Identification of relaxation modulus of viscoelastic materials from non-ideal ramp-test histories – problem and method

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Summary. The identification of the linear relaxation modulus of viscoelastic materials on the basis of the stress data from non-ideal ramp tests where a time-variable strain rate is followed by a constant strain is considered. The loading phase strain is described by the third order polynomial of time. The aim of the paper is to develop a method to approximate identification of relaxation modulus using such ramp strain histories. Middle point rule and generalized Simpson rule are used to derive a new method. The approximations of the relaxation modulus at successive time instants are determined on the basis on the stress measurements in at most three appropriately chosen sampling points. The properties of the relaxation modulus model determined according to the proposed method are examined under standard assumptions concerning the relaxation modulus of the material and the experiment. The method developed is a basis for synthesis of fast identification scheme.

Key words: relaxation test, finite ramp-time, relaxation modulus, identification method.

INTRODUCTION

In current engineering practice it is common to deal with either the uniaxial or shear time-dependent relaxation modulus of viscoelastic materials [1-5,7,9,11,12,15-18,27]. For linear viscoelastic materials the relaxation modulus is the stress, which is induced in the material when the unit step strain is imposed. Unfortunately, that deformation mode cannot be achieved experimentally without invoking stress waves [13]. Thus, the relaxation modulus is not directly accessible by means of straightforward measurement method. It is usually recovered from the experimental data of the stress relaxation process history. A common practice to identify the relaxation modulus is still to compute the modulus from the ideal step-strain case rule. Unfortunately, according to the „ten-times-rule", or equivalently, "factor-of-10'rule", this step-strain assumption is acceptable only if the time is at least ten times larger than the initial loading time, see Example given below. Thus, in practice quite often the first seconds of the relaxation data are ignored to account for the finite loading time of deformation [6].

To take into account the finite initial loading time in the real non-ideal relaxation tests a few methods have been proposed during the last several years [10,13,19-21,24-26,28]. Zapas and Phillips [28] developed a general method, which in the case of linear viscoelasticity takes the form of very simple rule, where the 'true' relaxation time is delayed of half loading time. For the case of constant loading rate a few methods for relaxation modulus identification has been proposed: the backward recursive method developed by Lee and Knauss [13], the differential rule proposed by Sorvari and Malinen [19] and the latest method based on the general trapezoidal rule presented in the papers [23, 24].

In practice, however, to inertia effects the assumption that the ramp loading is approximated to be linear may fail [6, 10, 26]. Following Flory and McKenna [6], see also [21, 26], it is assumed in this paper that the initial loading phase strain of the relaxation test is described by the third order polynomial of the time. To develop a fast method to approximate identification of relaxation modulus on the basis of such non-ideal ramp strain history data, in which the relaxation modulus at arbitrary time instant is determined using only few stress measurements, is the goal of the paper. Based on the mathematical properties of the problem and using two known numerical quadrature rules: midpoint rule and generalized Simpson's rule a new identification method is proposed, in which the approximation of the relaxation modulus at arbitrary time is determined on the basis of the stress measurements in at most three appropriately chosen sampling points. It is proved, under mild assumptions concerning the relaxation modulus of the material and the experiment that the resulted model is monotonically decreasing function with at most one discontinuity point.

PROBLEM STATEMENT

1. MATERIAL

We consider a linear viscoelastic material subjected to small deformations for which the uniaxial, nonaging and izotropic stress-strain equation can be represented by a Boltzmann superposition integral [11, 14]:

$$\sigma(t) = \int_{-\infty}^{t} G(t-\lambda)\dot{\varepsilon}(\lambda)d\lambda, \qquad (1)$$

where: $\sigma(t)$ and $\varepsilon(t)$ denotes the stress and stain, respectively, and $G(t)$ is the time-dependent linear (Boltzmann) relaxation modulus. By assumption, the exact mathematical description of the relaxation modulus $G(t)$ is completely unknown, but the value of $\sigma(t)$ can be measured with a certain accuracy for any given value of the time $t \in \mathcal{T}$, where $\mathcal{T} = [0, T]$ and $0 < T < \infty$.

