. Furthermore, the variable cannot be collinear and redundant (one variable cannot be a linear combination of other variables).

There is $d_{1.3bk}= d_{1.3zk} –K$ thus $d_{1.3bk}$ and $d_{1.3zk}$ cannot be together in the model. The correlations between variables are presented in the PCA diagram on the Figure 2 (in the coordinate of two first principal components). Moreover the vectors of traits *s* and *h* turned out to be the longest. This means that these two traits should be included in the model.

Table 1. Correlations coefficients among traits

	$d_{1.3zk}$	*k*	*h*	Zd_{10}	*s*	$d_{1.3bk}$	Zd_5	$g_{1.3}$
k	0.78	1						
h	0.83	0.59	1					
Zd_{10}	0.86	0.62	0.81	1				
s	-0.92	-0.75	-0.63	-0.74	1			
$d_{1.3bk}$	0.99	0.68	0.84	0.86	-0.9	1		
Zd_5	0.84	0.6	0.79	0.97	-0.73	0.839	1	
$g_{1.3}$	0.99	0.77	0.79	0.84	-0.88	0.977	0.82	1

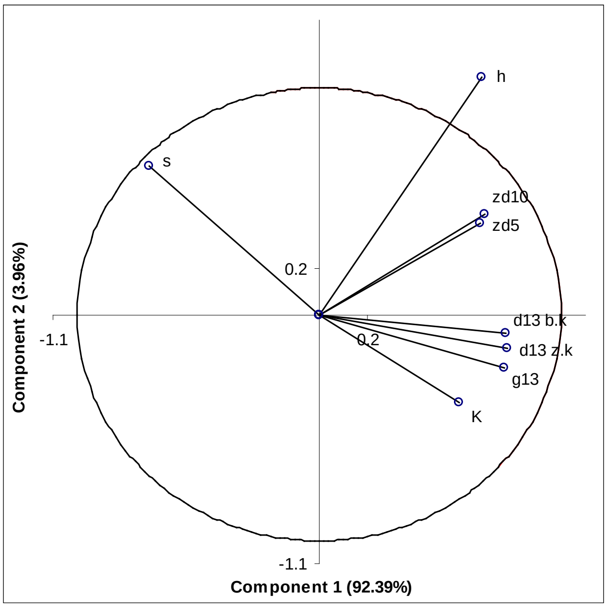


Fig. 2. PCA diagram for the two first principal components presented correlations between variables

Paying attention to above, relationships between the following sets of variables (1) $d_{1.3zk}$, *K*, *h*, Zd_{10}, *s*, (2) $d_{1.3zk}$, *K*, *h*, Zd_5, *s*, (3) $d_{1.3bk}$, *K*, *h*, Zd_{10}, *s*, (4) $d_{1.3bk}$, *K*, *h*, Zd_5, *s*, (5) $g_{1.3}$, *K*, *h*, Zd_{10}, *s* and (6) $g_{1.3}$, *K*, *h*, Zd_5, *s* can be proposed. The trait *K* was included in the models, despite the fact that it was strongly negatively correlated with *s*, since in the models (3) and (4) *K* is disabled from diameter and it is also interesting if this trait have the impact on discrimination. Moreover, the SAS procedure *STEPDISC* was used to select variables used for discrimination model. When using stepwise and backward selection methods, the matched model was the following (7) $d_{1.3zk}$, *h*, Zd_5, *s* and $g_{1.3}$. Meanwhile, when using the forward selection method the model (8) containing $d_{1.3zk}$, *h*, Zd_5, *s*, $g_{1.3}$. and Zd_{10} was established. In this case two variables Zd_5 and Zd_{10} are too strongly correlated (see Table 1 and Figure 2), thus they should not be in the model together.

For eight sets of variables (models) the partial lambda Wilks' statistic (Λ_+) were designated to evaluate discriminatory power of individual variables in the discriminant models. Results are shown in Table 2. In the (1) – (4) models the Λ_+ statistics evaluating the variable K was not significant, thus this variable should not be included to the model. In the model (8) Λ_+ statistics evaluating the variable Zd_{10} was not significant, whereas in the (5) and (7) sets all variables were significant. Then, the variable *K* was omitted in the sets of analyzed variables. Moreover, while omitting Zd_{10}, the model (8) was identical to model (7), thus model (8) was not considered.

