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# Speeds of movement of the point of gearing along contact lines in screw gear globoid cylindrical tooth gearing

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Summary. The mathematical model of speeds of movement of points of gearing along contact lines in a screw gear globoid cylindrical tooth gearing is stated in this article. Mathematical dependences for absolute and relative speeds of movement of points of gearing on contact lines on heads and legs of teeth of the leader and conducted wheels are received. Mathematical dependences for corners between vectors of speeds and the main coordinate lines are also received in the article.
Key words. Tooth gearing, globoid cylindrical, screw.

## INTRODUCTION

In drives of modern cars the screw tooth gearings, allowing to create a rational design of transmission gears find application, to improve smoothness of their work and to lower noise and dynamic characteristics.

Increase of load ability of tooth gearings, including screw, is an actual task.

## RESERCH ANALYSIS

Screw tooth gearings with evolvent gearing [2, 4, 5, 9, 13,] have dot contact and the increased slidings in the gearing, limiting application of these transfers. In work [22] it is shown that localization of a spot of contact also have dot contact and the increased sliding. In works [8, 14, 19, 22] it is specified that it is possible to increase load ability of screw tooth gearings synthesis by their screw gearing on the method developed by M.L. Novikov [11] and added in works [12, 15, 16].

**The purpose of article** is increase of load ability of screw tooth gearings by synthesis of screw globoid cylindrical transfer with screw gearing. In this regard in article the task of development of mathematical model of kinematics of a globoid cylindrical tooth gearing with Novikov's gearing is solved.

## RESULTS OF RESEARCH

We will write down the equations of surfaces of teeth on wheels of globoid cylindrical transfer with Novikov's gearing at which the working part of the main tool is outlined by arches of circles (an initial contour of GOST 15023-69). The equations of surfaces of teeth of leading (globoid) and conducted (cylindrical) wheels it is representable in systems of coordinates $O_1X_{11}Y_{11}Z_{11}$ and $O_2X_{22}Y_{22}Z_{22}$ respectively (Fig. 1).

Equations of surfaces of a head of tooth of a leading wheel:

$$x_{11} = (r_1 + R_1\cos\lambda_{11})\sin(\varphi_{11} + \varphi_1) + R_1 \times$$
$$\times\cos(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1,$$
$$y_{11} = (r_1 + R_1\cos\lambda_{11})\cos(\varphi_{11} + \varphi_1) - R_1 \times \quad (1)$$
$$\times\cos(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1,$$
$$z_{11} = p\varphi_1 ctg\gamma - R_1\sin\lambda_{11}\sin\gamma,$$

where: $r_1$ radius of an initial circle of a globoid.

$$r_1 = r_{10}\left(1 + u - u\cos\varphi_2^*\right).$$

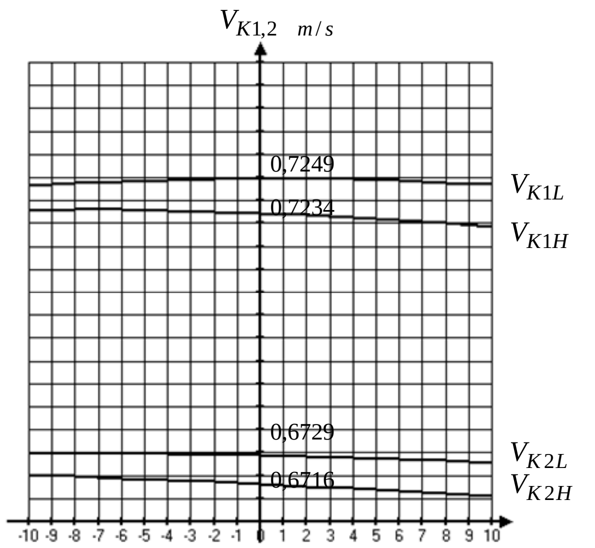


**Fig. 1.** Absolute speed of movement of a point of gearing along contact lines of a leading (globoid) wheel on heads $V_{K1H}$ and on the legs $V_{K1L}$, a conducted (cylindrical) wheel on heads $V_{K2H}$ and on legs $V_{K2L}$

