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Simulation model of wind turbine mechanical part

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Abstract: *Simulation model of wind turbine mechanical part.* In this paper a simulation model of wind turbine mechanical part has been proposed. The model has been realized in the LTSpice software. This software is was developed primarily for electronic circuit simulation. Change of equation system from mechanical to electrical domain has applied. Hence, arisen electrical circuit have been simulated. Results has shown that simulation of mechanical system in electronic simulation software is possible. Step response curves of wind turbine for a change of load torque has been presented. Moreover, an influence of blade pitch angle for an output mechanical power of turbine has been evaluated.

Key words: wind turbine, simulation, LTSpice, step response, coefficient of power, blade tip speed ratio

INTRODUCTION

In the last years a certain growth in the using of the green power was observed. In the Europe it was inspired by European Union policy: to 2020 renewable energy should be at least 20% overall energy mix [Directive 2009/28/EC]. Moreover, emission of CO_2 has to be limited, with a benefit for the natural environment. The most popular type of green energy used for the production of electrical current is wind energy, because of a high level of development. In 2015 there was a 23.4% increment of electrical energy production from wind farms in Poland [Leśniewski 2015]. If the real level of wind turbines utilization has to be increased, it will be preceded the verification of impact of these systems at electrical energy infrastructure. The cost of wind turbines or wind farms is very high, about 1,400 USD per 1 kW of installed power [Renewable Energy Technologies 2012]. Therefore, a computer simulation is the only way for research purposes. In this paper the LTSpice computer program has been applied. The main application of the Spice-class software is simulation of electronic circuits. If one can change an algebraic and differential equation system of the given dynamic system for an equivalent electrical circuit, simulation of mechanical system will be pursued.

MATHEMATICAL MODEL OF WIND TURBINE

The wind turbine is driven by a wind. Power of the wind (P_w) is given by formula [Akhmatov et al. 2013]:

$$P_w = \frac{1}{2} \rho A v^3 \tag{1}$$

$$A = \pi R^2 \tag{2}$$

where:
ρ – air density [kg·m^{-3}];
A – turbine area that is swept by turbine blade [m^2];
v – wind speed [m·s^{-1}];
R – turbine radius [m].

Typical value of air density (normal conditions) is 1.225 kg·m^{-3}.

Mechanical power (P_m) of wind turbine is given as [Pak et al. 2009]:

$$P_m = \frac{1}{2} C_p(\lambda, \beta) \rho A v^3 = C_p(\lambda, \beta) P_w \tag{3}$$

Mechanical power of turbine (P_m) is only some part of power of wind (P_w). This part is characterized by dimensionless and normalized coefficient of power (C_p). It is a function of blade tip speed ratio (λ) and blade pitch angle (β). The first one is given as [Gala Santos et al. 2014]:

$$\lambda = \frac{\omega R}{v} \tag{4}$$

where:
ω – angular velocity of turbine [rad·s^{-1}].

Blade pitch angle β [°] is angular deviation of blade chord above (or under) plane swept by blades.

Torque (T) produced by turbine is given as [Jelavić et al. 2007]:

$$T = \frac{P_m}{\omega} \tag{5}$$

From the numerical simulation point of view at the first step of calculations ω = 0 because of rotor is stopped. Hence, there is division by zero, and T is going to infinity. Formula (5) has to be transformed to (ω is calculated from equation (4)):

$$T = \frac{P_m}{\omega} = P_m \frac{R}{\lambda v} = \frac{1}{2} \frac{C_p}{\lambda} \rho \pi R^3 v^2 \tag{6}$$

Although for $\omega = 0$ also $\lambda = 0$, for a few beginning steps of simulation (<0.0001 s) λ has by assumption a very small value. This way one can avoid division-by-zero error.

Input parameter of the presented model is wind speed (v), and output parameters are torque (T) and angular velocity (ω). For the calculations necessary is knowledge of function C_p (λ, β). This function is unique for each turbine and often is given by manufacturer as a table of values [Manyonge 2012]. For modeling purposes, some empirical function have been developed. One of them, proposed in Heier [1998], Fujii et al. [2008], Samanvorakij et al. [2013], is:

$$C_p(\lambda, \gamma, \beta) = C_1 \left(\frac{C_2}{\lambda_i} - C_3 \beta - C_4 \right) e^{\frac{-C_s}{\lambda_i}} + C_6 \lambda \tag{7}$$

$$\lambda_i(\lambda, \beta) = \frac{1}{\dfrac{1}{\lambda + 0.08\beta} - \dfrac{0.035}{\beta^3 + 1}} \tag{8}$$

where constants are: $C_1 = 0.5$, $C_2 = 116$, $C_3 = 0.4$, $C_5 = 21$, $C_6 = 0.00068$.