Table 2. Evaluate of choosing variables for discriminant function

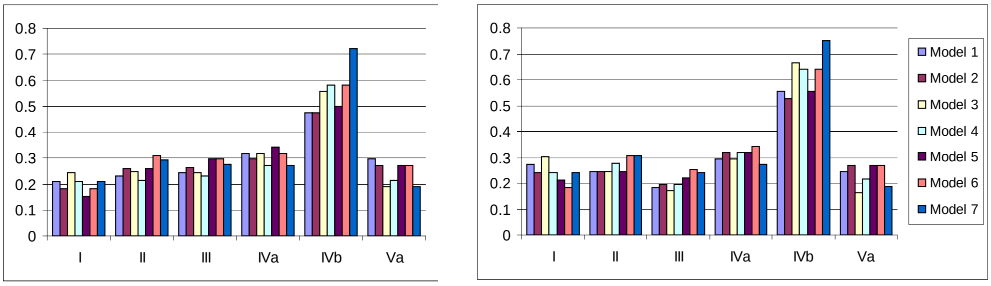
Model	variable	partial Λ_+	probability of remove	Model	variable	partial Λ_+	probability of remove
(1)	$d_{1.3zk}$	0.5136	<0.00001	(5)	$d_{1.3bk}$	0.5136	5.8E-12
	K	**0.9750**	**0.1913**		*K*	0.789	0.0005
	h	0.4783	<0.00001		*h*	0.4783	2E-13
	Zd_{10}	0.8345	<0.00001		Zd_{10}	0.8342	0.005
	s	0.5435	<0.00001		*s*	0.5435	7E-11
(2)	$d_{1.3zk}$	0.5013	<0.00001	(6)	$d_{1.3bk}$	0.5013	2E-12
	K	**0.9741**	**0.1739**		*K*	0.7834	0.0004
	h	0.4762	<0.00001		*h*	0.4762	2E-13
	s	0.5451	<0.00001		*s*	0.5451	8E-11
	Zd_5	0.7972	<0.00001		Zd_5	0.7972	0.0008
(3)	*K*	**0.9721**	**0.1402**	(7)	$d_{1.3zk}$	0.8286	0.000
	h	0.4915	<0.00001		*h*	0.8965	0.000006
	Zd_{10}	0.8245	<0.00001		*s*	0.9041	0.000018
	s	0.5667	<0.00001		Zd_5	0.7820	<0.00001
	$g_{1.3}$	0.4802	<0.00001		$g_{1.3}$	0.7787	<0.00001
(4)	*K*	**0.9713**	**0.1283**	(8)	$d_{1.3zk}$	0.820018	<0.00001
	h	0.4768	<0.00001		*h*	0.900044	0.00001
	Zd_5	0.5796	<0.00001		Zd_{10}	**0.974420**	**0.180992**
	s	0.7896	<0.00001		*s*	0.900922	0.000012
	$g_{1.3}$	0.4697	<0.00001		Zd_5	0.960791	0.039148
	-	-	-		$g_{1.3}$	0.769998	0.014381

For each model the number of useful discriminant function was established (see Methods). The results are presented in the Table 3 in which only significant cases are given. In all sets of traits the χ^2 statistics were significant at the level $\alpha = 0.05$, in the case of three discriminant function. All these results are similar, thus the probabilities of misclassification were designated for all considered cases. For this purpose the cross validation method described in the previous paragraph is used.

