Oil leaks intensity in variable-height gaps

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Summary. The paper deals with issues of oil flow intensity in variable-height flat and ring gaps in hydraulic machines. On the basis of the Navier-Stokes equations and the continuum equation a formulas are given describing leaks in gaps. It is demonstrated that the volumetric loss in gaps depends on the dimensions and exploitation parameters of hydraulic machines.
Key words: variable-height gaps, hydraulic oil leaks.

INTRODUCTION

Hydraulic systems are applied in numerous branches of industry [1, 3, 13, 19]. In hydraulic machines there are oil-filled spaces between surfaces of neighbouring parts, called gaps [2, 4, 11, 14, 15, 17, 20]. The phenomena occurring in gaps are of great practical importance, since most energy losses in hydrostatic machines is associated with complex processes taking place in gaps [9]. Examining and understanding these processes makes it possible for designers and constructors of hydraulic machines to create more and more effective, reliable and durable devices [6, 7, 12, 15].

In the majority of cases, a gap is not of constant height, which may be caused by the inevitable imprecision of construction, or non-uniform wear [14]. Fig. 1 presents a classification of variable-height gaps dealt with in this paper. Wedge-shaped gaps are a typical case of flat gaps. Cone-shaped gaps, on the other hand, are typical representatives of ring gaps. Such gaps occur between the piston and cylinder block in axial piston pumps and hydraulic motors.

Among variable-height gaps one can distinguish confusor gaps (the height decreases with the flow direction) and diffuser gaps (the height increases) [16, 18].

Each gap is a source of volumetric losses, and the leak can be caused by various kinds of flow [8]:
- pressure flow (there is a pressure difference between the ends of a gap)
- friction flow (the piston or the cylinder wall is moving),
- pressure-friction flow (both phenomena occur).

The gap height is usually 1 ÷ 50 µm, but the fluid flow is not disrupted even for 0.1 µm. Despite the fact that the gap height is so small, the laws of the fluid mechanics still hold [2].

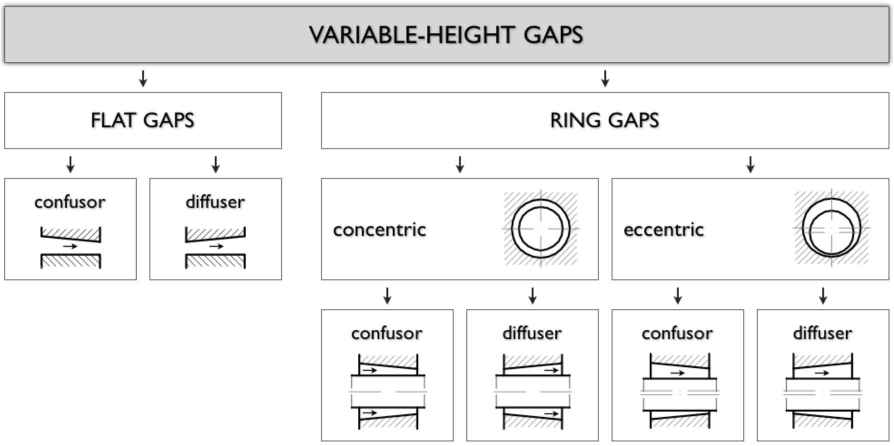

Fig. 1. Classification of variable-height gaps

APPLICATION OF THE NAVIER-STOKES EQUATIONS FOR DETERMINING VOLUMETRIC LOSSES IN LIQUID FLOWING THROUGH GAPS

Fig. 2 presents a diagram of a flat gap and a ring gap of a variable height h and the known length l. At the entrance to the gap of height h_1 the pressure is p_1 and at the exit from the gap of height h_2 the pressure is smaller and equals p_2. One of the walls of a gap can move with respect to the other with a relative velocity v. The direction of the liquid flow intensity Q is marked with an arrow.

