STEERABILITY AND STABILITY OF AUTOMOBILE NON-LINEAR MODEL

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S u m m a r i .The problem of the automobile course stability is considered. The change of the car steerability based on the analysis of non-linear bicycl model is analyzed. Generalized dependence determining "the steerability curve" is obtained. Conditions which caused the loss of stability with increasing speed while moving within the constant radius circle regarding the bifurcation set are analyzed.
K e y w o r d s : car, stability, model, slip, bifurcation set.

INTRODUCTION

The increased interest of researchers to study the dynamics of vehicles in recent years can be explained by increasing the number and growth rate of their movements, the increase in total traffic, the constant upgrading of vehicles, active introduction of automatic control elements in the various components of the car. This determines the need to improve the reliability and safety, requires an assessment of the impact of all the improvements introduced in the behavior of the car under different design solutions and traffic situations.

Studies of the dynamics of modern road transport differ considerably, not only from the classical, goes back to N. Zhukovsky [1], I.V. Rocard [2] and others, but also the methods of twenty - thirty years ago. [3] Development of computer technology allows us to consider vehicles with many degrees of freedom [4, 5]. Methods of computer algebra can write in an analytical form of the equation of motion, in terms of the vast specialist performs the conversion "by hand." The combination of these methods with the numerical analysis performed on high-speed computers to determine the movement in a variety of conditions.

At the same time, growth in the number of publications indicates a large number of intractable problems. Among them - the determination of the dependence of various dynamic properties of the system, such as stability and handling, or the nature of these properties on the loss of certain parameters of the problem; nonlabour-intensive and reliable identification of numerous regular and physical parameters of the problem and the study of the "sensitivity" of the mathematical model to a "depth modeling " in particular, the number and choice of degrees of freedom is not only necessary, but also sufficient for an adequate description of certain motions. Studies [6] showed that a reasonable reduction of the number of degrees of freedom is accompanied by significant savings in time and resources, has no significant effect on a number of important practical motion parameters. This points to the need for the most complete study of the properties of the simplest models and increase the number of degrees of freedom only when necessary.

As work on the dynamics of the most approaching the ideal choice for the complexity of the model, you can specify the work H. Troger [7 - 9], Lobas L.G., Sakhno V.P., Soltus A.P. [10 - 18]. In these works for the simplest models of plane equations of motion are written, developed a process of elimination reactions, studied the stability conditions linear motion systems, depending on the speed of movement, are the main mechanisms of instability associated with the birth of the pendulum and "serpentiform" movements, found a weak "sensitivity" of the results research to model the behavior of the driver's choice.

Based on the simulations, a numerical study of stationary motions under the influence of various stationary disturbing factors: constant cross wind, the side slope of the road, move in a circle. The first approximation built various sections in the space of parameters of the stability regions. Proved that nonequilibrium systems inherent bifurcation. Reaching the stage of bifurcation lost stability and control. Alternation of stability and instability - a common phenomenon in the evolution of any open system, and the process is irreversible. After passing the bifurcation probability of returning the system to its original state is very small.

Active car safety is largely depended on its steerability. Automobile accidents largely result from the loss of steerability [19]. The major part of accidents are caused by skidding and fixed trajectory deviation while maneuvering. Therefore, the preservation of qualities of automobile dynamic systems is of great importance for solving practical tasks. Our paper deals with investigating the automobile models with understeering and oversteering. Necessary and sufficient conditions for car steerability change are analyzed.

SETTING UP THE PROBLEM

The model with Rockar's flexible wheels has the following form [2, 20]:

$$R=\frac{V}{\omega^*}=\frac{k_1\cdot k_2\cdot l^2-m\cdot V^2\cdot(k_1\cdot a-k_2\cdot b)}{k_1\cdot k_2\cdot l\cdot\theta},\qquad(1)$$

where: V – longitudinal speed constituent of the of the automobile mass center;

ω^* – angle velocity of the automobile relative to the vertical axis;

k_1, k_2 – coefficients of slipping resistance;

l – wheelbase of the automobile;

m – mass of the automobile;

a, b – distance from the mass center to the front and rear axles of the automobile ($a+b=l$);

θ – rotational angle of steering wheels.

