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Design scheme for the stress relaxation experiment for maxwell model identification

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Summary. The problem of a weighted least-squares approximation of viscoelastic material by Maxwell model is discussed when the noise-corrupted time-measurements of the relaxation modulus obtained in relaxation test experiment are accessible for identification. In the previous paper it has been shown that even when the true relaxation modulus description is completely unknown, the optimal Maxwell model parameters can be derived from the measurement data sampled randomly according to appropriate randomization. In this paper an identification algorithm leading to the best model when only a relaxation modulus data are accessible is derived using a concept of random choice of the sampling instants. The stochastic-type convergence analysis is conducted both for corrupted as well as noise free relaxation modulus measurements and the exponential convergence rate is proved. Experimental numerical results for five-parameter Maxwell model are provided. Applying the scheme proposed the four-parameter Maxwell models of an confined cylindrical specimen of the beet sugar root are determined for a few sets of relaxation modulus measurement data and the convergence of the sequence of best model parameters is demonstrated. The procedure has been successfully tested using both artificial and experimental data.

Key words: relaxation test, Maxwell model, identification algorithm, experiment design

INTRODUCTION

The classical Maxwell model [14] is a viscoelastic body that stores energy like a linearized elastic spring and dissipates energy like a classical fluid dashpot. The generalized Maxwell model, which is used to describe the relaxation modulus of linear viscoelastic materials, consists of a spring and *n* Maxwell units connected in parallel. Maxwell models are used frequently to describe viscoelasticity of polymers [1, 6], concrete [13], soils [5], rocks [8], rubber [20], glass [17], foods [16] and biological materials [2, 9].

Maxwell models described by a sum of exponential functions. Fitting a sums of exponentials to empirical data is a very old problem in system identification theory [7, 10]. Although a lot of methods are known for determining exponential sum models, in particular for finding the optimal least-squares exponential sum approximations to sampled data, the efficient tools for Maxwell model determination are still desirable and this is the purpose of this study.

In empirical sciences there is an increasing use of mathematical models to describe various physical phenomena. The work in modelling of physical phenomena neatly partitions into two pieces: the work in acquiring information (measurements) from the real system, and the overhead involved in determining the best model. More often than not, this second part of the work is dominated by system identification methods. Whence, members of the physical systems researchers community have been consumers of system identification algorithms. Thus, their needs have set some research directions for the system identification community. Simultaneously, this symbiotic relationship has entered a new phase, in which the new advances in system identification contribute to building a better and better models and they are also setting new research agendas for the physical systems researchers community.

In the previous paper [19] an idea of measurement point-independent approximation of a relaxation modulus of linear viscoelastic materials within the class of generalized Maxwell models, when the integral weighted square error is to be minimized and the true material description is completely unknown, is presented. In this paper a simple identification algorithm providing the strongly consistent estimate of the optimal model is given. Next, the rate of convergence is discussed for the case when the measurements are perfect or corrupted by additive noises. The results of simulation experiments for five parameter Maxwell model are presented and the identification algorithm is applied for computing the Maxwell model of an specimen of the beet sugar root.

OPTIMAL IDENTIFICATION OF MAXWELL MODEL

In the previous paper [19] the problem of an approximation of relaxation modulus $G(t)$ of linear viscoelastic material by generalized Maxwell model:

$$G_M(t,\boldsymbol{g}) = \sum_{j=1}^{n} E_j e^{-v_j t} + E_\infty, \quad (1)$$

with the vector of model parameters defined as:

$$\boldsymbol{g} = [E_1 \ \ldots \ E_n \ v_1 \ \ldots \ v_n \ E_\infty]^T, \quad (2)$$

where: E_j, v_j and E_∞ are the elastic modulus, relaxation frequencies and equilibrium modulus, respectively, is considered under the assumption that the exact mathematical description of the true relaxation modulus $G(t)$ is completely unknown. The value of $G(t)$ can be, however, measured with a certain accuracy for any given value of the time $t \in T$, where $T=[0,T]$ and $0<T<\infty$ or $T=R_+$; here $R_+=[0,\infty)$. The restriction that the model parameters are nonnegative and confined must be given to satisfy the physical meaning, i.e. $\boldsymbol{g} \in G$, where the admissible set of parameters G is compact subset of the space R_+^{2n+1}.