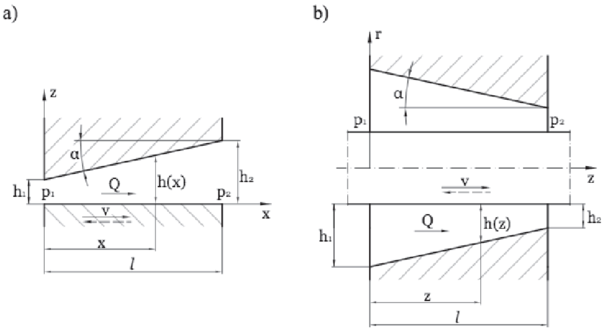

Fig. 2. Variable-height gaps: a) flat diffuser gap, b) ring confusor gap

The confusor and diffuser type gaps can be described by means of convergence m or convergence parameter k [8]:

$$m = tg\alpha = \frac{h_2 - h_1}{l}, \tag{1}$$

$$k = \frac{h_2 - h_1}{h_1}. \tag{2}$$

Pressure changes in the flat gap are accounted for by the Navier-Stokes equations and the flow continuum equation represented in the cylindrical coordinate system x, y, z [5, 10]:

$$\frac{\partial \mathrm{v}_x}{\partial t} + \mathrm{v}_x \frac{\partial \mathrm{v}_x}{\partial x} + \mathrm{v}_y \frac{\partial \mathrm{v}_x}{\partial y} + \mathrm{v}_z \frac{\partial \mathrm{v}_x}{\partial z} = \nu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2} + \frac{\partial^2 \mathrm{v}_x}{\partial y^2} + \frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{3}$$

$$\frac{\partial \mathrm{v}_y}{\partial t} + \mathrm{v}_x \frac{\partial \mathrm{v}_y}{\partial x} + \mathrm{v}_y \frac{\partial \mathrm{v}_y}{\partial y} + \mathrm{v}_z \frac{\partial \mathrm{v}_y}{\partial z} = \nu\left(\frac{\partial^2 \mathrm{v}_y}{\partial x^2} + \frac{\partial^2 \mathrm{v}_y}{\partial y^2} + \frac{\partial^2 \mathrm{v}_y}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{4}$$

$$\frac{\partial \mathrm{v}_z}{\partial t} + \mathrm{v}_x \frac{\partial \mathrm{v}_z}{\partial x} + \mathrm{v}_y \frac{\partial \mathrm{v}_z}{\partial y} + \mathrm{v}_z \frac{\partial \mathrm{v}_z}{\partial z} = \nu\left(\frac{\partial^2 \mathrm{v}_z}{\partial x^2} + \frac{\partial^2 \mathrm{v}_z}{\partial y^2} + \frac{\partial^2 \mathrm{v}_z}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial z}, \tag{5}$$

$$\frac{\partial \mathrm{v}_x}{\partial x} + \frac{\partial \mathrm{v}_y}{\partial y} + \frac{\partial \mathrm{v}_z}{\partial z} = 0. \tag{6}$$

The following assumptions were adopted in the analysis:
- the flow is laminar, uniform, steady and isothermal [9],
- inertia forces of the liquid are negligible,
- the liquid is incompressible and completely fills the gap,
- tangent stress is Newtonian,
- liquid particles directly adjacent to the moving surface retain its velocity,
- the gap surfaces are rigid,
- cavitation phenomena in gaps were not taken into consideration.

