

**WEAKLY NONLINEAR BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL SYSTEMS WITH IMPULSE ACTION**

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**Summary.** Coefficient obtained sufficient conditions for the existence and iterative algorithm of solutions of weakly nonlinear boundary value problems for ordinary differential equations with impulse action in general, when the number of boundary conditions does not coincide with the order of the differential system.

**Key words:** linear boundary value problem, Green’s matrix, weakly nonlinear boundary value problem, vector-function, vector-functional, pseudouniverse, Cauchy problem.

**RESULTS AND DISCUSSIONS**

**1. Linear boundary value problem.** Before we begin our exploration, we should find the criterion of existence and structure of general solution of linear inhomogeneous boundary value problem for systems of ordinary differential boundary value equations with pulse action in fixed times:

$$\dot{z} = A(t)z + \varphi(t), \quad t \neq \tau_i, \quad \Delta z|_{t=\tau_i} = S_i z(\tau_i - 0) + a_i, \tag{1}$$

$$lz = \alpha, \quad \tau_i, t \in [a, b] \tag{2}$$

Following next guessworks and denotations from [1], [2]:  $A(t)$  and  $\varphi(t)$  –  $(n \times n)$ -array and  $(n \times 1)$ -vector functions, the components of which belongs  $C([a, b] \setminus \{\tau_i\}_I)$  space of succession and piecewise continuous on the  $[a, b] / \{\tau_i\}_I$  function with breaks first order on the  $t$ , when  $t = \tau_i$ ;  $S_i$  –  $(n \times n)$ - constant matrix, such that  $[E + S_i]$  - nonsingular,  $a_i$  –  $n$ -vector-column of constants:

$a_i \in \mathbb{R}^n$ ;  $-\infty < \alpha < \tau_1 < \dots < \tau_i < \dots < \tau_p < b < +\infty$ ,  $i = 1, \dots, p$ ;  $l = col(l_1, \dots, l_m)$  – bounded linear  $m$ -vector-functional;  $\alpha = col(\alpha_1, \dots, \alpha_m) \in \mathbb{R}^m$ . Denote by  $X(t)$  normal ( $X(a) = E$ ) fundamental matrix, which corresponds to the (1) homogeneous ( $\varphi(t) = 0$ ,  $a_i = 0$ ) system with impulse action, and in the

capacity of Green’s matrix  $K(t, \tau)$  Cauchy problem for this system we take the next:

$$K(t, \tau) = \begin{cases} X(t)X^{-1}(\tau), & \text{if } 0 \leq \tau \leq t \leq T, \\ 0, & \text{if } 0 \leq \tau < t \leq T. \end{cases} \tag{3}$$

Let  $Q = LX(\cdot)$  -  $(m \times n)$ -array, obtained by substituting the boundary conditions  $X(t)$ ,  $Q^+$  - single pseudouniverse of it  $(n \times m)$ -matrix. Denote by  $P_Q(n \times m)$ -matrix (orthoprojector  $P_Q^2 = P_Q = P_Q^*$ ), projecting  $\mathbb{R}^n$  on the null-space  $N(Q)$  of matrix  $Q$ :  $P_Q: \mathbb{R}^n \rightarrow N(Q)$ ; similarly  $P_{Q^*} - (m \times m)$ -matrix:  $\mathbb{R}^m \rightarrow N(Q^*)$ . Next, denote by  $P_{Q_r}(n \times r)$ -matrix, columns of which  $r$ -linearly independent columns of matrix  $P_Q(r = n - n_1, n_1 = rank Q)$ ;  $P_{Q_d^*} - (d \times m)$ -matrix, rows of which -  $d$  linearly independent rows of matrix  $P_{Q^*}(d = m - n_1)$ ;  $X_r(t) = X(t)P_{Q_r}$ .

The next theorem is value.