Let $T_1,\ldots,T_N$ are independent random variables with a common probability density function $\rho(t)$ whose support is $\mathcal{T}$. Let $\bar{G}_i = G(T_i) + Z_i$ denote measurements of the relaxation modulus $G=G(T_i)$ obtained in a certain stress relaxation test performed on the specimen of the material under investigation, $i=1,\ldots,N$. Here Z_i are additive measurement noises.

As a measure of the model (1) accuracy the global approximation error of the form:

$$Q(\boldsymbol{g}) = \int_T [G(t) - G_M(t,\boldsymbol{g})]^2 \rho(t)\,dt, \quad (3)$$

where a chosen weighting function $\rho(t) \geq 0$ is a density on $\boldsymbol{T}$, i.e., $\int_T \rho(t)dt = 1$, is applied. From practical reasons the global index $Q(\boldsymbol{g})$ is replaced by the following empirical one:

$$Q_N(\boldsymbol{g}) = \frac{1}{N}\sum_{i=1}^{N} [\bar{G}_i - G_M(T_i,\boldsymbol{g})]^2. \quad (4)$$

Therefore, the classical least-squares problem for Maxwell model is obtained.

The problem of the relaxation modulus $G(t)$ optimal approximation within the class of Maxwell models (1) consists in determining the admissible parameter minimizing $Q(\boldsymbol{g})$:

$$\boldsymbol{g}^* = arg \min_{g \in G} Q(\boldsymbol{g}), \quad (5)$$

Here $arg\ min\ \boldsymbol{g} \in \boldsymbol{G}\ Q(\boldsymbol{g})$ denotes the vector $\boldsymbol{g}$ that minimizes $Q(\boldsymbol{g})$ on the set $\boldsymbol{G}$. The respective empirical task is as follows:

$$\boldsymbol{g}_N = arg \min_{g \in G} Q_N(\boldsymbol{g}). \quad (6)$$

Under standard assumptions concerning the relaxation modulus and noises:

Assumption 1. The relaxation modulus $G(t)$ is bounded on $\mathcal{T}$, i.e. $sup_{t \in T} G(t) \leq M < \infty$,

Assumption 2. The measurement noises Z_i are bounded, i.e. $|Z_i| \leq \delta < \infty$ for $i=1,\ldots,N$,

Assumption 3. $\{Z_i\}$ is a time-independent sequence of independent identically distributed (i.i.d.) random variables with zero mean and a common finite variance σ^2,

it is shown in [19], that Maxwell model which is asymptotically (when the number of measurements tends to infinity) independent on the particular sampling instants t_i can be derived from the set of relaxation modulus time-data by introducing a simple randomization. Namely, it is proved that if the Assumptions 1-3 hold and $T_1,\ldots,T_N$ are independently, at random selected from $\mathcal{T}$, each according to probability distributions with density $\rho(t)$, then both for perfect as well as for the additive noise corrupted relaxation modulus measurements:

$$\boldsymbol{g}_N \to \boldsymbol{g}^* \qquad w.p.1 \quad as \quad N \to \infty \quad (7)$$

and for all $t \in \mathrm{T}$:

$$G_M(t,\boldsymbol{g}_N) \to G_M(t,\boldsymbol{g}^*) \qquad w.p.1 \quad as \quad N \to \infty, \quad (8)$$

where $w.p.1$ means "with probability one". Thus, the model parameter $\boldsymbol{g}_N$ is strongly consistent estimate of the parameter $\boldsymbol{g}^*$. Moreover, since the model $G_M(t,\boldsymbol{g})$ is Lipschitz on G uniformly in $t \in \mathrm{T}$, then the almost sure convergence of $\boldsymbol{g}_N$ to $\boldsymbol{g}^*$ in (7) implies that:

$$\sup_{t \in T} \left| G_M(t,\boldsymbol{g}_N) - G_M(t,\boldsymbol{g}^*) \right| \to 0 \qquad w.p.1 \quad as \quad N \to \infty. \quad (9)$$

Thus, $G_M(t,\boldsymbol{g}_N)$ is a strongly uniformly consistent estimate of the best model $G_M(t,\boldsymbol{g})$.

Summarizing, when the Assumptions 1-3 are satisfied, the arbitrarily precise approximation of the optimal Maxwell model (with the parameter $\boldsymbol{g}^*$) can be obtained (almost everywhere) as the number of mesurements N grows large, despite the fact that the real description of the relaxation modulus is completely unknown.