In the same system of coordinates, the equations of a surface of a leg of tooth of a leading wheel has an appearance:

$$x_{12} = (r_1 + R_2\cos\lambda_{12})\sin(\varphi_{11} + \varphi_1 - \xi_1) +$$
$$+ R_2\cos(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1,$$
$$y_{12} = (r_1 + R_2\cos\lambda_{12})\cos(\varphi_{11} + \varphi_1 - \xi_1) - \quad (2)$$
$$- R_2 cos(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1,$$
$$z_{12} = p(\varphi_1 ctg\gamma - \xi_1) - R_2\sin\lambda_{12}\sin\gamma.$$

The equations of a surface of a head of tooth of a conducted (cylindrical) wheel in system $O_2X_{22}Y_{22}Z_{22}$ it is representable in the following look:

$$x_{21} = (r_2 + R_1\cos\lambda_{21})\sin(\varphi_{22} + \varphi_2) + R_1 \times$$
$$\times\cos(\varphi_{22} + \varphi_2)\sin\lambda_{21}\cos\beta_2,$$
$$y_{21} = (r_2 + R_1\cos\lambda_{21})\cos(\varphi_{22} + \varphi_2) - R_1 \times \quad (3)$$
$$\times\cos(\varphi_{22} + \varphi_2)\sin\lambda_{21}\cos\beta_2,$$
$$z_{21} = p\varphi_1 tg\gamma - R_1\sin\lambda_{21}\sin\gamma,$$

In the same system the equation of surfaces of legs of teeth of a conducted wheel will have an appearance:

$$x_{22} = (r_2 + R_2\cos\lambda_{22})\sin(\varphi_{22} + \varphi_2 - \xi_2) +$$
$$+ R_2\cos(\varphi_{22} + \varphi_2 - \xi_2)\sin\lambda_{22}\cos\beta_2,$$
$$y_{22} = (r_2 + R_2\cos\lambda_{22})\cos(\varphi_{22} + \varphi_2 - \xi_2) - \quad (4)$$
$$- R_2\cos(\varphi_{22} + \varphi_2 - \xi_2)\sin\lambda_{22}\cos\beta_2,$$
$$z_{22} = p(\varphi_1 tg\gamma - \xi_2) - R_2\sin\lambda_{22}\sin\gamma.$$

In the equations (1) … (4):

– $\varphi_1$ and $\varphi_2$ – corners of rotation of the leader and conducted wheels,

– $R_1$ and $R_2$ – radiuses of curvature of profiles of heads and legs of teeth respectively,

– $\gamma$ –a corner of lead of the line of tooth of a globoid wheel to the face plane,

– $p$ – the parameter of the screw of the central screw line,

– $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ – independent variables, angles of rotation of radiuses of main circles,

– $\varphi_{11}$ and $\varphi_{22}$ – the corners defining the provision of face sections of heads of teeth with the face plane of the leader and conducted wheels,

– $\beta_1$ and $\beta_2$ – tilt corners of lines of teeth and an axis of rotation,

– $\xi_1$ and $\xi_2$ – the corners defining the provision of profiles of legs of teeth concerning profiles of heads of these teeth in the face plane of surfaces of teeth of the leader and conducted wheels respectively.

Systems (1) … (4) are the equations the screw of surfaces with a circle in the section located at a corner $\gamma$ to the face plane.

Having determined the first private derivatives of the equations (1) … (4) by parameter $\varphi_1$ and having increased them by angular speeds of rotation of wheels $w_{1,2}$, we will find projections to mobile axes of coordinates $O_1X_{11}Y_{11}Z_{11}$ and $O_2X_{22}Y_{22}Z_{22}$ speeds of points of gearing at their movement on contact lines of leading globoid and conducted cylindrical wheels. These projections will be equal:

− on heads of teeth of a leading wheel:

$$\begin{aligned} &V_{X11} = w_1\big[r_1'\sin(\varphi_{11}+\varphi_1)+ \\ &+(r_1+R_1\cos\lambda_{11})\cos(\varphi_{11}+\varphi_1)- \\ &-R_1\sin(\varphi_{11}+\varphi_1)\sin\lambda_{11}\cos\beta_1\big], \\ &V_{Y11} = w_1\big[r_1'\cos(\varphi_{11}+\varphi_1)- \\ &-(r_1+R_1\cos\lambda_{11})\sin(\varphi_{11}+\varphi_1)+ \\ &+R_1\sin(\varphi_{11}+\varphi_1)\sin\lambda_{11}\cos\beta_1\big], \\ &V_{Z11} = w_1 p\cdot tg\beta_1, \end{aligned} \tag{5}$$