Table 3. Characteristics of discriminant function in the models

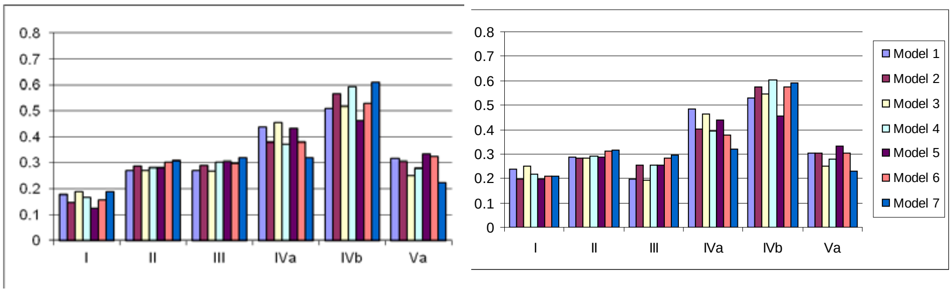
Model	Deleted elements	eigenvalue	canonical R	Λ_k	χ^2	d.f.	p
	0	7.9940	0.9428	0.050032	886.5450	20	<0.0001
(1)	1	1.1237	0.7274	0.449992	236.3637	12	<0.0001
	2	0.0425	0.2018	0.955631	13.4335	6	0.0366
	0	8.2009	0.9441	0.047856	899.7067	20	<0.0001
(2)	1	1.1249	0.7276	0.440321	242.7944	12	<0.0001
	2	0.0598	0.2376	0.935656	19.6862	6	0.0031
	0	7.9137	0.9422	0.046920	905.5543	20	<0.0001
(3)	1	1.2718	0.7482	0.418234	258.0272	12	<0.0001
	2	0.0484	0.2149	0.950165	15.1313	6	0.0193
	0	8.0906	0.9434	0.044974	918.0951	20	<0.0001
(4)	1	1.2753	0.7487	0.408842	264.7502	12	<0.0001
	2	0.0660	0.2488	0.930247	21.4025	6	0.0016
	0	7.9948	0.9428	0.048784	892.5148	25	<0.0001
(5)	1	1.1413	0.7301	0.438802	243.4051	16	<0.0001
	2	0.0593	0.2365	0.939612	18.4061	9	0.0307
	0	8.2009	0.9441	0.046619	905.9283	25	<0.0001
(6)	1	1.1420	0.7302	0.428936	250.1251	16	<0.0001
	2	0.0763	0.2663	0.918802	25.0242	9	0.0029
	0	9.1833	0.9496	0.037265	972.1031	25	<0.0001
(7)	1	1.3140	0.7536	0.379485	286.3222	16	<0.0001
	2	0.1201	0.3274	0.878128	38.4039	9	<0.0001

The a posteriori probabilities of error classifications are presented in the Figure 3 and 4. The most of misclassifications were recorded in class IVb, whereas the a priori misclassification probabilities were either equal or proportional (Figure 3 and 4). This result is independent of the model. Least misclassification was observed in class I or Va in depending on the used model. In the case of proportional a priori probabilities the small misclassification probability was also observed in class III. Using the second method of cross validation (Figure 4) all a posteriori probabilities were smaller than using the first method.



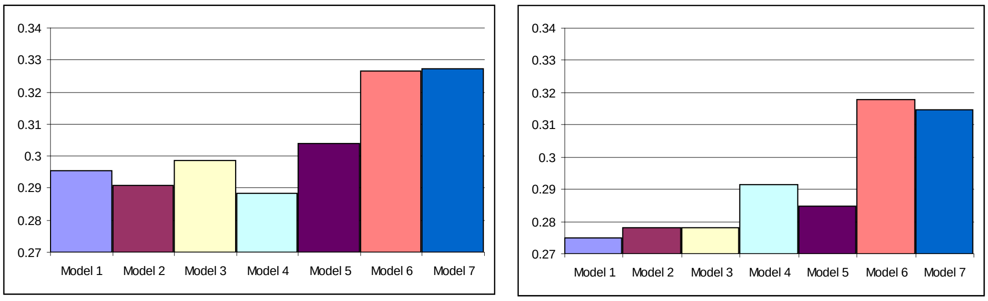
a) b)

Fig. 3. A posteriori probabilities of misclassification, designated separately for each Kraft class and method, if cross validation method is used for individual observations; a) a priori probabilities are equal, b) a priori probabilities are proportional



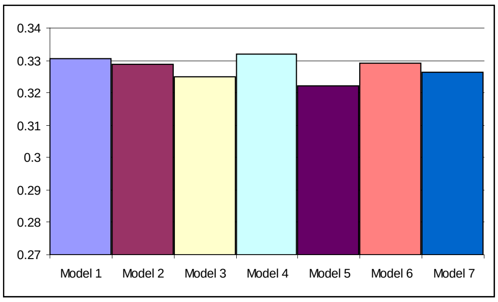
a) b)

Fig. 4. A posteriori probabilities of misclassification, designated separately for each Kraft class and method, if cross validation method is used for half of the randomly selected observations; a) a priori probabilities are equal, b) a priori probabilities are proportional