IDENTIFICATION ALGORITHM

Taking into account the convergence results (7)-(9) the calculation of the approximate value $\boldsymbol{g}_N$ of optimal Maxwell model parameter $\boldsymbol{g}^*$ involves the following steps.

1. Select randomly from the set T the sampling instants $t_1,\ldots,t_N$, each t_i independently, according to the probability distribution on T with the density given by the weighting function $\rho(t)$ in (3).
2. Perform the stress relaxation test, record and store the relaxation modulus measurements $\bar{G}(t_i)$, $i=1,\ldots,N$, corresponding to the chosen points $t_i \geq 0$.
3. Solve the optimization task (6) and compute the Maxwell model parameter $\boldsymbol{g}_N$.

4. In order to ascertain if $\boldsymbol{g}_N$ is a satisfactory approximation of $\boldsymbol{g}^*$ enlarge the set of data to the extent $\overline{N} >> N$ repeating Steps 1 and 2.
5. Execute Step 3 for the new set of data determining $\boldsymbol{g}_{\overline{N}}$.
6. Examine if $\left\| \boldsymbol{g}_N - \boldsymbol{g}_{\overline{N}} \right\|_2 < \varepsilon$, for ε, a small positive number. If not, put $N = \overline{N}$ and go to Step 4. Otherwise, stop the procedure taking $\boldsymbol{g}_N$ as the approximate value of $\boldsymbol{g}^*$.

Remark. The stopping rule from Step 6 can be replaced by a less restrictive one, based on testing, whether $\left| Q_{\overline{N}}\left(\boldsymbol{g}_{\overline{N}}\right) - Q_N\left(\boldsymbol{g}_N\right) \right| < \varepsilon$ holds. Both the stopping rules considered correspond with those commonly used in the numerical minimization techniques.

CONVERGENCE ANALYSIS

Taking account of (7) the question immediately arises how fast does $\boldsymbol{g}_N$ tend to $\boldsymbol{g}^*$ as N grows large. The distance between the Maxwell model parameters $\boldsymbol{g}_N$ and $\boldsymbol{g}^*$ will be estimated in the sense of quality difference $|Q(\boldsymbol{g}^*)-Q(\boldsymbol{g}_N)|$. We shall examine how fast, for given $\varepsilon>0$, does $P\left\{\left|Q\left(g^*\right)-Q\left(g_N\right)\right| \geq \varepsilon\right\}$ tend to zero as N increases. On the basis of inequality (11) from [4] we obtain for noise case the following bound:

$$P\left\{\left|Q_N(g)-Q\left(g^*\right)\right| \geq \varepsilon\right\} \leq 2\,exp\left(-N\varepsilon^2/8\hat{M}^2\right), \tag{10}$$

where $\hat{M}$ is such that the next estimation is valid for any $\boldsymbol{g}\in\mathsf{G}$:

$$\left|\left[G(T_i)+Z_i-G_M(T_i,g)\right]^2-\left[Q(g)+\sigma^2\right]\right| \leq \tilde{M}+\delta^2+2c\delta+\sigma^2=\hat{M}. \tag{11}$$

Here a positive constant $\tilde{M}$ is such that:

$$\left[G(T_i)-G_M(T_i,g)\right]^2-Q(g)\leq\tilde{M} \tag{12}$$

for any $\boldsymbol{g}\in\mathsf{G}$, $i=1,\dots,N$, and a positive constant c is defined by the inequality $|G(T_i)-G_M(T_i,\boldsymbol{g})|\leq c$ which holds for every $\boldsymbol{g}\in\mathsf{G}$, $i=1,\dots,N$. The existence of $\tilde{M}$ follows immediately from the Assumptions 1 and 2, Property 3 and the fact that, the weighting function $\rho(t)\geq0$ is a density on T. In view of Assumption 1 and Property 2 the positive constant c there exists and can be evaluated without a difficulty. Note also, that in the noiseless case the inequality (10) takes especially simple form:

$$P\left\{\left|Q_N(g)-Q\left(g^*\right)\right| \geq \varepsilon\right\} \leq 2\,exp\left(-N\varepsilon^2/8\tilde{M}^2\right). \tag{13}$$