− on legs of teeth of a leading wheel:

$$\begin{aligned} &V_{X12} = w_1\big[r_1'\sin(\varphi_{11}+\varphi_1-\xi_1)+ \\ &+(r_1+R_2\cos\lambda_{12})\cos(\varphi_{11}+\varphi_1-\xi_1)- \\ &-R_2\sin(\varphi_{11}+\varphi_1-\xi_1)\sin\lambda_{12}\cos\beta_1\big], \\ &V_{Y12} = w_1\big[r_1'\cos(\varphi_{11}+\varphi_1-\xi_1)- \\ &(r_1+R_2\cos\lambda_{12})\sin(\varphi_{11}+\varphi_1-\xi_1)+ \\ &+R_2\sin(\varphi_{11}+\varphi_1-\xi_1)\sin\lambda_{12}\cos\beta_1\big], \\ &V_{Z12} = w_1 p\cdot tg\beta_1, \end{aligned} \tag{6}$$

– on heads of teeth of a conducted wheel:

$$\begin{aligned} &V_{X21} = w_2\left[(r_2+R_1\cos\lambda_{21})\cos\left(\varphi_{22}+\frac{\varphi_1}{u}\right)\frac{1}{u}-\right. \\ &\left.-R_1\sin\left(\varphi_{22}+\frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right], \\ &V_{Y21} = w_2\left[-(r_2+R_1\cos\lambda_{21})\sin\left(\varphi_{22}+\frac{\varphi_1}{u}\right)\frac{1}{u}+\right. \\ &\left.+R_1\sin\left(\varphi_{22}+\frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right], \\ &V_{Z21} = w_2 p\cdot tg\beta_2, \end{aligned} \tag{7}$$

− on heads of teeth of a conducted wheel:

$$\begin{aligned} &V_{X22} = w_2\left[(r_2+R_2\cos\lambda_{22})\cos\left(\varphi_{22}+\frac{\varphi_1}{u}-\xi_1\right)\times\right. \\ &\left.\times\frac{1}{u}-R_2\sin\left(\varphi_{22}+\frac{\varphi_1}{u}-\xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right], \\ &V_{Y22} = w_2\left[-(r_2+R_2\cos\lambda_{22})\sin\left(\varphi_{22}+\frac{\varphi_1}{u}-\xi_1\right)\times\right. \\ &\left.\times\frac{1}{u}+R_2\sin\left(\varphi_{22}+\frac{\varphi_1}{u}-\xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right], \\ &V_{Z22} = w_2 p\cdot tg\beta_2, \end{aligned} \tag{8}$$

From expressions (5) … (8), using formulas of communication of coordinates $O_1X_1Y_1Z_1$ and $O_1X_{11}Y_{11}Z_{11}$ for a driving wheel, $O_2X_2Y_2Z_2$ and $O_2X_{22}Y_{22}Z_{22}$ for a conducted wheel, we will define projections of vectors of speeds of points of gearing to motionless axes of coordinates. For a leading globoid wheel we will receive:

− on heads of teeth of a driving wheel:

$$\begin{aligned} &V_{X1H} = w_1\big[r_1'\sin(\varphi_{11}+\varphi_1)+ \\ &+(r_1+R_1\cos\lambda_{11})\cos(\varphi_{11}+\varphi_1)- \\ &-R_1\sin(\varphi_{11}+\varphi_1)\sin\lambda_{11}\cos\beta_1\big], \\ &V_{Y1H} = w_1\big[r_1'\cos(\varphi_{11}+\varphi_1)- \\ &-(r_1+R_1\cos\lambda_{11})\sin(\varphi_{11}+\varphi_1)+ \\ &+R_1\sin(\varphi_{11}+\varphi_1)\sin\lambda_{11}\cos\beta_1\big], \\ &V_{Z1H} = w_1 p\cdot tg\beta_1, \end{aligned} \tag{9}$$