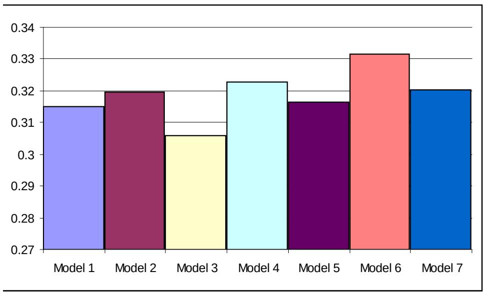


a) b)

Fig. 5. A posteriori probabilities of misclassification, designated together for all Kraft classes and separately for each method, if cross validation method is used for individual observations; a) a priori probabilities are equal, b) a priori probabilities are proportional



a)



b)

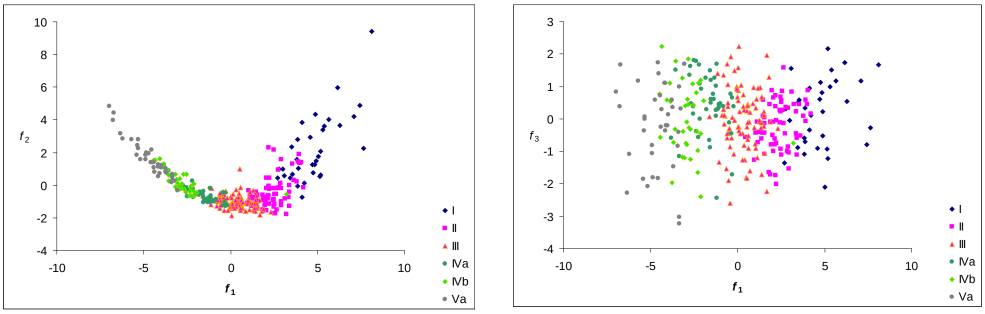
Fig. 6. A posteriori probabilities of misclassification, designated together for all Kraft classes and separately for each method, if cross validation method is used for half of the randomly selected observations; a) a priori probabilities are equal, b) a priori probabilities are proportional

The evaluation of a posteriori misclassification probabilities for individual model but jointly for all Kraft classless (Figure 5) is following: the models (1) – (5) were better than the models (6) – (7) if the fist way of cross validation was used. In the case of validation on the second way (Figure 6) the models (3) and (5) were the best. For these models the probabilities of misclassification into individual class were often also small. In this connection the models (3) and (5) appeared to be among the best (especially when a priori probabilities were proportional). The standardized coefficients of three significant discriminant functions for these models are given in Table 4. On the basis of absolute value of these coefficients the impact of several variable on the discriminant variables can be precise. In the model (3) the height and slenderness had the highest influence on the first discriminant variable f_1, and basal area had the biggest impact on the second variable f_2, while Zd_{10} on the third variable. In the model (5) the height and Zd_{10} had the strongest influence on the first discriminant variable, the slenderness and breast height diameter without bark had impact on the second variable and the most influence on the third variable had breast height diameter without bark.

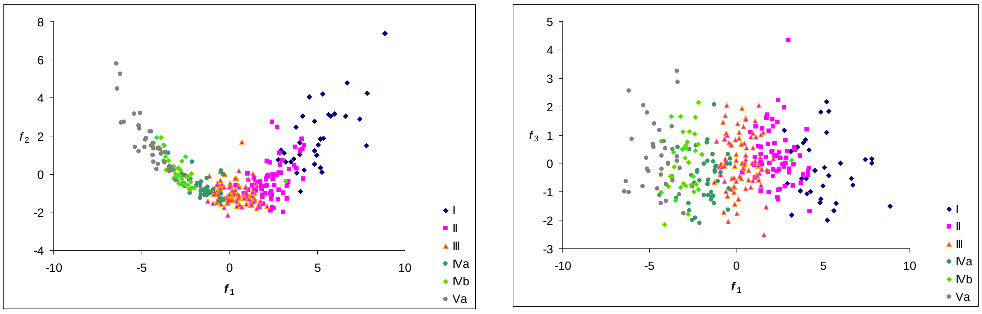
Table 4. Standardized coefficients of discriminant function in models (3) and (5).