The inequalities (13) and (10) show some connections between the convergence rate and the number of measurements N as well as the measurement noises. In particular, if ε is fixed, then the bound (10) tends to zero at exponential rate as N increases. Note also, that the rate of convergence is the higher the lower is $\hat{M}$, thus by (11) the lower are δ and σ^2, i.e. the measurement noises are weaker. This is not a surprise since, with large noises, the measurements are not much adequate to the true relaxation modulus. Notice, however, that for fixed $\varepsilon>0$ both for noiseless as noise case:

$$P\left\{\left|Q_N(g)-Q\left(g^*\right)\right| \geq \varepsilon/N^{\alpha}\right\} \leq 2\,exp\left(-N^{(1-2\alpha)}\varepsilon^2/8\hat{M}^2\right)$$

with $0\leq\alpha<1/2$ still tends to zero as $N\to\infty$ in quasi-exponential rate.

We now present the results of the numerical experiments. Both the asymptotic properties (as the number of measurements $N\to\infty$) as well as the influence of the measurement noises on solution will be studied.

EXPERIMENTAL STUDIES

Consider viscoelastic material whose relaxation modulus is described by:

$$G(t)=\frac{1}{2}e^{-20t+18t^2}erfc\left(3\sqrt{2}t-5\sqrt{2}/3\right)+0.2, \tag{14}$$

where $erfc(t)$ is the complementary error function [11]. The time interval $\mathsf{T}=[0;0,8]$ seconds has been taken for the experiment in view of the modulus $G(t)$ (14) course. The measurement points t_i are selected randomly, each t_i independently according to the uniform distribution on T. The 5 parameter Maxwell model of the form:

$$G_M(t)=E_1e^{-v_1t}+E_2e^{-v_2t}+E_\infty,$$

has been taken for numerical studies. The relaxation modulus $\overline{G}(t_i)=G(t_i)+z(t_i)$ has been sampled in N sampling instants during the time period T.

In order to study the influence of the noises on the Maxwell model parameters $\{z_i\}$ have been generated independently by random choice with normal distribution with zero mean value and variance σ^2; $\sigma=0.005,0.0075,0.01,0.02$ is taken for experiment. Such measurement noises are even strongest than the true disturbances recorded for the plant materials (see [18; Chapter 5.5.4]). For the analysis of asymptotic properties of the scheme $N=50,100,500,1000,5000,10000$ has been used in the experiment. The experiment and next the computation of the optimal model have been repeated $n=100$ times for every pair (N,σ^2). The distance between the optimal model parameters: 'empirical' $\boldsymbol{g}_N$ and 'ideal' $\boldsymbol{g}^*$ has been estimated by standardized mean error defined for n element sample as:

$$ERR\left(N,\sigma^2\right)=\frac{1}{n}\sum_{j=1}^{n}\left\|g_{N,j}-g^*\right\|_2\Big/\left\|g^*\right\|_2, \tag{15}$$

where $\boldsymbol{g}_{N,j}$ denote the model parameter $\boldsymbol{g}_N$ determined for $j-th$ experiment repetition for a given pair (N,σ^2), $j=1,\dots,n$. Here $\|\cdot\|_2$ denotes the Euclidean norm in the space R^{2n+1}. Relationship of $ERR(N,\sigma^2)$ (15) on N and σ^2 is depicted in Figure 1. The ranges of variation of the index $ERR(N, \sigma^2)$ obtained in the simulation experiment are given in table 1. In Figure 1 we can see that for small and middle noises and $N \geq 500$ the error $ERR(N, \sigma^2)$ does not depend essentially on the number of measurements. For small and middle noises the index $ERR(N, \sigma^2)$ do not exceed 2%, however, for large noises ($\sigma = 0.02$) this error exceeds 10%.