2. EXPERIMENT

A classical manner of studying viscoelasticity for such materials is by two-phase stress relaxation test, where the strain increases during the loading time interval $[0, t_R]$ until a predetermined strain ε_0 is reached at ramp-time t_R, after which that strain ε_0 is maintained constant at that value [13]. In ideal ramp-test [13] the strain increases along a constant strain rate path. However, the constant strain rate in the loading phase is usually unrealizable due to experimental limitations [19, 26]. Following Flory and McKenna [6], see also [21; Ramp III] and [26], we assume that the strain in non-ideal ramp-test is described by the function:

$$\varepsilon(t) = \begin{cases} 0 & for \ \ t < 0 \\ \frac{a}{3}\left(t - \frac{t_R}{2}\right)^3 + bt + c & for \ \ 0 \le t < t_R \\ \varepsilon_0 & for \ \ t \ge t_R \end{cases}, \qquad (2)$$

where: the non-ideal ramp-strain parameters are: $a = -\frac{3}{4}\varepsilon_0\left(\frac{2}{t_R}\right)^3$, $b = \frac{3}{2}\frac{\varepsilon_0}{t_R}$ and $c = -\frac{1}{4}\varepsilon_0$. The strain $\varepsilon(t)$ (2) is shown in Figure 1, where the ideal step-strain $\varepsilon_0(t)$ and the ideal ramp-test strain $\varepsilon_I(t)$ corresponding to linear loading phase strain, are also depicted.

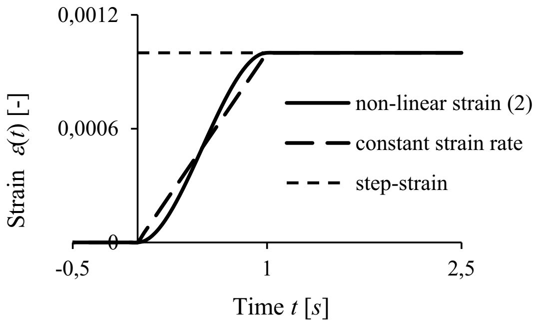

Fig. 1. Step-strain, ideal and non-ideal ramp strain; $t_R = 1$ [s], $\varepsilon_0 = 0{,}001$ [$-$]

3. IDENTIFICATION

Suppose that the non-ideal ramp test (2) performed on the real material resulted in a set of the stress measurements. Identification consists in estimating of the relaxation modulus of viscoelastic material described by the equation (1) using the stress measurements. A common practice is to calculate the relaxation modulus by the rule $G(t) = \sigma(t)/\varepsilon_0$, which in view of (1) is valid only for infinitely short initial loading time (for ideal step-strain). According to the „ten-times-rule" that step-strain assumption is acceptable only for $t \ge 10t_R$. Thus, for the times lower than the ten loading time t_R, the classical rule $G(t) = \sigma(t)/\varepsilon_0$ may fail. To illustrate the errors of such approach the following example is considered.

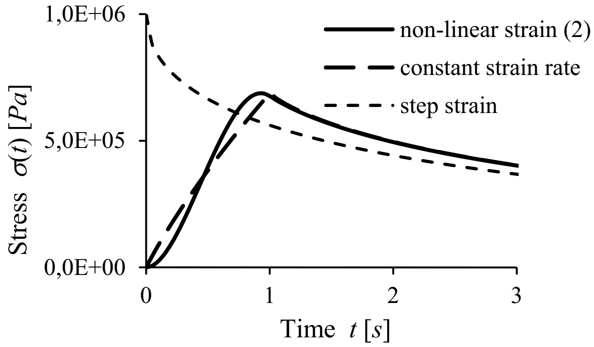

Fig. 2. Stress in ideal and non-ideal ramp-tests and in step-strain relaxation test