	Model (3) – three discriminant variables explain 99.96% of the variation (the first 86.90%)				**Model (5) – three discriminant variables explain 99.95% of the variation (the first - 85.67%)**				
variable	***h***	***Zd*₁₀**	***s***	***g*₁.₃**	***d*₁.₃bk**	***K***	***h***	***Zd*₁₀**	***s***
f_1	0.6189	0.4412	-0.6133	0.0192	0.2549	0.0965	0.4753	0.4285	-0.3686
f_2	-1.0083	0.0372	1.3068	1.6922	2.2550	0.8108	-1.5880	0.0514	2.3199
f_3	0.6534	-0.7215	0.4841	0.4376	-0.7548	0.3138	-0.3919	0.6412	-0.6318

The transformed observations of analyzed trees are presented in the system of the first two discriminant variables as well as in the system of the first and the third discriminant variable in the Figure 7. This transformed data with little error can be separated linearly. In the models (3 and 5) explicitly two groups of trees can distinguish. Namely suppressed stand (class IVa, IVb, Va) and dominant stand (I, II, III). The above is visible in the figure presenting system of two first discriminant variables as well as at the first and third discriminant variables.



model (3)



model (5)

Fig. 7. The transformed observations of analyzed trees in the system of the first two discriminant variables (left) and in the system of the first and the third discriminant variable (right) in models (3) and (5)

The discriminant variables in the model (3) are the following

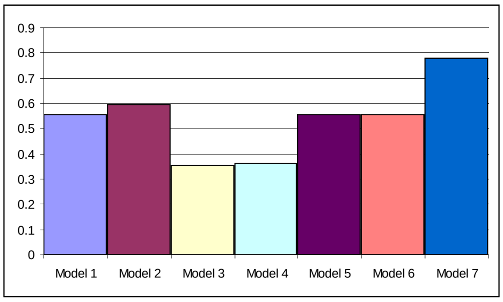
$$\begin{aligned} f_1 &= \ \ 1.234h + 0.934Zd_{10} - 1.063s + 0.045g_{1.3}, \\ f_2 &= -2.011h + 0.079Zd_{10} + 2.264s + 4.026g_{1.3}, \\ f_3 &= \ \ 1.303h - 1.527Zd_{10} + 0.839s + 1.039g_{1.3} \end{aligned} \tag{4.1}$$

and in the model (5) are

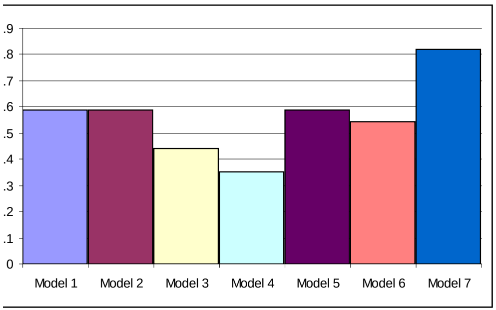
$$f_1 = \ \ 0.637d_{1.3bk} + 0.133K + 0.948h + 0.907Zd_{10} - 0.637s,$$

$$f_2 = 5.631d_{1.3bk} + 1.118K - 3.166h + 0.109Zd_{10} + 4.019s,$$

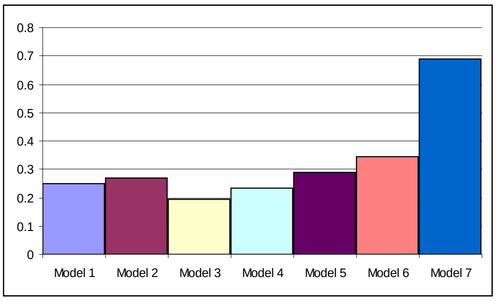
$$f_3 = -1.095d_{1.3bk} + 0.433K - 0.781h + 1.357Zd_{10} - 1.095s.$$



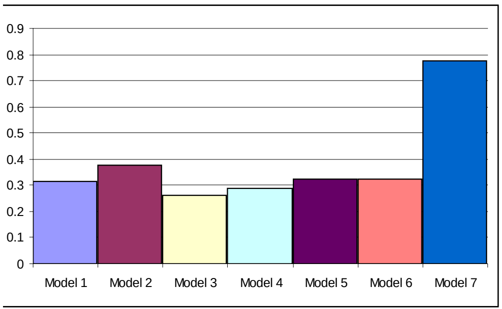
a) 33 old trees



b) 33 old trees



a) 44 old trees



b) 44 old trees

Fig. 8. A posteriori probabilities of misclassification, designated together for all Kraft classes and separately for each method, if cross validation method is used for half of the randomly selected observations; a) a priori probabilities are equal, b) a priori probabilities are proportional