The form (1) is valid at small angles θ and determines the trajectory radius of the point on the longitudinal axle of the automobile (fig. 1). The speed of the point numerically coincides with the longitudinal constituent V of the mass center.

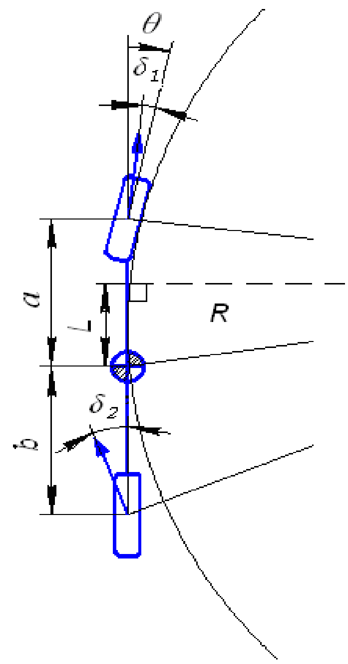


Fig. 1. Parameters determining the position of the automobile at fixed turn (Rockar's flexible wheels): the turn centre and relative position of the automobile

The position of the automobile is determined by the trajectory radius R and distance L (from indicated point to the centre of the automobile mass).

$$R=\frac{V}{\omega^*}=\frac{l}{\theta}\cdot\left(1-\frac{V^2}{V_{кр}^2}\right),$$

$$L=\left|\frac{u^*}{\omega^*}\right|=\left|\frac{b\cdot l\cdot k_2-a\cdot m\cdot V^2}{l\cdot k_2}\right|.\qquad(2)$$

Motion within the constant radius R at various velocity results from the form (1), obtained by the solution relative to θ

$$\theta=\frac{l\cdot\left(1-V^2/V_{kp}^2\right)}{R}=\frac{l}{R}+\frac{\bar{k}_2-\bar{k}_1}{\bar{k}_1\cdot\bar{k}_2}\cdot\frac{a_y}{g}.\qquad(3)$$

Followed by the equation (3), the gradient of the understeering [21] in the case of non-linear slipping hypothesis is defined by

$$K_{us}=\frac{\bar{k}_2-\bar{k}_1}{\bar{k}_1\bar{k}_2}.$$

THE INVESTIGATION OF A NONLINEAR AUTOMOBILE STEERABILITY MODEL

For a linear automobile model with flexible Rocker's wheels the steerability (1 – understeer, 2 – neutral, 3 – oversteer) is known to be determined by the relation of the dimensionless coefficients of slipping resistance for front and rear axles: 1 - $\bar{k}_2 > \bar{k}_1$; 2 - $\bar{k}_2 = \bar{k}_1$; 3 - $\bar{k}_2 < \bar{k}_1$.

At automobile's moving within radius R ith various velocity V (longitudinal velocity constituent of the mass center) Ackerman's angle should remain constant

$$l / R = \theta + \delta_2 - \delta_1. \tag{4}$$

Therefore, the turn angle of steering wheels is determined by

$$\theta = l / R + (\delta_1 - \delta_2). \tag{5}$$

The geometric approach to the finding of the second constituent in the form (5) is given below.

A number of equations determining the set of stationary regimes for a bicycle model of the automobile can be changed by the following equation [11]

$$\bar{Y}(\delta_2 - \delta_1) = \frac{V^2}{g \cdot l}(\theta + \delta_2 - \delta_1), \tag{6}$$

where: $\bar{Y} = \bar{Y}(\delta_2 - \delta_1)$ – a stationary curve determined by dimensionless dependences of slipping forces for axles (fig. 2)

$$\bar{Y} = \bar{Y}_1(\delta_1) = \bar{Y}_2(\delta_2). \tag{7}$$

Sloping angle tangent of the mobile straight line is proportional to the square of the longitudinal velocity constituent of the automobile mass center $\frac{V^2}{g \cdot l}$, turn angle of steering wheels θ determines the parallel displacement of a straight line.