Example. Let us consider viscoelastic material whose relaxation modulus is described by the KWW model (Kohlrausch, Williams and Watts) of the form [6, 19]:

$$G(t) = G_0 e^{-(t/\tau)^{\beta}}, \qquad (3)$$

where: following [6] $G_0 = 10^9$ [Pa], the dimensionless parameter $\beta = 0{,}5$ [$-$] and the relaxation time $\tau = 3$ [s]. The strain $\varepsilon_0 = 0{,}001$ [–] and the ramp-time $t_R = 1$ [s]. The stress $\sigma(t)$ resulting for KWW material for non-ideal ramp test (2) is plotted in Figure 2, where the related stress $\sigma_0(t)$ for ideal step-strain relaxation test and the material response $\sigma_I(t)$ after the application of a constant loading strain rate are also given. The differences between the three signals $\sigma(t)$, $\sigma_I(t)$ and $\sigma_0(t)$ are characterized by the relative absolute percent errors summarized in Table 1 for some selected points of time.

The errors are big at short time, and decrease with the time $t > t_R$. The differences for the two signals $\sigma(t)$ and $\sigma_0(t)$ at the time point t_R exceed 20% of the ideal relaxation test stress $\sigma_0(t)$, compare Figure 2, at the

Table 1. Stress differences between the ideal and non-ideal ramp-test and step strain relaxation test

Time t	$t_R/6$	$t_R/4$	$t_R/3$	$t_R/2$	$2t_R/3$	$3t_R/4$	$0{,}95t_R$	t_R	$3t_R$	$5t_R$	$10t_R$	$100t_R$
Errors	Ramp time $t_R = 1\ [s]$											
$\frac{\lvert\sigma(t)-\sigma_0(t)\rvert}{\sigma_0(t)}$ [%]	91,738	82,162	69,844	40,191	9,831	3,165	20,566	20,502	9,296	6,939	4,767	1,458
$\frac{\lvert\sigma(t)-\sigma_I(t)\rvert}{\sigma_I(t)}$ [%]	54,25	35,355	19,313	3,901	14,851	15,602	4,251	1,543	0,118	0,056	0,023	1,64E-3
	Ramp time $t_R = 2\ [s]$											
$\frac{\lvert\sigma(t)-\sigma_0(t)\rvert}{\sigma_0(t)}$ [%]	91,349	81,133	67,844	35,428	1,871	12,511	31,121	30,575	13,45	9,983	6,822	2,07
$\frac{\lvert\sigma(t)-\sigma_I(t)\rvert}{\sigma_I(t)}$ [%]	53,704	34,449	18,085	5,518	16,361	16,813	3,937	2,442	0,199	0,098	0,041	3,13E-3
	Ramp time $t_R = 5\ [s]$											
$\frac{\lvert\sigma(t)-\sigma_0(t)\rvert}{\sigma_0(t)}$ [%]	90,508	78,871	63,377	24,519	16,69	34,432	55,845	53,957	22,271	16,335	11,043	2,07
$\frac{\lvert\sigma(t)-\sigma_I(t)\rvert}{\sigma_I(t)}$ [%]	52,604	32,66	15,667	8,705	19,263	19,063	2,965	4,654	0,417	0,211	0,091	3,13E-3

time $t = 10t_R$ the differences are of 5% degree and they are lesser than 1,5% only at $t > 100t_R$. However, as the two curves approach each other at sufficiently long times greater than $100t_R$, the difference is not negligible as $t \le 100t_R$. Thus, the „ten-times-rule", according to which the relaxation modulus is calculated as $G(t) = \sigma(t)/\varepsilon_0$ for $t \ge 10t_R$, may fail. The errors for the signal $\sigma_I(t)$ are not as big as for $\sigma_0(t)$, but the accuracy in $\sigma(t)$ approximation is also insufficient, especially in short time region $t < t_R$.

Thus, both using the ramp-test data as ideal step-strain data and calculating the relaxation modulus using the formula $G(t) = \sigma(t)/\varepsilon_0$, as well as even applying the known rules derived for ideal ramp test of ε_0/t_R loading rate, leads to unacceptable errors. What is especially important, these errors are unacceptably big in the time intervals of the greatest dynamics of the stress relaxation process. The presented results convincingly prove, that using the ramp test data $\sigma(t)$ as an ideal step-strain data $\sigma_0(t)$ and even as the ideal ramp test data $\sigma_I(t)$, in many cases fails to give satisfactory approximation of the relaxation modulus of the material.