Another formula for objective function was given by [Anderson et al. 1983]:

$$C_p = (\lambda - 0.022\beta^2 - 5.6) e^{-0.17\lambda} \tag{9}$$

In this paper formula (7) have been used. On Figure 1 this function is illustrated. When blade pitch angle is going up, values of C_p are bigger. Each of the curves has only one maximum for different values of blade tip speed ratio.

In the real wind turbine, torque is developed by rotor with blades drives an electric generator [Hansen et al. 2001]. However in this paper, modeling of gen-

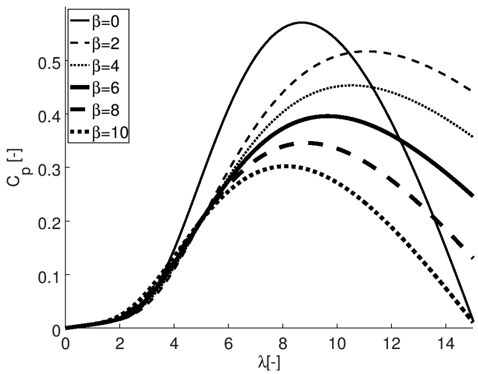

FIGURE 1. Characteristics of C_p (λ, β), blade pitch angle (β) [°]

erator is beyond of the scope. The turbine rotor drives a load, which is characterized from a mechanical point of view as the load torque (T_l). Mathematical description of this system is based on the one-mass lumped model [Muyeen et al. 2009]:

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = T - T_l \tag{10}$$

where:

J – mass moment of inertia [kg·m^2].

Because of electric nature of the LTSpice, above equation has to be changed to electric counterpart. Differential equation for an electric inductor is [Bolkowski 1986]:

$$L\frac{\mathrm{d}i}{\mathrm{d}t} = u \tag{11}$$

where:
L – inductance [H];
i – momentary value of electric current [A];
u – momentary value of voltage [V].

If a circuit has more than one voltage source, the right part of equation (11) will be a sum of source voltages (by the second law of Kirchoff). The equivalent equation for (10), but in the electric domain is:

$$L\frac{\mathrm{d}i}{\mathrm{d}t} = u - u_l \tag{12}$$

where (from the mathematical point of view, despite different phenomena nature):
L – inductance [H] = J – equivalent of mass moment of inertia [kg·m^2];
i – current [A] = ω – angular speed [rad·s^{-1}];
u – voltage [V] = T – torque [Nm].

Circuit diagram, which is realization of formula (12) is presented on Figure 2.

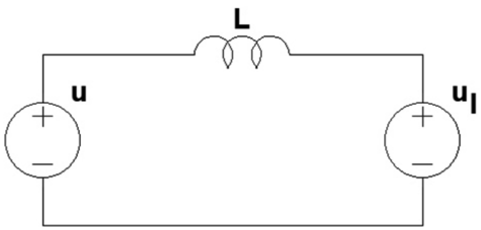

FIGURE 2. Electrical circuit as realization of formula (12)

The circuit (Fig. 2) has to be simulated easily in Spice-class software for electric circuit. Angular speed (current) is calculated by numerical integration of equation (12).

SIMULATION MODEL OF WIND TURBINE

Model of wind turbine which has been simulated in LTSpice is presented on the Figure 3. Voltage sources and elements are presented on Figure 3 realize mathematical formulas and constant param-