Points of intersection of a straight line and a stationary curve correspond to stationary regimes of the model (ordinate of the point of intersection Y determines the specific side acceleration of the mass center s for a corresponding stationary regime, its abscissa – the difference between slipping angles on the axis $(\delta_2 - \delta_1)$.

With current value of Y the difference $(\delta_2 - \delta_1)$ is determined by the function *G(Y)*, converse to the function $Y(\delta_2 - \delta_1)$.

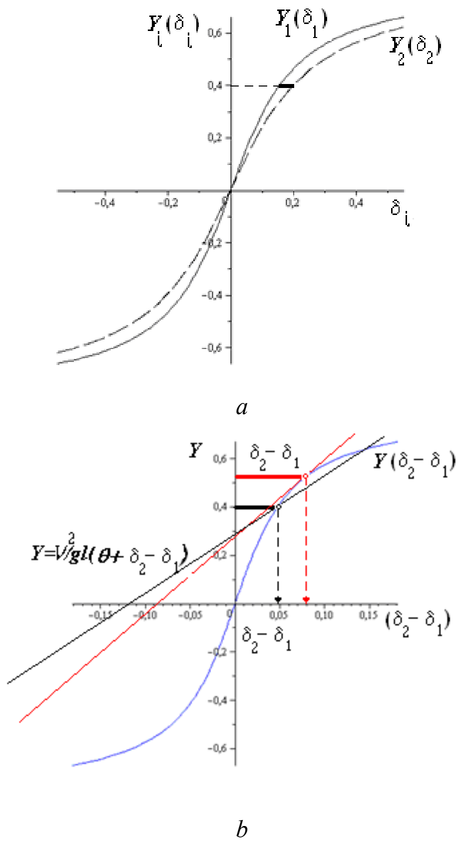


Fig. 2. Drawing a stationary curve $\bar{Y} = \bar{Y}(\delta_2 - \delta_1)$: a - diagrams of slipping forces for front and rear axles, as functions of slipping angles; b - multiple roots of the equation (6)

Initial dependences are $Y_1 = Y_1(\delta_1)$, $Y_2 = Y_2(\delta_2)$.

Having accepted these dependences for δ_i, we have $\delta_1 = F_1(Y)$, $\delta_2 = F_2(Y)$, $(\delta_2 - \delta_1) = G(Y) = F_2(Y) - F_1(Y)$.

For drawing the curve $G(Y)$ (fig. 3) diagrams of slipping angles dependences δ_1, δ_2 for front and rear axles are initial ones, as functions of slipping forces Y.

Having determined the function $G(Y)$, we have θ by the equation (5):

$$\theta = l / R + \delta_1 - \delta_2 = l / R - G(Y). \tag{8}$$

In the case of the linear slipping hypothesis ($\delta_2 - \delta_1 = (\frac{\bar{k}_2 - \bar{k}_1}{\bar{k}_1 \bar{k}_2}) \cdot Y$) we have the earlier determined equation [22]

$$\theta = l / R + (\frac{\bar{k}_2 - \bar{k}_1}{\bar{k}_1 \bar{k}_2}) a_y / g. \tag{9}$$

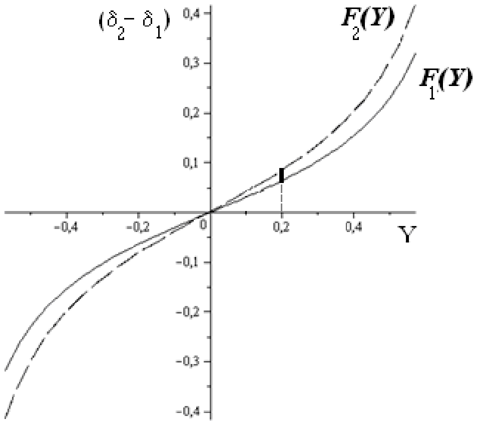
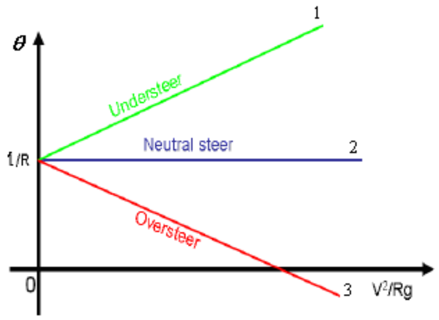