**Theorem 1.** If linear inhomogeneous boundary value problem (1), (2) with pulse action is satisfies the conditions indicated above and  $rank Q = n_1 \leq \min(n, m)$ . Then, similiary (1), (2) homogeneous ( $\varphi(t) = 0$ ,  $a_i = 0, \alpha = 0$ ) boundary value problem has  $r = n - n_1$  and only  $r$  independent decisions. Inhomogeneous boundary value problem (1), (2) solvable for those and only those  $\varphi(t) \in C([a, b] \setminus \{\tau_i\}_I)$ ,  $a_i \in \mathbb{R}^n, \alpha \in \mathbb{R}^m$ , which satisfies the condition

$$P_{Q_d^*} \left\{ \alpha - l \int_a^b K(\cdot, \tau) \varphi(\tau) d\tau - l \sum_{i=1}^p k(\cdot, \tau_i + 0) a_i \right\} = 0 \tag{4}$$

and has  $r$ -parametric family of solutions  $z_0(t, c_r)$  on the  $C^{-1}([a, b] \setminus \{\tau_i\}_I)$  space of piecewise continuously differentiable vector-functions, which has discontinuities of the first kind on the  $t$ , when  $t = \tau_i$

$$z_0(t, c_r) = X_r(t)c_r + \left( G \begin{bmatrix} \varphi \\ a_i \end{bmatrix} \right) (t) + X(t)Q^+ \alpha, \tag{5}$$

Where  $(G \begin{bmatrix} * \\ * \end{bmatrix})(t)$  – generalized Green’s function of boundary value problem (1), (2), which has the form

$$[K \begin{bmatrix} \varphi \\ a_i \end{bmatrix}](t) \stackrel{\text{def}}{=} (\int_a^b K(t, \tau) * d\tau - X(t)Q^+l \int_a^b K(\cdot, \tau) * d\tau, [K \sum_{i=1}^p K(t, \tau_{i+0}) * - K \sum_{i=1}^p K(\cdot, \tau_{i+0}) *] \begin{bmatrix} \varphi(\tau) \\ a_i \end{bmatrix})(t) \quad (6)$$

**2. Weakly nonlinear boundary value problem.** Consider the boundary value problem

$$\begin{aligned} \dot{z} &= A(t)z + \varphi(t) + \varepsilon Z(z, t, \varepsilon), \\ t &\neq \tau_i \in [a, b], \\ \Delta z|_{t=\tau_i} &= S_i z(\tau_i - 0) + a_i + \varepsilon I_i(z(\tau_i - 0, \varepsilon)\varepsilon), \\ lz &= \alpha + \varepsilon I(z(\cdot, \varepsilon), \varepsilon), i = 1, \dots, P. \end{aligned} \quad (7)$$

Find the conditions of existence, and an algorithm for constructing solution  $z_0(t, \varepsilon)$ :

$z(\cdot, \varepsilon) \in C^1([a, b] \setminus \{\tau_i\}_I), z(t, \cdot) \in C[0, \varepsilon_0]$  of problem (7), turns when  $\varepsilon = 0$  to the generating solution  $z_0(t, c_r)$  (5) of generation boundary value problem (1), (2) obtained from (7) when  $\varepsilon = 0$ . We assume that the conditions of theorem is following. Furthermore, nonlinear  $n$ -vector function  $Z(z, t, \varepsilon)$  is that, then

$$\begin{aligned} Z(\cdot, t, \varepsilon) &\in C^1[|z - z_0| \leq q], \\ Z(z, \cdot, \varepsilon) &\in C([a, b] \setminus \{\tau_i\}_I), Z(z, t, \cdot) \in C[0, \varepsilon_0]; \\ I_i(z(\tau_i - 0, \varepsilon), \varepsilon), I(z(\cdot, \varepsilon), \varepsilon) &- \text{nonlinear } n\text{- and } m\text{-vector functionals in the first argument } z \text{ continuously differentiable (according to Frechet), and as the vector-functions of second argument permanently on the } \varepsilon \in [0, \varepsilon_0]. \end{aligned}$$

A necessary condition for the existence of the desired solution of the boundary value problem (7) is the following in the next.