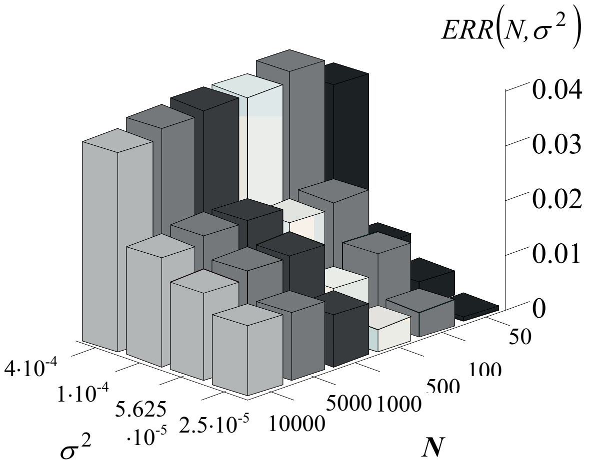

Fig. 1. The index $ERR(N, \sigma^2)$ as a function of N and σ^2

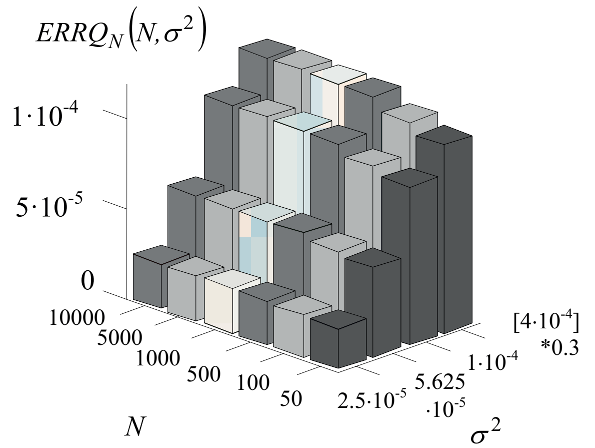

Fig. 2. The mean square approximation error of the relaxation modulus measurements $ERRQ_N(N, \sigma^2)$; the notation [4·10⁻⁴]*0.3 is used to describe the function $0.3 \cdot ERRQ_N(N, \sigma^2)$ for $\sigma^2 = 4 \cdot 10^{-4}$.

To estimate the approximation error of the relaxation modulus measurements for n-element sample the following index (cf. definition (4)) is taken:

$$ERRQ_N\left(N,\sigma^2\right) = \frac{1}{nN}\sum_{j=1}^{n} Q_N\left(\mathrm{g}_{N,j}\right) = = \frac{1}{nN}\sum_{j=1}^{n}\sum_{i=1}^{N}\left[\bar{G}_{i,j} - G_M\left(t_{i,j},\mathrm{g}_{N,j}\right)\right]^2,$$

where $\bar{G}_{i,j} = G\left(t_{i,j}\right) + z_{i,j}$ denote the relaxation modulus measurements for $j-th$ experiment repetition for a given pair (N, σ^2), $j = 1,\ldots,n$. The index $ERRQ_N(N, \sigma^2)$ as a function of N and σ^2 is depicted in Figure 2. We can see that $ERRQ_N(N, \sigma^2)$ does not depend essentially on the number of measurements both for small as well as large noises. The algorithm ensures very good quality of the measurements approximation even for large noises (see table 1).

The mean integral error of the relaxation modulus $G(t)$ (14) approximation is defined as:

$$ERRQ\left(N,\sigma^2\right) = \frac{1}{n}\sum_{j=1}^{n} Q\left(\boldsymbol{g}_{N,j}\right),$$

where the global integral error is given by (3). The error $ERRQ(N, \sigma^2)$ is decreasing function of the number of sampling points and the number of model summands as depicted in Figure 3. The interpretation of Figure 3 becomes quite clear when we take into account the convergence analysis conducted. As we have shown, the global integrated index $Q(\boldsymbol{g})$ converges exponentially both with the increase of the number of measurements N as well as with the decrease of the noise variance σ^2 – compare the inequality (10) and the definition of $\hat{M}$(10).

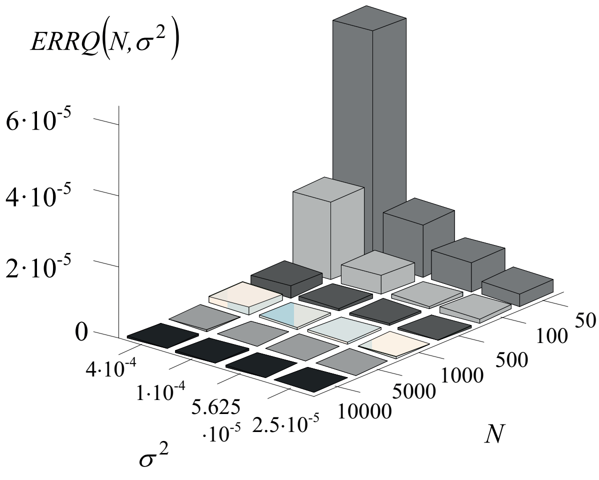

Fig. 3. Mean integral approximation error of the relaxation modulus $ERRQ(N, \sigma^2)$