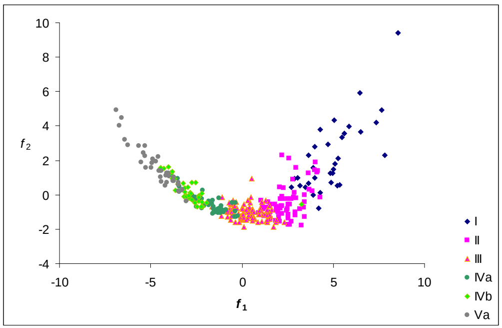
In the next step, validation of proposed models, the data from other stands were tested using seven received models. The data from 33 old trees and 44 old trees were used. In the result of calculation the probabilities of misclassification were designated and they are presented in the Figure 8. These probabilities in the case of the first group of tree were usually bigger than 50%, only in the case of models (3) and (4) they were slightly smaller. More promising results were obtained in the case of the second group of trees (44 old). In both these groups of data models (3) and (4) seemed to be the most useful. The discriminant variables in the model (4) were of the form (the first function explains 85.70% variability, the second 3.51% and third 0.70%)

$$f_1 = \quad 1.234h + 0.921zd_5 - 0.975s + 0.186g_{13},$$

$$f_2 = -2.037h + 0.124zd_5 + 2.283s + 4.001g_{13}, \qquad (4.2)$$

$$f_3 = -1.193h - 1.629zd_5 - 0.486s - 0.904g_{13}$$

and the transformed observations of analyzed trees in the system of the first two discriminant variables as well as in the system of the first and the third discriminant variable are presented in the Figure 9. Similarly as on the basis of Figure 7, in this case transformed data can be separated linearly into two groups: suppressed stand (class IVa, IVb, Va) and dominant stand (I, II, III), as well.



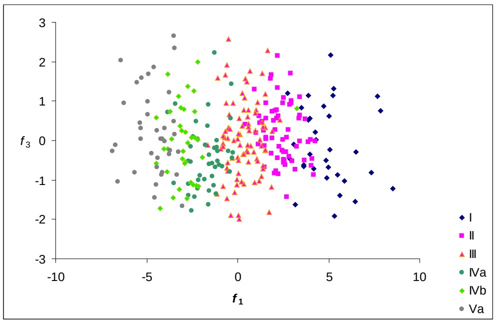


Fig. 9. The transformed observations of analyzed trees in the system of the first two discriminant variables (left) and in the system of the first and the third discriminant variable (right) in the model (4)

5. Conclusions and discussion

In the paper written by Thessler et al. (2008) among others the discriminant analysis were used to distinguish floristically different forest types. Authors showed that the non-parametric k nearest neighbors (k-nn) classifier can distinguish classes better than the linear discriminant analysis. In the paper written by Jing et al. (2015), a Fisher discriminant analysis method was put forward based on multivariate statistical analysis to assess the status of forest fire risk points.

In the paper by Grala and Kaźmierczak (2011) the problem of allocation of trees to Ktaft classes was considered. The discrimination analysis were used to indicate the discriminant power of traits which can be measured on the felled trees.

In this study also the discrimination method was used to divide on trees into Ktaft classes but basing on characteristics measured on standing trees. The models (3), (4) and (5) occur among the best to determine the condition of the trees. Most commonly the (3) model is considered the best if the cross validation method is used to evaluate of the misclassification probability, as well as if calculated model is used for others data sets. Thus the traits: height, ten (or five) year increments at the breast height, slenderness and breast height diameter without bark can be designated on the stand trees. Using these four trait and equations (4.1) or (4.2) the Kraft classes can be determined for trees aged between 35-45 years. Moreover, on the basis of Figures 7 and 9 it can be concluded that tress easily may be divided into two following classes: suppressed stands and dominant stands. In the first (suppressed) group there are the trees from Kraft classes V, IVb and IVa. The trees from second group (I, II and III Kraft classes) can be separated linearly. Thought Kraft Classes I and II in the left diagrams are quite mixed. Unfortunately indication of the group explicitly separated from the rest is almost impossible. In the next research the neighbour index will be designated and included to models.

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