*Theorem 2.* If weakly nonlinear boundary value problem (7) with pulse action in fixed times has solution  $z(t, \varepsilon): z(\cdot, \varepsilon) \in C^1([a, b] \setminus \{\tau_i\}_I), z(t, \cdot) \in C[0, \varepsilon_0]$  turns when  $\varepsilon = 0$  to the one of solutions  $z_0 = z_0(t, c_r)$ , generating for (7) boundary value problem (1), (2) with the constant  $c_r = c_r^* \in R^r (r = n - \text{rank} Q = n - n_1)$ . Then, the vector constant  $c_r^*$  satisfies the equation

$$F(c_r) = P_{Q_d^*} \{I(z_0(\cdot, c_r), 0) - \int_a^b K(\cdot, \tau) Z(z_0(\tau, c_r), \tau, 0) d\tau$$

$-l \sum_{i=1}^l K(t, \tau_i + 0) I_i(z_0(\begin{bmatrix} \tau_i - \\ -0, c_r \end{bmatrix}), 0)\} = 0$  (8) will be called the equation for generating amplitudes boundary value problem (7) with impulse action.

Find sufficient conditions for the existence of the desired solution of the boundary value problem (7). After using in (7) the change of variables  $z(t, \varepsilon) = z_0(t, c_r^*) + x(t, \varepsilon)$ , we obtain the problem of finding solutions

$$\begin{aligned} x(t, \varepsilon): x(\cdot, \varepsilon) &\in C^1([a, b] \setminus \{\tau_i\}_I), \\ x(t, \cdot) &\in C[0, \varepsilon_0], x(t, 0) = 0 \text{ of boundary value problem} \end{aligned}$$

$$\begin{aligned} \dot{x} &= A(t)x + \varepsilon Z(z_0(t, c_r^*) + x(t, \varepsilon), t, \varepsilon) \quad t \neq \tau_i, \\ \Delta x|_{t=\tau_i} &= S_i x(\tau_i - 0) + \varepsilon I_i(z_0(\tau_i - 0, c_r^*) + x(\tau_i - 0, \varepsilon)\varepsilon), \\ lz &= \varepsilon I(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon). \end{aligned} \quad (9)$$

Taking into account conditions on the nonlinearity, we have the following expansion in the vicinity  $x = 0, \varepsilon = 0$ :

$$Z(z_0 + x, t, \varepsilon) = Z(z_0, t, 0) + A_1(t)x + R(x, t, \varepsilon),$$

$$A_1(t) = \frac{\partial Z(z, t, 0)}{\partial z}|_{z=z_0(t, c_r^*)},$$

$$R(0, t, 0) = 0, \frac{\partial R(0, t, 0)}{\partial x} = 0;$$

$$\begin{aligned} I_i(z_0(\tau_i - 0, c_r^*) + x(\tau_i - 0, \varepsilon), \varepsilon) &= \\ &= I_i(z_0(\tau_i - 0, c_r^*), 0) + \\ &+ A_{1i}x(\tau_i - 0, \varepsilon) + \\ &+ R_i(x(\tau_i - 0, \varepsilon), \varepsilon), \end{aligned}$$

$$A_{1i}(t) = \frac{\partial I(z, 0)}{\partial z}|_{z=z_0(\tau_i - 0, c_r^*)},$$

$$R(0, 0) = 0, \frac{\partial R_i(0, 0)}{\partial x} = 0;$$

$$\begin{aligned} I(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon) &= \\ &= I(z_0(\cdot, c_r^*), 0) + l_1 x(\cdot, \varepsilon) \\ &+ R_0(x(\cdot, \varepsilon), \varepsilon), \end{aligned}$$

$l_1 x(\cdot, \varepsilon)$  - the linear part of the vector functional

$$\begin{aligned} I(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon); R(0, 0) &= \\ &= 0, \frac{\partial R_0(0, 0)}{\partial x} = 0. \end{aligned}$$