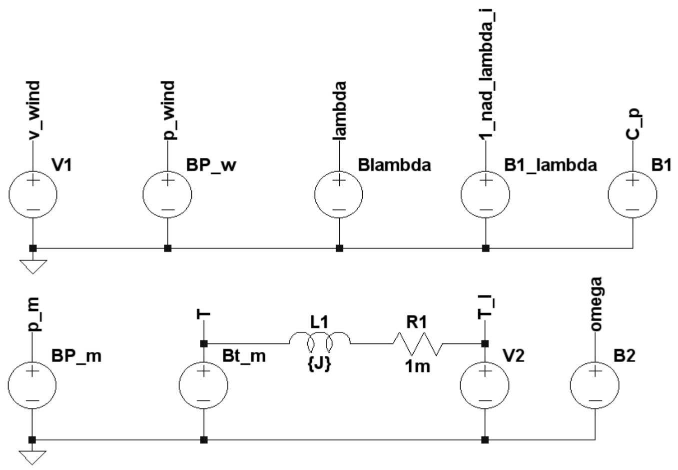

FIGURE 3. Simulation model of wind turbine

eters: *V*1 – input parameter, wind speed function of time, *BP*_w – power of wind (1), *Blambda* – blade tip speed ratio – λ (4), *B1_lambda* – λ_i (8), *B*1 – C_p (9), *BP_m* – P_m (3), *Bt_m* – output parameter, *T* (5), *V*2 – load torque T_l function of time, *B*2 – output parameter, angular speed (ω). There is a new element, resistor *R*1 of very small value 1 mΩ, which is added from numerical reasons. Without *R*1 a worse stability of calculation algorithm has been observed, with a longer time of calculations. Moreover, resistance *R*1 has a mechanical counterpart, i.e. damping coefficient (*D*) [Nms] [Latek 1982]. Very small resistance indicate a very small, negligible damping of rotating wind turbine rotor shaft. Mechanical, nominal parameters of turbine are based on literature [Errami et al. 2010, Samanvorakij et al. 2013] and are presented in the Table. It is rather small, horizontal wind turbine and a rated power (P_n) is equal 8.5 kW.

TABLE. Parameters of wind turbine

Parameter	Value
P_n [kW]	8.5
R [m]	3.0
Number of blades (*n*)	3
λ [-]	8.5
T [Nm]	200
J [kg·m^2]	4
β [°]	0

RESULTS AND DISCUSSION

On Figures 4–7 some parameters of wind turbine versus time at start-up at constant $v = 6$ m·s^{-1} and $v = 12$ m·s^{-1} are presented. After initial rotor runaway, step load torque (T_l) change from 0 to 100 Nm at 2 s is applied (for 6 m·s^{-1}) and T_l change 0–200 Nm at 1.5 s ($v = 12$ m·s^{-1}). Angular speed (ω) variability of time, at the same torque step change, but at time 1.0 s is illustrated on Figure 8.

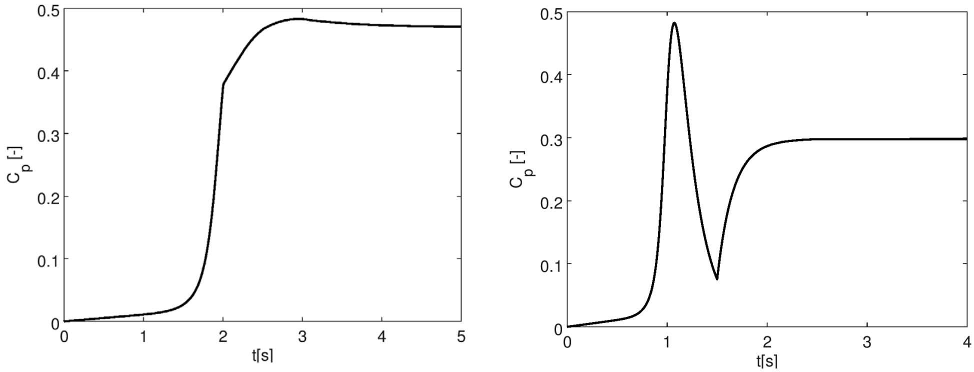

FIGURE 4. Coefficient of power (C_p) versus time (t) at v = 6 m·s^{-1} (left), at 12 m·s^{-1} (right)