When $\mathrm{v}_x = \mathrm{v}_x(x,z)$, $\mathrm{v}_y = 0$ and $\mathrm{v}_z = 0$ is assumed, equations (3 ÷ 6) become simplified:

$$\mathrm{v}_x \frac{\partial \mathrm{v}_x}{\partial x} = \nu\left(\frac{\partial^2 \mathrm{v}_x}{\partial x^2} + \frac{\partial^2 \mathrm{v}_x}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{7}$$

$$0 = \frac{\partial p}{\partial y}, \tag{8}$$

$$0 = \frac{\partial p}{\partial z}, \tag{9}$$

$$\frac{\partial \mathrm{v}_x}{\partial x} = 0. \tag{10}$$

It follows from equations (7 ÷ 10) that $\mathrm{v}_x = \mathrm{v}_x(z)$ and $p = p(x)$. Thus, the liquid motion in the gap is described by the following differential equation:

$$0 = \nu\frac{\partial^2 \mathrm{v}_x}{\partial z^2} - \frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{11}$$

With the dynamic viscosity coefficient μ, equation (11) becomes:

$$\frac{\partial^2 \mathrm{v}_x}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}. \tag{12}$$

To obtain a formula for the liquid flow velocity in the gap, equation (12) was integrated twice and boundary conditions for a gap with a moving wall were assumed:

$$\mathrm{v}_x = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(z^2 - hz\right) - \frac{\mathrm{v}}{h}z + \mathrm{v}. \tag{13}$$

The liquid flow intensity through a gap of width b can be represented as:

$$Q = b\int_0^h \mathrm{v}_x dz. \tag{14}$$

The pressure of the liquid in the gap is:

$$p(x) = \int \frac{dp}{dx}dx. \tag{15}$$

The differential dp/dx occurring in equation (15) was obtained as:

$$\frac{dp}{dx} = \frac{6\mu \mathrm{v}}{h^2} - \frac{12\mu Q}{bh^3}, \tag{16}$$

where h is the gap height equal to:

$$h = \left(h_2 - h_1\right)\frac{x}{l} + h_1. \tag{17}$$

Substituting (16) into (15) and taking (17) into consideration:

$$p(x) = \int\left[\frac{6\mu \mathrm{v}}{\left(\left(h_2 - h_1\right)\frac{x}{l} + h_1\right)^2} - \frac{12\mu Q}{b\left(\left(h_2 - h_1\right)\frac{x}{l} + h_1\right)^3}\right]dx. \tag{18}$$

After integrating equation (18), determining the integration constant and applying transformations:

$$p(x) = \frac{6\mu x}{h_1 h}\left(\pm \mathrm{v} - \frac{Q}{b}\cdot\frac{h+h_1}{hh_1}\right) + p_1 . \tag{19}$$

Further to the operations stated above, assuming the boundary conditions as $x=l$, $p=p_2$, $h=h_2$, the leak flow intensity Q was obtained from (19):

$$Q = \frac{b\left(p_1 - p_2\right)\left(h_1 h_2\right)^2}{6\mu l\left(h_1 + h_2\right)} \pm \frac{\mathrm{v} b h_1 h_2}{h_1 + h_2} . \tag{20}$$

An analogous procedure was conducted for the ring gap and the following formula was obtained for the leak flow intensity in a cone-shaped gap [8]:

$$Q = \frac{\pi d h_1\left(p_1 - p_2\right)}{12\mu l}\left[1 + 1.5\left(\frac{e}{h_1}\right)^2 + 0.5k\left(3 + 0.5k - 0.25k^2\right) + 0.75k\left(\frac{e}{h_1}\right)^2 + \frac{1}{8}\frac{k^4}{\sqrt{\left(2+k\right)^2 - 4\left(\frac{e}{h_1}\right)^2}}\right] \pm$$
$$\pm \frac{\pi d h_1 \mathrm{v}}{4}\left(2 + k - \frac{k^2}{\sqrt{\left(2+k\right)^2 - 4\left(\frac{e}{h_1}\right)^2}}\right), \tag{21}$$

where: e is a centrifugal displacement of the piston with diameter d.