Fig. 3. The curve $G(Y)$

The change car steerability occurring sometimes in our life [22] (fig. 4) cannot be explained by the linear slipping hypothesis.



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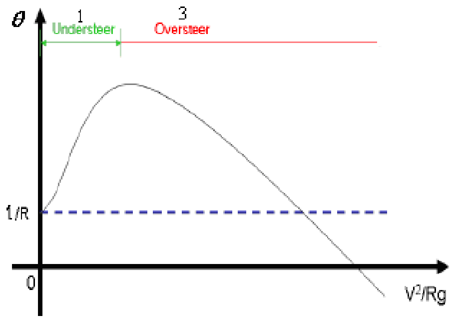


b

Fig. 4. Dependence of controllability: *a* - diagram of the steerability corresponding to the linear slipping hypothesis; *b* - virtual alteration of the automobile steerability

In our paper we try to explain such impossibility by the analysis of the non-linear bicycle model regarding nonlinearity of sideways slipping forces (monotone dependences with the function of saturation $\bar{Y}_i(\delta_i) = \bar{k}_i \delta_i (1 + \bar{k}_i^2 \delta_i^2 / \varphi_i^2)^{-1/2}$ are regarded).

Analytical conditions for change car steerability are provided for indicated dependences determining slipping forces: $\varphi_1 > \varphi_2$ is necessary and adequate (inadequate turn – excessive turn) to the model with understeering $\bar{k}_2 > \bar{k}_1$; $\varphi_1 < \varphi_2$ is necessary and adequate (excessive turn – inadequate turn) to the model with oversteering. Both cases are realized by the specific sideway acceleration value determined by

$$V^2 / Rg = \varphi_1 \varphi_2 \sqrt{\frac{(\bar{k}_1)^{2/3} - (\bar{k}_2)^{2/3}}{\varphi_2^2 (\bar{k}_1)^{2/3} - \varphi_1^2 (\bar{k}_2)^{2/3}}} . \qquad (10)$$

We are going to draw the diagram of the steerability for definite numeric values (for oversteering).

The distance between the front and rear axles is

$$l = 5 \text{ m}.$$

Dimensionless coefficients of slipping resistance:

$$\bar{k}_1 = 3{,}300;\ \bar{k}_2 = 2{,}526.$$

The critical velocity for linear motion is

$$V_{kp} = \sqrt{9{,}8 \cdot l \cdot \bar{k}_1 \cdot \bar{k}_2 \big/ (\bar{k}_1 - \bar{k}_2)} = 22{,}98 \text{ m/s}.$$

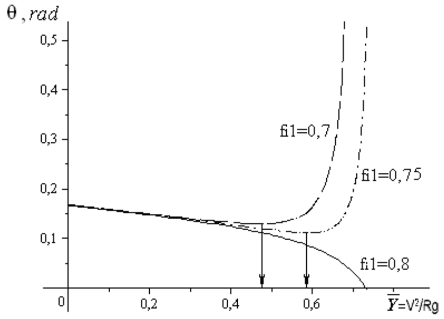
The influence of the changeable clutch coefficient variation for the front axle in the sideway direction $\varphi_1 = \{0.8, 0.75, 0.7\}$ (clutch coefficient for the rear axle in the sideway direction is $\varphi_2 = 0.8$) on the diagram of the steerability (R = 30,5 m) in the case of the non-linear slipping hypothesis is illustrated.