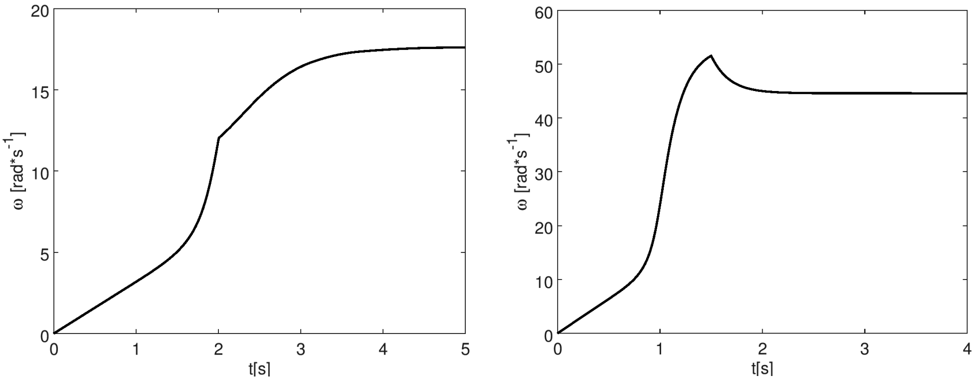

FIGURE 5. Angular speed of rotor (ω) versus time (t) at v = 6 m·s^{-1} (left), at 12 m·s^{-1} (right)

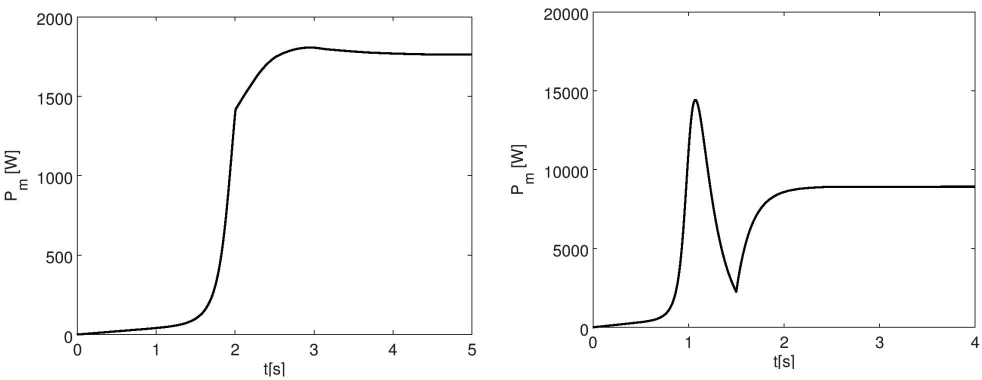

FIGURE 6. Mechanical power of rotor (P_m) versus time (t) at v = 6 m·s^{-1} (left), at 12 m·s^{-1} (right)

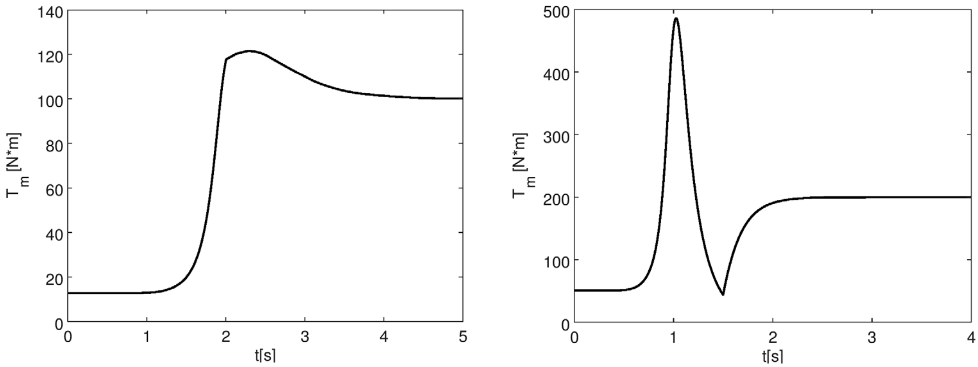

FIGURE 7. Torque of rotor (T) versus time (t) at $v = 6$ m·s^{-1} (left), at 12 m·s^{-1} (right)

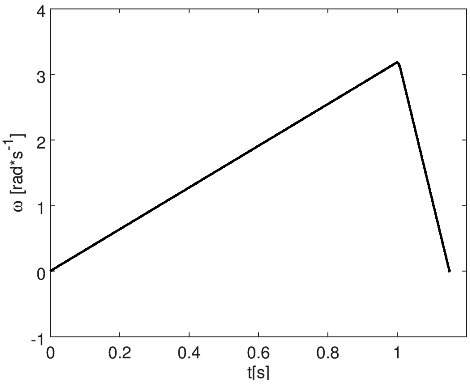

FIGURE 8. Angular speed of rotor (ω) versus time (t) at $v = 6$ m·s^{-1}, step load change at $t = 1.0$ s

Initial torque of turbine rotor (Fig. 7) is very small. The value of torque is going up faster over time for the higher wind speed (the first 1.0–1.5 s on Fig. 7). If the load torque is applied to fast, angular velocity of turbine rotor is going down and rotor stall (Fig. 8).