RESULTS OF SIMULATION EXPERIMENTS ON LEAK FLOW INTENSITY OF OIL IN VARIABLE-HEIGHT GAPS

Simulation experiments on leaks in typical hydraulic oil were performed for a flat diffuser gap (convergence parameter k = 0.8) and for a ring confusor gap (convergence parameter k = -0.5). It was assumed that the oil flow is of pressure-friction type. One of the gap walls is moving with respect to the other with velocity v = 1 [m/s] in the direction reverse to the oil flow in the gap. Similarly, the piston is moving in the cylinder with velocity v = 1 [m/s] in the direction reverse to the oil flow in the gap.

The following parameters were assumed in the computations for the flat gap:
- pressure at the gap entrance p1 = 25 [MPa],
- pressure at the gap exit p2 = 0 [MPa],
- the gap length l = 0,032 [m],
- the gap width b = 0,01 [m],
- dynamic viscosity coefficient μ = 0.0253 [Pas].

For the ring gap, the parameters were as follows:
- pressure at the gap entrance p1 = 25 [MPa],
- pressure at the gap exit p2 = 0 [MPa],
- the gap length l = 0,02 [m],
- the piston diameter d = 0,015 [m],
- dynamic viscosity coefficient μ = 0.0253 [Pas].

Fig. 3 presents the results obtained in a flat diffuser gap for various dimensions and exploitation parameters. It can be observed that leaks grow with decreasing values of the dynamic viscosity coefficient of oil (Fig. 3a). At the same time, as the mean gap height increases, leaks grow exponentially (Fig. 3b). When the gap wall is moving in the direction reverse to that of oil flow, increase in the gap wall velocity contributes to decreasing leaks (Fig. 3c). Besides, leaks grow with the growth of the pressure difference between the two ends of the gap (Fig. 3d).

Fig 4 presents the results obtained in a ring confusor gap for various dimensions and exploitation parameters. By changing the value of the centrifugal displacement of the piston from 0 (concentric position in the cylinder) to 16 μm (the maximal displacement of the piston in the case under consideration), its impact on volumetric losses was examined. The analysis demonstrate that as the piston displacement increases, the value of leaks in the gap increases too (Fig. 4a). The leaks are the greater, the bigger the mean gap height is (Fig. 4b). Increase in the gap length l is accompanied by decreasing leaks (Fig. 4c). Oil viscosity is a crucial factor: when viscosity increases, leaks decrease (Fig. 4d). Besides, leaks grow with the growth in the pressure difference between the two ends of the gap (Fig. 4e). When the piston is moving in the reverse direction to that of oil flow, then increase in the velocity of the piston motion causes leaks to decrease (Fig. 4f). Thus, it is evident that piston velocity influences the quantity of leaks due to the operation of molecular friction forces in oil.

CONCLUSIONS

The study leads to the following conclusions:
1. Computational models discussed in the present analysis provide adequate tools for determining the flow intensity of oil leaks in flat and ring gaps.
2. The intensity of oil leaks depends on a number of parameters, such as dimensions or exploitation factors. Their occurrence negatively affects the efficiency of hydraulic machines and devices.
3. The laminar flow occurs in hydraulic gaps, and the Reynolds number increases with the growth of leaks.