We enlist below the values of specific sideway acceleration causing the change of the car steerability based on the analytical form (10):

for $\varphi_1 = 0{,}75$ we have $\bar{Y} = 0{,}59$, $V = 13{,}28$ m/s;

for $\varphi_1 = 0{,}7$ - $\bar{Y} = 0{,}47$, $V = 11{,}88$ m/s.

Stationary circular motion regimes ($R = 30{,}5$ m) with increasing longitudinal velocity of the mass center is realized. The loss of stability takes place at $V = 13.03$ m/s [23] (fig. 6); unstable circularregimes correspond to the dotted lines of diagram.



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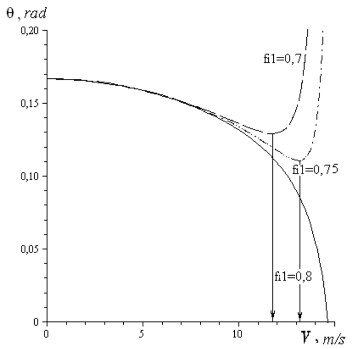

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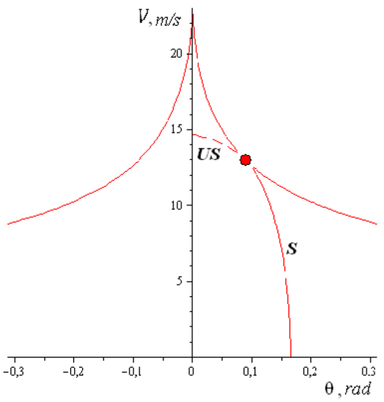
Fig. 5. Diagrams of the steerability for the non-linear slipping hypothesis with various changeable clutch coefficients in the sideway direction for the front axle: *a*- dependence $\theta=\theta(a_y/g)$; *b* - dependence $\theta=\theta(V)$



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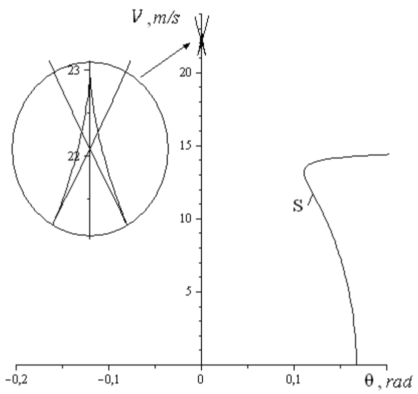

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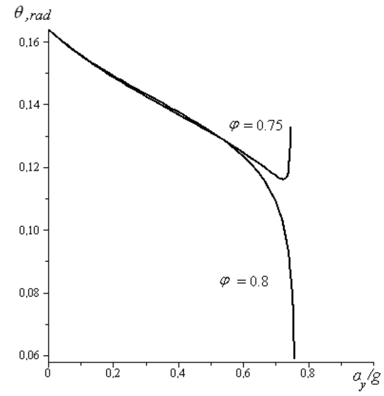
Fig. 6. Diagrams of the stability within the plain of the model controlling values: a - $\varphi_1=0.8$; *b*- $\varphi_1=0.75$

Next, we consider a dependence of the type $\overline{Y}_i(\delta_i)=\gamma_i\delta_i\cdot(1+(|\delta_i|-\beta_i)^2/\beta_i^2)^{-1/2}$, which guarantee the nonmonotonicity of slipping forces (unlike monotone dependences at considerable slipping angle the function has descending sections). Parameters γ_i and β_i are due to keeping geometrical characteristics of the monotone dependences, $\overline{Y}_i(\delta_i)=\bar{k}_i\delta_i(1+\bar{k}_i^2\delta_i^2/\varphi_i^2)^{-1/2}$, enabling the constancy of the critical velocity for rectilinear motion, coordination of maximum values of dimensionless slipping forces: $\gamma_i=\bar{k}_i\cdot\sqrt{2}$, $\beta_i=\varphi_i/(2\cdot\bar{k}_i)$.