− on legs of teeth of a leading wheel:

$$\begin{aligned} &V_{X1L} = w_1\big[r_1'\sin(\varphi_{11}+\varphi_1-\xi_1)+ \\ &+(r_1+R_2\cos\lambda_{12})\cos(\varphi_{11}+\varphi_1-\xi_1)- \\ &-R_2\sin(\varphi_{11}+\varphi_1-\xi_1)\sin\lambda_{12}\times \\ &\times\cos\beta_1\big], \\ &V_{Y1L} = w_1\big[r_1'\cos(\varphi_{11}+\varphi_1-\xi_1)- \\ &-(r_1+R_2\cos\lambda_{12})(r_1+R_2\cos\lambda_{12})+ \\ &+R_2\sin(\varphi_{11}+\varphi_1-\xi_1)\sin\lambda_{12}\times \\ &\times\cos\beta_1\big], \\ &V_{Z1L} = w_1 p\cdot tg\beta_1, \end{aligned} \tag{10}$$

− on heads of teeth of a conducted wheel:

$$V_{X2H} = w_2\left[(r_2 + R_1\cos\lambda_{21})\cos\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\times \frac{1}{u} - R_1\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right],$$

$$V_{Y2H} = w_2\left[-(r_2 + R_1\cos\lambda_{21})\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\times \frac{1}{u} + R_1\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right],$$

$$V_{Z2H} = w_2 p\cdot ctg\beta_1, \qquad (11)$$

− heads of teeth of a conducted wheel:

$$V_{X2L} = w_2\left[(r_2 + R_2\cos\lambda_{22})\cos\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\times \frac{1}{u} - R_2\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$

$$V_{Y2L} = w_2\left[-(r_2 + R_2\cos\lambda_{22})\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\times \frac{1}{u} + R_2\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$

$$V_{Z2L} = w_2 p\cdot ctg\beta_1. \qquad (12)$$

Absolute value of speed of movement of a point of gearing along contact lines of a leading (globoid) wheel:

− on heads of teeth:

$$V_{K1H} = \sqrt{V_{X1H}^2 + V_{Y1H}^2 + V_{Z1H}^2} = w_1\cdot\sqrt{K_{VK1H}}, \qquad (13)$$

where:

$$K_{VK1H} = [r_1'\sin(\varphi_{11} + \varphi_1) + (r_1 + R_1\cos\lambda_{11})\times\cos(\varphi_{11} + \varphi_1) - R_1\sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1]^2 + [r_1'\cos(\varphi_{11} + \varphi_1) - (r_1 + R_1\cos\lambda_{11})\sin(\varphi_{11} + \varphi_1) + R_1\sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1]^2 + p^2\cdot tg^2\beta_1,$$

− on legs of teeth:

$$V_{K1L} = \sqrt{V_{X1L}^2 + V_{Y1L}^2 + V_{Z1L}^2} = w_1\cdot\sqrt{K_{VK1L}}, \qquad (14)$$

where:

$$K_{VK1H} = [r_1'\sin(\varphi_{11} + \varphi_1 - \xi_1) + (r_1 + R_2\cos\lambda_{12})\times\cos(\varphi_{11} + \varphi_1 - \xi_1) - R_2\sin(\varphi_{11} + \varphi_1 - \xi_1)\times\sin\lambda_{12}\cos\beta_1]^2 + [r_1'\cos(\varphi_{11} + \varphi_1 - \xi_1) - + [-(r_1 + R_2\cos\lambda_{12})\sin(\varphi_{11} + \varphi_1 - \xi_1) + R_2\sin(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1]^2 + p^2\cdot tg^2\beta_1,$$

− conducted (cylindrical) wheel:

− on heads of teeth:

$$V_{K2H} = \sqrt{V_{X2H}^2 + V_{Y2H}^2 + V_{Z2H}^2} = w_2\cdot\sqrt{K_{VK2H}}, \qquad (15)$$

where:

$$K_{VK2H} = \left[(r_2 + R_1\cos\lambda_{21})\cos\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\times\frac{1}{u} - R_1\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right]^2 + \left[-(r_2 + R_1\cos\lambda_{21})\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u} + R_1\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right]^2 + p^2\cdot tg^2\beta_2,$$

− on legs of teeth:

$$V_{K2L} = \sqrt{V_{X2L}^2 + V_{Y2L}^2 + V_{Z2L}^2} = w_2\cdot\sqrt{K_{VK2L}}, \qquad (16)$$

where:

$$K_{KV2L} = \left[(r_2 + R_2\cos\lambda_{22})\cos\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} - R_2\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right]^2 + \left[-(r_2 + R_2\cos\lambda_{22})\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} + R_2\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right]^2 + p^2\cdot tg^2\beta_2,$$

where: $r_1' = r_{10} u \sin\varphi_2^*$.