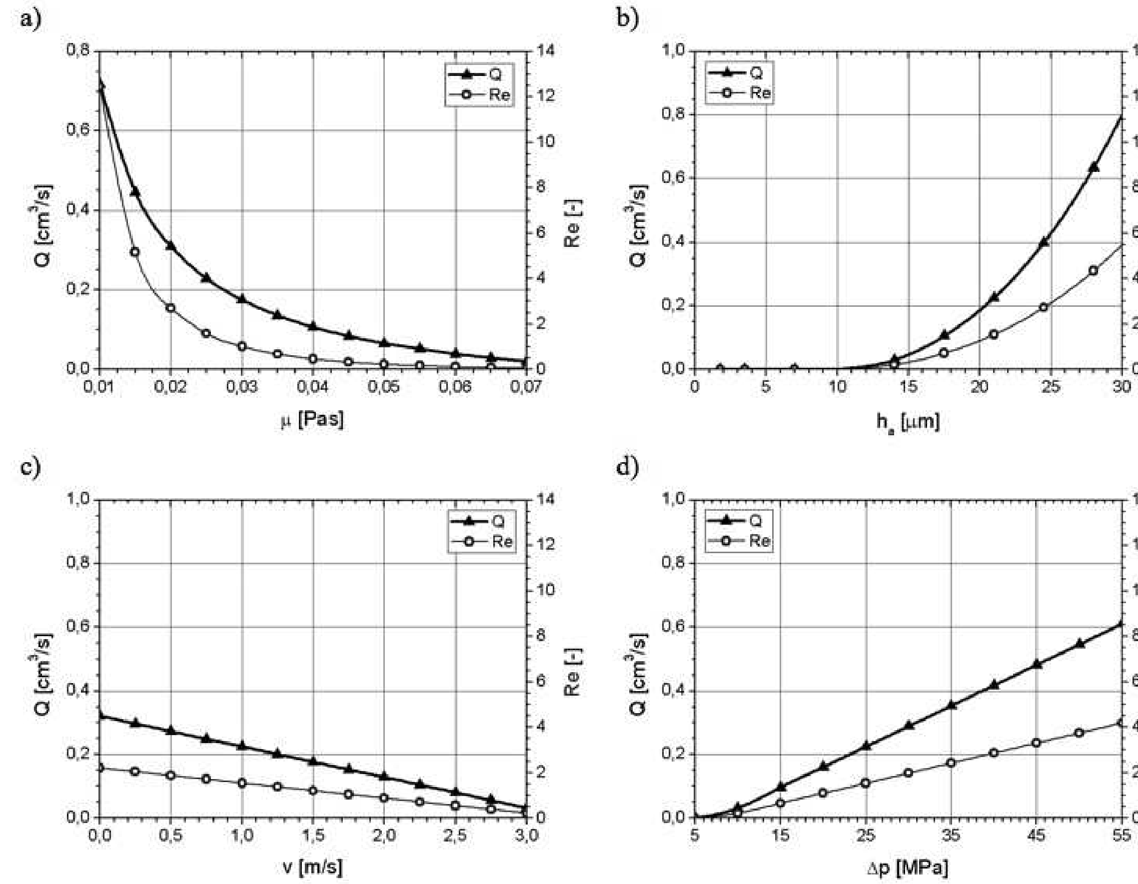

Fig. 3. Leaks and Reynolds numbers in a flat gap depending on: a) dynamic viscosity coefficient μ of oil, b) mean gap height $h_a = (h_1+h_2)/2$, c) velocity v of one of the gap walls, d) pressure drop Δp between the two ends of the gap