Table 1. The ranges of variation of the approximation error indices used in the simulation experiment for the number of measurements $N \geq 500$

	$\sigma = 0{,}005$	$\sigma = 0{,}0075$	$\sigma = 0{,}01$	$\sigma = 0{,}02$
$ERR(N, \sigma^2)$	0,00408÷0,0128	0,0087÷0,01757	0,0182÷0,0213	0,12175÷0,1284
$ERRQ_N(N, \sigma^2)$	2,47E-5÷2,51E-5	5,56E-5÷5,63E-5	9,88E-5÷9,9E-5	1,17E-4÷1,18E-4
$ERRQ(N, \sigma^2)$	0,0189÷0,0193	9,98E-3÷0,018	9,167E-3÷0,017	1,2E-3÷4,518E-3

Table 2. Maxwell model (16) parameters and the values of identification index $Q_N(\boldsymbol{g}_N)$; random choice of the sampling instants

N	$Q_N(\boldsymbol{g}_N)$	Model parameters			
		$E_{1,N}$ [MPa]	$E_{2,N}$ [MPa]	$v_{1,N}$ [s^{-1}]	$v_{2,N}$ [s^{-1}]
15	7,914E-5	10,2607	0,5257	4,6823E-4	0,0489
20	2,978E-4	10,3447	0,5097	5,4625E-4	0,0737
25	8,38E-4	10,4025	0,5292	6,3646E-4	0,1004
40	2,233E-3	10,3763	0,5886	5,8039E-4	0,0963
50	1,867E-3	10,3553	0,6049	5,5898E-4	0,0886
75	4,575E-4	10,3388	0,5736	5,3957E-4	0,0793
100	2,527E-4	10,2973	0,5409	4,9971E-4	0,0598
150	2,596E-4	10,2879	0,5363	4,8541E-4	0,0582
200	2,912E-4	10,2936	0,5342	4,9343E-4	0,0587
250	3,476E-4	10,2877	0,5427	4,7722E-4	0,0584
300	2,541E-4	10,284	0,5398	4,8225E-4	0,0561
350	2,615E-4	10,2785	0,5369	4,7334E-4	0,0557
400	2,851E-4	10,2786	0,5383	4,7207E-4	0,0564

The next example shows how the proposed identification scheme can be used in the Maxwell model identification of real material.

MAXWELL MODEL OF THE SUGAR BEET SAMPLE

Let us consider again the sample of the root of sugar beet Janus variety [3] studied in the example in [19]. The stress relaxation experiment performed by Gołacki and co-workers is described in details in [3] and the way how the experiment data has been preliminary proceeded is circumscribed in paper [19]. The sampling points t_i have been generated independently by random choice with uniform distribution in time interval [0;95] seconds, consecutively, for N from the set $\mathsf{N}1 = \{15,20,25,40,50, 75,100,150,200,250,300,350,400\}$, and the respective relaxation modulus measurements have been selected from the whole set of measurement data. Next, the optimal four-parameter Maxwell models:

$$G_M(t) = E_1 e^{-v_1 t} + E_2 e^{-v_2 t}, \tag{16}$$

were determined for each N. The parameters of the optimal models and the respective values of empirical index $Q_N(\boldsymbol{g})$ (4) are given in Table 2. The fast convergence of the model parameters and the model quality index is illustrated in Figures 4 and 5, respectively. In Figure 6 the distance $d_N = \|\boldsymbol{g}_N - \boldsymbol{g}_{[N]}\|_2$, between the successive model parameters $\boldsymbol{g}_N$ is also shown as a function of N, where $[N]$ is a direct predecessor of N in the set N.