Directing cosines of a vector of speed of movement of a leg of gearing are equal:

− on heads of teeth of a driving wheel:

$$\cos\alpha_{K1H} = \frac{V_{X1H}}{V_{K1H}} = \frac{1}{\sqrt{K_{VK1H}}} \cdot \left[r_1' \sin(\varphi_{11} + \varphi_1) + \right.$$
$$+ (r_1 + R_1 \cos\lambda_{11})\cos(\varphi_{11} + \varphi_1) -$$
$$\left. - R_1 \sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1\right],$$
$$\cos\beta_{K1H} = \frac{V_{Y1H}}{V_{K1H}} = \frac{1}{\sqrt{K_{VK1H}}} \cdot \left[r_1' \cos(\varphi_{11} + \varphi_1) - \right.$$
$$- (r_1 + R_1 \cos\lambda_{11})\sin(\varphi_{11} + \varphi_1) +$$
$$\left. + R_1 \sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1\right],$$
$$\cos\gamma_{K1H} = \frac{V_{Z1H}}{V_{K1H}} = \frac{p \cdot tg\beta_1}{\sqrt{K_{VK1H}}}, \qquad (17)$$

− on legs of teeth of a leading wheel:

$$\cos\alpha_{K1L} = \frac{V_{X1L}}{V_{K1L}} = \frac{1}{\sqrt{K_{VK1L}}} \times$$
$$\times \left[r_1' \sin(\varphi_{11} + \varphi_1 - \xi_1) + \right.$$
$$+ (r_1 + R_2 \cos\lambda_{12})\cos(\varphi_{11} + \varphi_1 - \xi_1) -$$
$$\left. - R_2 \sin(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1\right],$$
$$\cos\beta_{K1L} = \frac{V_{Y1L}}{V_{K1L}} = \frac{1}{\sqrt{K_{VK1L}}} \times$$
$$\times \left[r_1' \cos(\varphi_{11} + \varphi_1 - \xi_1) - \right. \qquad (18)$$
$$- (r_1 + R_2 \cos\lambda_{12})\sin(\varphi_{11} + \varphi_1 - \xi_1) +$$
$$\left. + R_2 \sin(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1\right],$$
$$\cos\gamma_{K1L} = \frac{V_{Z1L}}{V_{K1L}} = \frac{p \cdot tg\beta_1}{\sqrt{K_{VK1L}}},$$

− on heads of teeth of a conducted wheel:

$$\cos\alpha_{K2H} = \frac{V_{X2H}}{V_{K2H}} = \frac{1}{\sqrt{K_{VK2H}}} \cdot \left[(r_2 + R_1 \cos\lambda_{21}) \times \right.$$
$$\left. \times \cos\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u} - R_1 \sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right],$$

$$\cos\beta_{K2H} = \frac{V_{Y2H}}{V_{K2H}} = \frac{1}{\sqrt{K_{VK2H}}} \times \left[-(r_2 + R_1 \cos\lambda_{21}) \times \right.$$
$$\times \sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u} + R_1 \sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21} \times$$
$$\left. \times \cos\beta_2\right],$$
$$\cos\gamma_{K2H} = \frac{V_{Z2H}}{V_{K2H}} = \frac{p \cdot ctg\beta_1}{\sqrt{K_{VK2H}}}, \qquad (19)$$

− on legs of tooth of a conducted wheel:

$$\cos\alpha_{K2L} = \frac{V_{X2L}}{V_{K2L}} = \frac{1}{\sqrt{K_{VK2L}}} \times$$
$$\times \left[(r_2 + R_2 \cos\lambda_{22})\cos\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} - \right.$$
$$\left. - R_2 \sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$
$$\cos\beta_{K2L} = \frac{V_{Y2L}}{V_{K2L}} = \frac{1}{\sqrt{K_{VK2L}}} \times$$
$$\times \left[-(r_2 + R_2 \cos\lambda_{22})\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} + \right. \qquad (20)$$
$$\left. + R_2 \sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$
$$\cos\gamma_{K2L} = \frac{V_{Z2L}}{V_{K2L}} = \frac{p \cdot ctg\beta_1}{\sqrt{K_{VK2L}}}.$$

The relative speed of movement of the contact points located on contact lines of heads and legs of teeth of the leader and conducted wheels we will determine as a vector difference of speeds of movement of points of gearing along contact lines of the leader and conducted wheels:

− on heads of teeth of the leader and legs of teeth of the conducted:

$$V_{XS1} = V_{X1H} - V_{X2L} = K_{VXS1}, \qquad (21)$$

where:

$$K_{VXS1} = w_1\left[r_1' \sin(\varphi_{11} + \varphi_1) + (r_1 + R_1 \cos\lambda_{11}) \times \right.$$
$$\left. \times \cos(\varphi_{11} + \varphi_1) - R_1 \sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1\right] -$$
$$- w_2\left[(r_2 + R_2 \cos\lambda_{22})\cos\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} - \right.$$
$$\left. - R_2 \sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$

$$V_{YS1} = V_{Y1H} - V_{Y2L} = K_{VYS1}, \qquad (22)$$

where:

$$K_{VYS1} = w_1\left[r_1' \cos(\varphi_{11} + \varphi_1) - (r_1 + R_1 \cos\lambda_{11}) \times \sin(\varphi_{11} + \varphi_1) + R_1 \sin(\varphi_{11} + \varphi_1)\sin\lambda_{11}\cos\beta_1\right] - w_2\left[-(r_2 + R_2\cos\lambda_{22})\sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u} + R_2 \sin\left(\varphi_{22} + \frac{\varphi_1}{u} - \xi_1\right)\frac{1}{u}\sin\lambda_{22}\cos\beta_2\right],$$

$$V_{ZS1} = V_{Z1H} - V_{Z2L} = K_{VZS1}, \qquad (23)$$

where:

$$K_{VZS1} = p(w_1 tg\beta_1 - w_2 tg\beta_2),$$

− on legs of teeth of a leading wheel and heads of teeth of the conducted:

$$V_{XS2} = V_{X1L} - V_{X2H} = K_{VXS2}, \qquad (24)$$

where:

$$K_{VXS2} = w_1\left[r_1' \sin(\varphi_{11} + \varphi_1 - \xi_1) + (r_1 + R_2\cos\lambda_{12})\cos(\varphi_{11} + \varphi_1 - \xi_1) - R_2 \times \sin(\varphi_{11} + \varphi_1 - \xi_1)\cdot \sin\lambda_{12}\cos\beta_1\right] - w_2\left[(r_2 + R_1\cos\lambda_{21})\cos\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u} - R_1 \sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right].$$

$$V_{YS2} = V_{Y1L} - V_{Y2H} = K_{VYS2}, \qquad (25)$$

where:

$$K_{VYS2} = w_1\left[r_1' \cos(\varphi_{11} + \varphi_1 - \xi_1) - \cdot - (r_1 + R_2\cos\lambda_{12})\sin(\varphi_{11} + \varphi_1 - \xi_1) + R_2 \sin(\varphi_{11} + \varphi_1 - \xi_1)\sin\lambda_{12}\cos\beta_1\right] - w_2\left[-(r_2 + R_1\cos\lambda_{21})\sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u} + R_1 \sin\left(\varphi_{22} + \frac{\varphi_1}{u}\right)\frac{1}{u}\sin\lambda_{21}\cos\beta_2\right],$$

$$V_{ZS2} = V_{Z1L} - V_{Z2H} = K_{VZS2}, \qquad (26)$$

where:

$$K_{VZS1} = p(w_1 tg\beta_1 - w_2 tg\beta_2).$$

The absolute value of relative speed of movement of a point of gearing will be equal:
− on heads of teeth of a leading wheel and legs of teeth of a conducted wheel:

$$V_{S1} = \sqrt{V_{XS1}^2 + V_{YS1}^2 + V_{ZS1}^2} = \sqrt{K_{VXS1}^2 + K_{VYS1}^2 + K_{VZS1}^2}, \qquad (26)$$

− on legs of teeth of a leading wheel and heads of teeth of a conducted wheel:

$$V_{S2} = \sqrt{V_{XS2}^2 + V_{YS2}^2 + V_{ZS2}^2} = \sqrt{K_{VXS2}^2 + K_{VYS2}^2 + K_{VZS2}^2}, \qquad (27)$$

Arrangements of vectors of speeds of points of gearing on a head and on a leg of a tooth in globoid cylindrical transfer at their movement along contact lines are defined by directing cosines of a vector of relative speed of movement of a point of gearing.