The wind power (P_w) at 6 m·s^{-1} is equal 3.7 kW, at 12 m·s^{-1} – 30 kW. It result from Figure 4 that C_p is equal (in the final state) about 0.47 and 0.29, respectively. Hence, only about a half or one-third of wind power is extracted by the turbine rotor: 1.7 or 8.8 kW, depending on wind speed. Very close values of mechanical power of turbine are seen on Figure 6 at final state. Time of the start-up and angular speed of rotor depend on the wind speed (Fig. 5). The specific maxima of functions of C_P and P_m at wind speed 12 m·s^{-1} (for $t = 1$ s, Figs 4 and 7) are not the result of division-by-zero effect avoiding or step load torque change. Explanation of this fact demands further research.

Change of blade pitch angle β (0–12°) allows to control the mechanical power of rotor (P_m) in range 6–10 kW (Fig. 9). The maximum value of the power occurs for $\beta = 4°$, because of relatively high value of $\lambda \approx 12$ (Fig. 1).

Moreover, the maximum of curve of turbine rotor torque, is moved to the left, toward beginning of time (Fig. 9). Hence initial torque will be going up, if pitch angle is going up, to 12°.

CONCLUSIONS

In this paper the simulation model of the wind turbine mechanical part was presented. The model included the rotor with blades, shaft and mechanical load. Results showed, that mechanical system of wind turbine has to be simulated in a Spice-class electronic software rela-

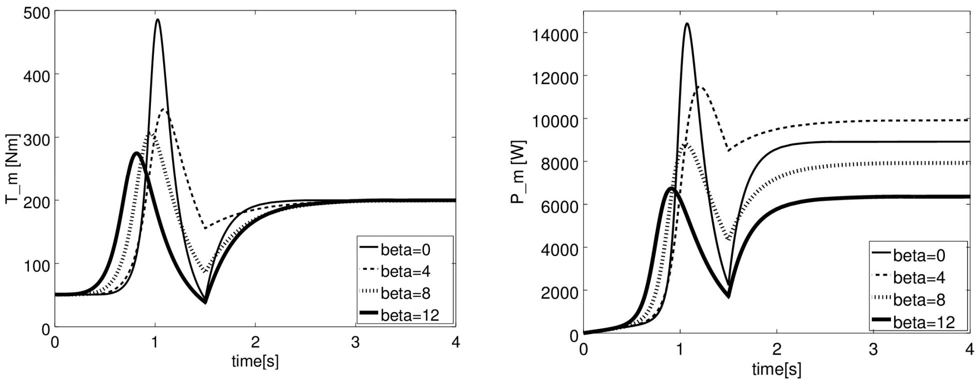

FIGURE 9. Torque (T) (left) and power (P_m) (right) of rotor versus time (t) at v = 12 m·s^{-1} and blade pitch angle (β) [°]. Step load change at t = 1.5 s

tively easy. Change of equation system from mechanical to electrical domain is required only. The mechanical load has to be applied to the shaft after rotor startup, due a small initial torque of horizontal wind turbine with three blades. In the real wind turbine is has to be realized on the side of electrical generator, by the control of an electric load. Because of a rated power o generator, mechanical power of turbine rotor will be reduced, if a wind speed archives relatively high values. The way of reduction is change a blade pitch angle toward higher values.

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Streszczenie: *Model symulacyjny części mechanicznej turbiny wiatrowej.* W artykule opisano model symulacyjny części mechanicznej turbiny wiatrowej. Model został zrealizowany w oprogramowaniu LTSpice, przeznaczonym do symulacji obwodów elektronicznych. Układ równań opisujący model został przekształcony na równoważny obwód elektryczny. Uzyskane rezultaty potwierdziły możliwość symulacji układów mechanicznych w oprogramowaniu przeznaczonym dla obwodów elektrycznych. Wyznaczono charakterystyki odpowiedzi na skokową zmianę momentu obciążenia oraz oszacowano wpływ kąta nachylenia łopat na moc turbiny wiatrowej.

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