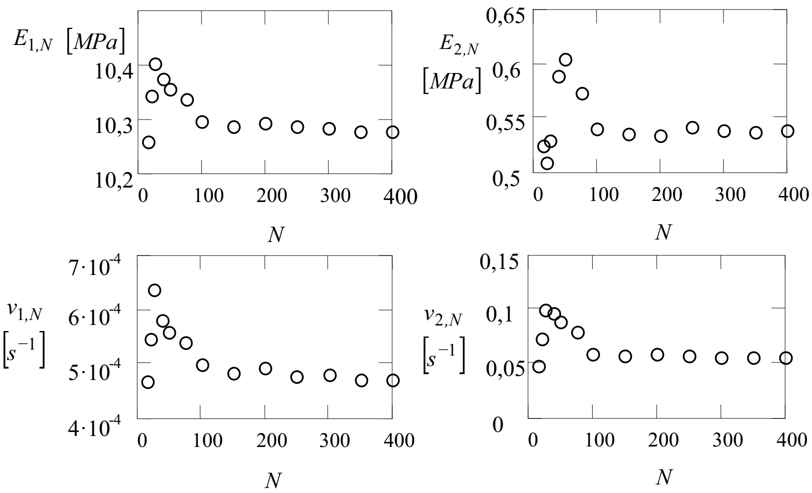

Fig. 4. The optimal Maxwell model parameters as a function of the number of measurements N; random choice of the sampling instants

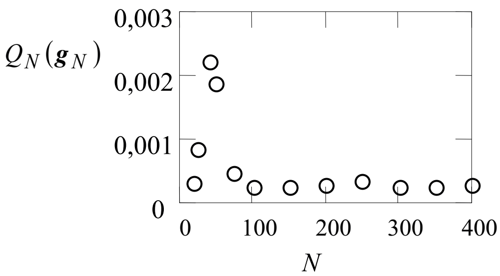

Fig. 5. The identification index $Q_N(\boldsymbol{g}_N)$ as a function of the number of measurements N; random choice of the sampling instants

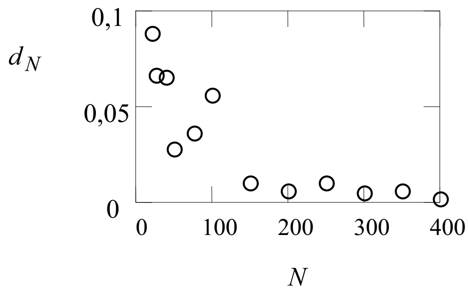

Fig. 6. The distance d_N between the two successive Maxwell model parameters $\boldsymbol{g}_N$ as a function of the number of measurements N; random choice of the sampling instants

The fitting of the relaxation modulus computed according to the best Maxwell models $G_M(t,\boldsymbol{g}_N)$ to experiment data is shown for a few values of the number of measurements in Figure 7, where the measurements $\overline{G}(t_i)$ are also marked. It can be seen from Figure 7 that four parameter Maxwell models are necessary for almost excellent fitting the data, if the time instants t_i are chosen in appropriate way.

The differences in the convergence speed between the cases of random and equidistant experiment data are demonstrated by comparison of Figures 5,6 and 2 in [19], Figures 7 and 3 in [19]. Thus, the general conclusion is that the choice of the sampling instants has fundamental meaning for the Maxwell model obtained.

FINAL REMARKS

1. The approximation of the optimal Maxwell model can be derived from relaxation modulus data sampled randomly according to respective randomization. The approximate model parameters are strongly consistent estimate of the parameters of that Maxwell model, which is independent of particular sampling instants used in relaxation test.
2. It is worth of noticing that the resulting identification procedure is very useful in application because it does not require any other experimental technique more sophisticated than the independent random sampling of time instants t_i from the set T according to a stationary rule.
3. When the set $\{t_i\}$ is opened to manipulation during the data collection, it is an important experiment design issue to take appropriate sampling instants. Therefore,