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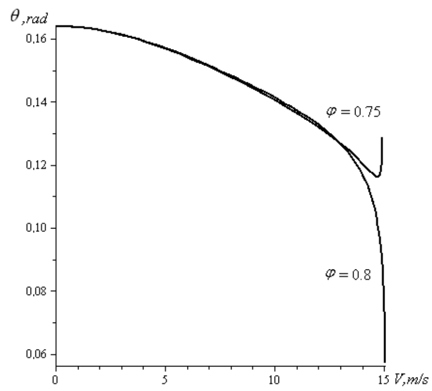

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Fig. 7. Diagrams of the steering for the non-linear slipping hypothesis with changeable clutch coefficients variation in the sideway direction for the front axle: *a* - dependence $\theta=\theta(a_y/g)$; *b* - dependence $\theta=\theta(V)$

The influence of the changeable clutch coefficient variation for the front axle in the sideway direction $\varphi_1=\{0{,}8;0{,}75\}$ (clutch coefficient for the rear axle in the sideway direction is $\varphi_2=0{,}8$) on the steering diagram (R = 30,5 m) in the case of the non-linear slipping hypothesis is illustrated.

We enlist below the values of specific sideway acceleration causing the change of the car steerability: for $\varphi_1 = 0{,}75$ we have $\overline{Y} = 0{,}72$, $V = 14{,}7$ m/s.

Stationary circular motion regimes ($R = 30{,}5$ m) with increasing longitudinal velocity of the centre of mass being realized, the loss of stability takes place at $V = 14{,}38$ m/s (fig. 8); unstable circularregimes correspond to the dotted lines of diagram.

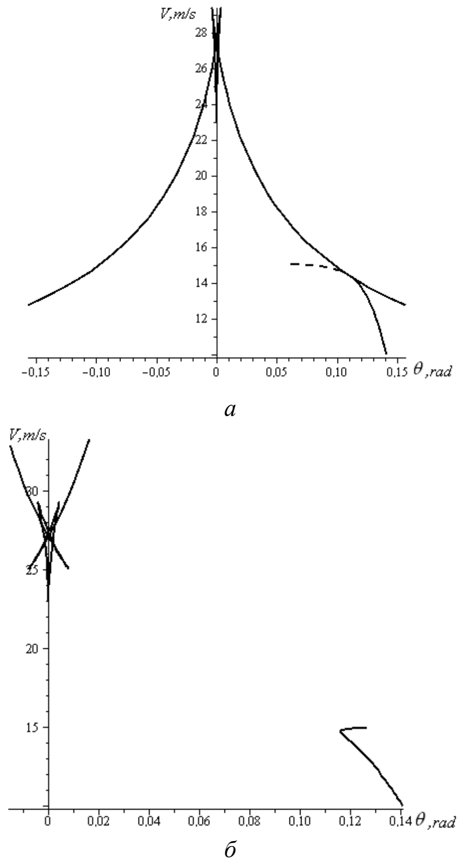

Fig. 8. Diagrams of the stability within the plain of a model of controlling values of a model: a - $\varphi_1 = 0{,}8$; *b* - $\varphi_1 = 0{,}75$

CONCLUSIONS

Analytical regimes for the change of the car model steerability of the automobile are obtained. Diagrams of the steering for the non-linear slipping hypothesis for changeable cutch coefficient variations in the sideway direction on the front axle are given.

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УПРАВЛЯЕМОСТЬ И УСТОЙЧИВОСТЬ НЕЛИНЕЙНОЙ МОДЕЛИ АВТОМОБИЛЯ

Александр Кравченко, Владимир Вербицкий, Валерий Хребет, Наталья Вельмагина

Аннотация. Рассматриваются проблемы курсовой устойчивости движения автомобиля. В работе анализируется смена свойств управляемости на основе анализа нелинейной велосипедной модели, которая учитывает нелинейность сил бокового увода. Получена обобщенная зависимость, определяющая «кривую управляемости». Проанализированы условия потери устойчивости для случая движения с возрастающей скоростью по окружности постоянного радиуса на основе построения бифуркационного множества.

Ключевые слова: автомобиль, устойчивость, модель, боковой увод, бифуркационное множество.