Considering the non-linearity in (9) formally as heterogeneity and applying to (9) theorem 1 for solution  $x(t, \varepsilon)$  of weakly nonlinear boundary value problem (9) get the

$$x(t, \varepsilon) = X_r(t)c + x^{(1)}(t, \varepsilon). \quad (10)$$

where the unknown constant vector  $c = c(\varepsilon) \in R^r$  determined by the condition (4) of the existence of solution of weakly nonlinear boundary value problem (9)

$$P_{Q_d^*} \{I(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon) - l \int_a^b K(\cdot, \tau) Z(z_0(\tau, c_r^*) + x(\tau, \varepsilon), \tau, \varepsilon) d\tau - l \sum_{i=1}^p K(\cdot, \tau_i + 0) I_i(z_0(\tau_i - 0, c_r^*) + x(\tau_i - 0, \varepsilon), \varepsilon)\} = 0, \quad (11)$$

and the unknown vector function  $x^{(1)}(t, \varepsilon)$  via generalized Green's function  $\left(G \begin{bmatrix} * \\ * \end{bmatrix}\right)(t)$  is

$$x^{(1)}(t, \varepsilon) = \varepsilon X(t) Q^+ I(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon) + \varepsilon \left(G \begin{bmatrix} Z(z_0(\tau, c_r^*) + x(\tau, \varepsilon), \tau, \varepsilon) \\ I_i(z_0(\cdot, c_r^*) + x(\cdot, \varepsilon), \varepsilon) \end{bmatrix}\right)(t) \quad (12)$$

$$x^{(1)}(t, \varepsilon) = \varepsilon X(t) Q^+ [I(z_0(\cdot, c_r^*), 0) + l_1 (X_r(\cdot) c + x^{(1)}(\cdot, \varepsilon)) + R_0(x(\cdot, \varepsilon), \varepsilon)] + \left(G \begin{bmatrix} Z(z_0(\tau, c_r^*), \tau, 0) + A_1(\tau) (X_r(\tau) c + x^{(1)}(\tau, \varepsilon)) + R(x, \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0, c_r^*), 0) + A_{1i} (X_r(\tau_i - 0) c + x^{(1)}(\tau_i - 0, \varepsilon)) + R_i(x(\tau_i - 0), \varepsilon) \end{bmatrix}\right)(t),$$

where  $B_0 = P_{Q_d^*} \{l_1 X_r(\cdot) - l \int_a^b K(\cdot, \tau) A_1(\tau) X_r(\tau) d\tau - l \sum_{i=1}^p K(\cdot, \tau_i + 0) A_{1i} X_r(\tau_i - 0)\} -$   $(d \times r)$ -matrix ( $r = n - n_1, d = m - n_1, n_1 = \text{rank } Q$ ).

Denote by  $P_{B_0}$   $(r \times r)$ -matrix (orthoprojector), which projecting  $\mathbb{R}^r$  on the  $N(B_0)$ ,

and by  $P_{B_0^*}$   $(d \times d)$ -matrix, which projecting  $\mathbb{R}^d$  on the  $N(B_0^*)$ . Then, when

$$P_{B_0} = 0, \quad P_{B_0^*} P_{Q_d^*} = 0 \quad (14)$$

the second equation of the system operator (13) is uniquely solvable for  $c$ :

$$c = -B_0^+ P_{Q_d^*} \{l_1 x^{(1)}(\cdot, \varepsilon) + R_0(x(\cdot, \varepsilon), \varepsilon) - l \int_a^b K(\cdot, \tau) [A_1(\tau) x^{(1)}(\tau, \varepsilon) + R(x(\tau, \varepsilon), \tau, \varepsilon)] d\tau - l \sum_{i=1}^p K(\cdot, \tau_i + 0) [A_{1i} x^{(1)}(\tau_i - 0, \varepsilon) + R_i(x(\tau_i - 0), \varepsilon)]\},$$

$B_0^+$   $(-r \times d)$ -pseudouniverse.

As a result, the obtained solutions for the redundant system operator apply the method of simple iterations. Thus, the following theorem.

$$l_1 (X_r(\cdot) c_k + x_k^{(1)}(\cdot, \varepsilon)) + R_0(x_k(\cdot, \varepsilon), \varepsilon) +$$

$$+ \left(G \begin{bmatrix} Z(z_0(\tau, c_r^*), \tau, 0) + A_1(\tau) (X_r(\tau) c_k + x_k^{(1)}(\tau, \varepsilon)) + R(x_k, \tau, \varepsilon) \\ I_i(z_0(\tau_i - 0, c_r^*), 0) + A_{1i} (X_r(\tau_i - 0) c_k + x_k^{(1)}(\tau_i - 0, \varepsilon)) + R_i(x_k(\tau_i - 0), \varepsilon) \end{bmatrix}\right)$$