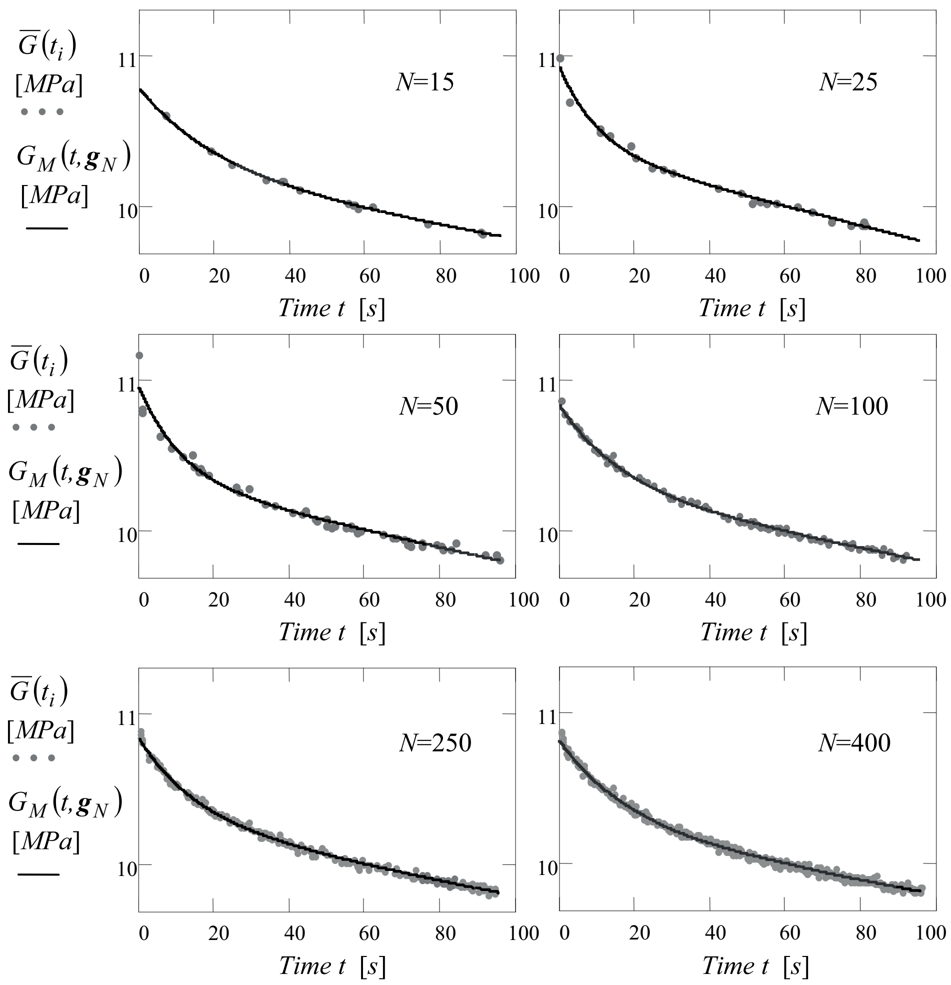

Fig. 7. The relaxation modulus measurements $\overline{G}(t_i)$ (points) and the approximate Maxwell models $G_M(t,\boldsymbol{g}_N)$ (solid line); random choice of the sampling instants

new deeper insight in the stress relaxation experiment can be achieved. The approach proposed lies, in fact, in widely understood data mining framework [15].

4. The paper is concerned with the Maxwell model but the proposed experiment design scheme can also be successfully applied to identification of generalized Kelvin-Voigt model of the creep compliance measurements obtained in creep test [14].

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O PLANOWANIU EKSPERYMENTU DLA IDENTYFIKACJI MODELU MAXWELLA NA PODSTAWIE TESTU RELAKSACJI NAPRĘŻĘŃ

S t r e s z c z e n i e . Przedmiotem pracy jest problem optymalnej, w sensie najmniejszej sumy kwadratów aproksymacji modułu relaksacji materiałów liniowo lepkosprężystych uogólnionym modelem Maxwella na podstawie zakłóconych pomiarów modułu relaksacji zgromadzonych w teście relaksacji naprężeń. Zaproponowano algorytm identyfikacji oparty o koncepcję odpowiedniej randomizacji punktów pomiarowych prowadzący do wyznaczenia modelu Maxwella o parametrach asymptotycznie niezależnych od punktów pomiarowych nawet wówczas, gdy rzeczywisty opis modułu relaksacji jest całkowicie nieznany. Przeprowadzono analizę zbieżności modelu dla dokładnych i zakłóconych pomiarów i wskazano na jego wykładniczą zbieżność. Dla pięcio-parametrowego modelu Maxwella podano wyniki eksperymentów numerycznych. Wyznaczono cztero-parametrowe modele Maxwella próbki buraka cukrowego i pokazano zbieżność ciągu parametrów modelu optymalnego.

S ł o w a k l u c z o w e : test relaksacji naprężeń, model Maxwella, algorytm identyfikacji, planowanie eksperymentu