For heads of teeth of the leading and legs of teeth conducted wheels we will receive:

$$\cos\alpha_{S1} = \frac{V_{XS1}}{V_{S1}} = \frac{K_{VXS1}}{\sqrt{K_{VXS1}^2 + K_{VYS1}^2 + K_{VZS1}^2}},$$
$$\cos\beta_{S1} = \frac{V_{YS1}}{V_{S1}} = \frac{K_{VYS1}}{\sqrt{K_{VXS1}^2 + K_{VYS1}^2 + K_{VZS1}^2}}, \qquad (28)$$
$$\cos\gamma_{S1} = \frac{V_{ZS1}}{V_{S1}} = \frac{K_{VZS1}}{\sqrt{K_{VXS1}^2 + K_{VYS1}^2 + K_{VZS1}^2}}.$$

For legs of teeth of the leading and heads of teeth conducted wheels we will receive:

$$\cos\alpha_{S2} = \frac{V_{XS2}}{V_{S2}} = \frac{K_{VXS2}}{\sqrt{K_{VXS2}^2 + K_{VYS2}^2 + K_{VZS2}^2}},$$
$$\cos\beta_{S2} = \frac{V_{YS2}}{V_{S2}} = \frac{K_{VYS2}}{\sqrt{K_{VXS2}^2 + K_{VYS2}^2 + K_{VZS2}^2}}, \qquad (29)$$

$$\cos\gamma_{S2} = \frac{V_{ZS2}}{V_{S2}} = \frac{K_{VZS2}}{\sqrt{K_{VXS2}^2 + K_{VYS2}^2 + K_{VZS2}^2}}.$$

From Fig. 2 follows that the size of absolute speed of relative movement of points of gearing of screw tooth globoid cylindrical gearing a variable and changes from the maximum value on the average the section of a globoid wheel to minimum on both sides from this section for 0,28%.

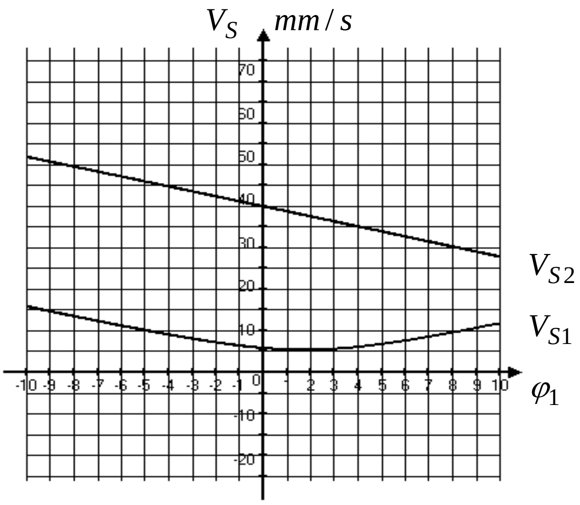


**Fig. 2.** The absolute value of relative speed of movement of a point of gearing of the leader $V_{S1}$ and conducted $V_{S2}$ wheels

## CONCLUSIONS

The received mathematical dependences allowed to establish:

1. It is established that The absolute and relative speeds of movement of points of contact of teeth in a screw globoid cylindrical tooth gearing variables and which sizes change for 0,28% during removal from the plane of average (throat) section of a globoid wheel.

2. It is established that the difference of absolute speeds of movement of points of contact on a leg and a head of a leading (globoid) cogwheel makes 1,5%.

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## СКОРОСТИ ДВИЖЕНИЯ ТОЧКИ ЗАЦЕПЛЕНИЯ ВДОЛЬ КОНТАКТНЫХ ЛИНИЙ В КРУГОВИНТОВОЙ ЗУБЧАТОЙ ГЛОБОИДНО-ЦИЛИНДРИЧЕСКОЙ ПЕРЕДАЧЕ

*Николай Утутов, Наталья Плясуля*

Аннотация. В статье изложена математическая модель скоростей движения точек зацепления вдоль контактных линий в круговинтовой глобоидно-цилиндрической зубчатой передаче. Получены математические зависимости для обсалютных и относительных скоростей движения точек зацепления по контактным линиям на головках и ножках зубьев ведущего и ведомого колес. Также получены математические зависимости для углов между векторами скоростей и основными координатными линиями.

Ключевые слова: зубчатая передача, глобоидно-цилиндрическая, круговинтовая.