The boundary value problem (7) in this case has only been accessed by  $\varepsilon = 0$  to the generating  $z_0(t, c_r^*)$  (5) solution  $z(t, \cdot) \in C[0, \varepsilon_0]$  which is determined by an iterative process (15) and of formula  $z_k(t, \varepsilon) = z_0(t, c_r^*) + x_k(t, \varepsilon), k = 0, 1, 2, \dots$

Considering that the vector constant  $c_r^* \in \mathbb{R}^r$  need to satisfy the equation for generating amplitudes of the boundary value problem (7), taking into account the expansion of nonlinearities in order to find solutions of the boundary value problem (7) we obtain the following equivalent operator system:

$$x(t, \varepsilon) = X_r(t) c + x^{(1)}(t, \varepsilon), \\ B_0 c = -P_{Q_d^*} \{l_1 x^{(1)}(\cdot, \varepsilon) + R_0(x(\cdot, \varepsilon), \varepsilon) - l \sum_{i=1}^p K(\cdot, \tau_i + 0) [A_{1i} x^{(1)}(\tau_i - 0, \varepsilon) + R_i(x(\tau_i - 0), \varepsilon)]\}, \quad (13)$$

*Theorem 3.* Suppose boundary value problem (9) satisfies the above conditions, so that  $\text{rank } Q = n_1$  and generating the corresponding boundary value problem (1), (2) subject to (4) ( $d = m - n_1$ ), and only when it has  $r$ -parametrical family ( $r = n - n_1$ ) generating solutions  $z(t, c_r)$  (5). Then for every vector values  $c_r = c_r^* \in \mathbb{R}^r$ , satisfying the equation (8) for generating amplitudes, under the condition (14) the boundary value problem (9) has a unique solution  $x(t, \varepsilon): x(t, \cdot) \in C[0, \varepsilon_0]$  turn to zero when  $\varepsilon = 0$ . This solution can be determined by converging on the  $[0, \varepsilon_0]$  iterative process

$$c_k = -B_0^+ P_{Q_d^*} \{l_1 x^{(1)}(\cdot, \varepsilon) + R_0(x_k(\cdot, \varepsilon), \varepsilon) - l \int_a^b K(\cdot, \tau) [A_1(\tau) x^{(1)}(\tau, \varepsilon) + R(x(\tau, \varepsilon), \tau, \varepsilon)] d\tau - l \sum_{i=1}^p K(\cdot, \tau_i + 0) [A_{1i} x_k^{(1)}(\tau_i - 0, \varepsilon) + R_i(x_k(\tau_i - 0), \varepsilon)]\}, \quad (15)$$

$$x_{k+1}^{(1)}(t, \varepsilon) = \varepsilon X(t) Q^+ [I(z_0(\cdot, c_r^*), 0) +$$

Note that in the case of boundary value problems of the Fredholm's type  $m = n$  if  $P_{B_0} = 0$ , then  $P_{B_0^*} = 0$ , and so  $P_{B_0^*} P_{Q_d^*} = 0$  in (14) is automatically value.

## CONCLUSIONS

Built equation for generating amplitudes of weakly nonlinear boundary value problems with pulse action that determines the amplitude of the generating solutions to the desired and gives a necessary condition of its existence.

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**СЛАБОНЕЛИНЕЙНЫЕ КРАЕВЫЕ  
ЗАДАЧИ ДЛЯ СИСТЕМ  
ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ  
С ИМПУЛЬСНЫМ ВОЗДЕЙСТВИЕМ**

**Аннотация.** Коэффициенты получены с  
достаточных условий для существования и  
итерационного алгоритма решения через  
слабо нелинейные краевые задачи для  
обыкновенных дифференциальных  
уравнений с импульсным воздействием в  
общем случае, когда число граничных  
условий не совпадает с порядком  
дифференциальной системы.

**Ключевые слова:** линейный краевой  
задачи, матрица Грина, слабо нелинейные  
краевые задачи, вектор-функция, вектор-  
функциональны, задача Коши.