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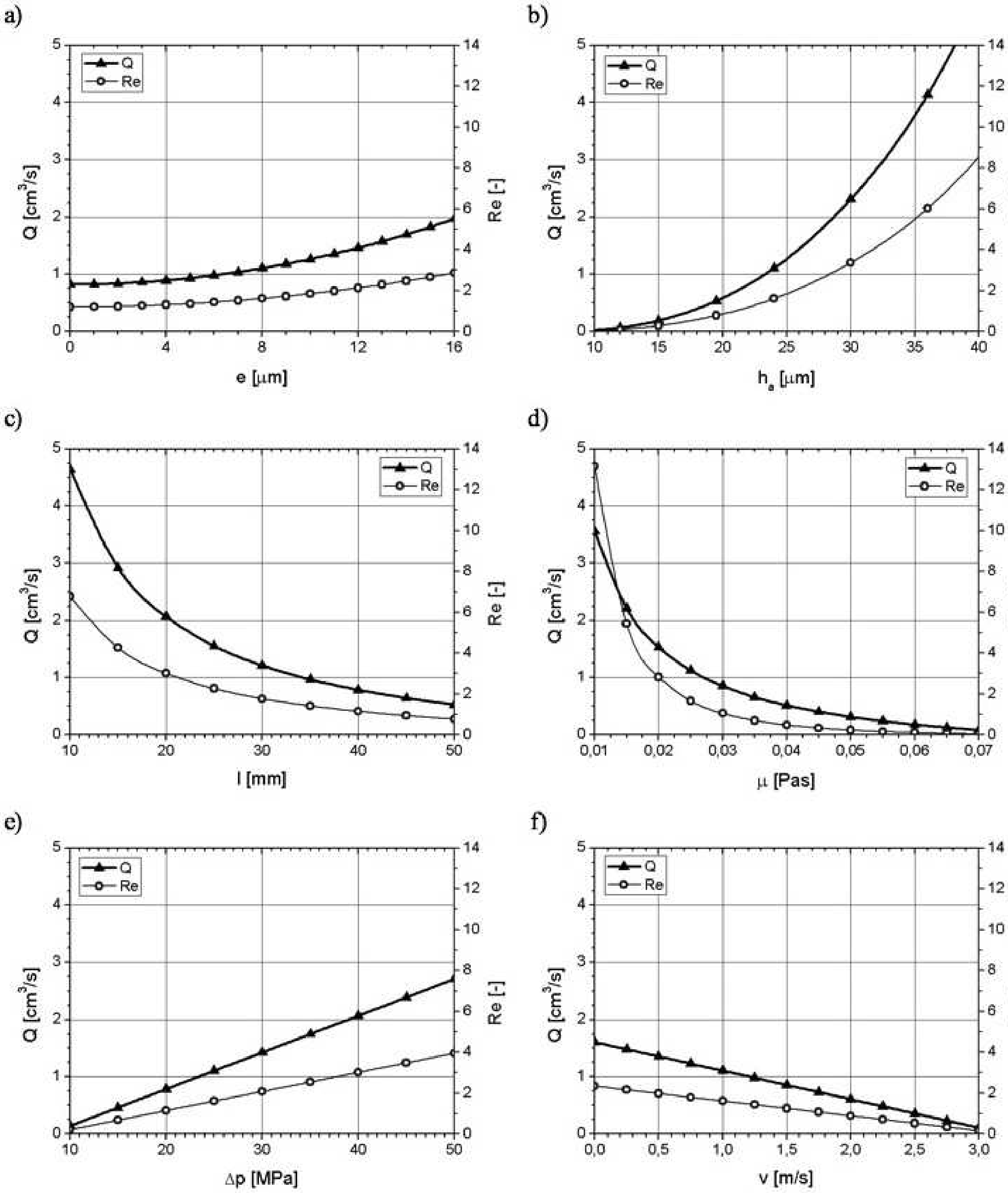

Fig. 4. Leaks and Reynolds numbers in a ring gap depending on: a) centrifugal displacement *e* of the piston, b) mean gap height $h_a = (h_1+h_2)/2$, c) gap length *l*, d) dynamic viscosity coefficient μ of oil, e) pressure drop Δp between the two ends of the gap, f) piston velocity

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NATĘŻENIE PRZEPŁYWU PRZECIEKÓW OLEJU W SZCZELINACH O ZMIENNEJ WYSOKOŚCI

Streszczenie. W artykule przedstawiono problematykę związaną z natężeniem przepływu oleju przez szczeliny płaskie i pierścieniowe o zmiennej wysokości, które występują w różnego rodzaju maszynach hydraulicznych. W oparciu o równania Naviera-Stokesa i równanie ciągłości wyznaczono zależności określające przecieki w szczelinach. Rezultaty obliczeń strat wolumetrycznych występujących w szczelinach przedstawiono w zależności od parametrów geometrycznych i eksploatacyjnych.

Słowa kluczowe: szczelina o zmiennej wysokości, przecieki oleju hydraulicznego.