NEW METHOD

In this paper the following assumption will be taken.
Assumption. The relaxation modulus $G(t)$ is double differentiable function such that:

$$G(t) \ge 0, -\frac{dG(t)}{dt} \ge 0,\ \frac{d^2G(t)}{dt^2} \ge 0 \quad for \quad t > 0. \tag{4}$$

The above assumption seems to be quite natural. In particular, it takes account of the course of the experimentally recorded relaxation modulus. This assumption, taken for example in [8, 20, 22], is satisfied by commonly used rheological models, such that Maxwell, Zener, KWW and Peleg models. Note also that from (4) it follows immediately that $G(t)$ strictly monotonically non-increasing (decreasing) continuous convex function.

On the basis of (1) and (2), taking into account that $\dot{\varepsilon}(\lambda) = 0$ for any $\lambda < 0$, the stress during the initial loading phase of the stress relaxation test in the interval of time $0 < t < t_R$ is described by the following equation:

$$\sigma(t) = \int_0^t G(t-\lambda)\left[a\left(\lambda - \frac{t_R}{2}\right)^2 + b\right]d\lambda,$$

which, taking into account the definitions of the parameters a and b, can be rewritten as:

$$\sigma(t) = \int_0^t \Phi(\lambda, t, t_R)d\lambda, \tag{5}$$

where the integrand:

$$\Phi(\lambda, t, t_R) = aG(t-\lambda)\lambda(\lambda - t_R), \tag{6}$$

is introduced for brevity. We approximate the integral (6) by respective quadrature. Unfortunately, no general methods can be recommended for numerical integration. The choice of the suitable method must be done on a case-by case basis, depending on the integrand function properties. Let us notice, that $\Phi(0, t, t_R) = 0$, and since $a < 0$, then for an arbitrary $0 < \lambda < t_R$ we have $\Phi(\lambda, t, t_R) > 0$; here $t_R > 0$, $0 < t \le t_R$. The partial derivative is given by the expression:

$$\frac{\partial\Phi(\lambda,t,t_R)}{\partial\lambda} = -a\dot{G}(t-\lambda)\lambda(\lambda - t_R) + aG(t-\lambda)(2\lambda - t_R).$$

Thus, in view of the Assumption, it is clear that if $\lambda < t_R/2$, then $\partial\Phi(\lambda, t, t_R)/\partial\lambda > 0$ and $\partial\Phi(\lambda, t, t_R)/\partial\lambda \ge 0$ at $\lambda = t_R/2$ for any time variable $0 < t \le t_R$. Hence, if $t \le t_R/2$, that is $\lambda \le t_R/2$, then the integrand $\Phi(\lambda, t, t_R)$ is monotonically increasing function of λ. The course of an exemplary integrand $\Phi(\lambda, t, t_R)$ as a function of the variable $0 \le \lambda \le t$ for KWW material (3), ramp-time $t_R = 1\ [s]$ and time instant $t = \frac{2}{5}t_R$ is show in Figure 3(a). Thus the simplest method of numerical integration, the midpoint rule, is appropriate for numerical approximation of the integral

(5) for $t \leq t_R/2$. Moreover, it is known that for Volterra's equation of the first kind (1), the midpoint rule is numerically stable. By applying the middle-point rule to integral of the right-hand side of (5) we obtain:

$$\sigma(t) \approx aG\left(\frac{t}{2}\right)\frac{t}{2}\left(\frac{t}{2} - t_R\right)t,$$

whence, for $0 < t \leq t_R/2$, the following expression follows immediately:

$$G^{(NM)}\left(\frac{t}{2}\right) = \frac{{t_R}^3}{3\varepsilon_0\left(t_R - \frac{t}{2}\right)t^2}\sigma(t).$$

It is easy to verify that the above implies for $0 < t \leq t_R/4$ the rule:

$$G^{(NM)}(t) = \frac{{t_R}^3}{12\varepsilon_0(t_R - t)t^2}\sigma(2t). \qquad (7)$$

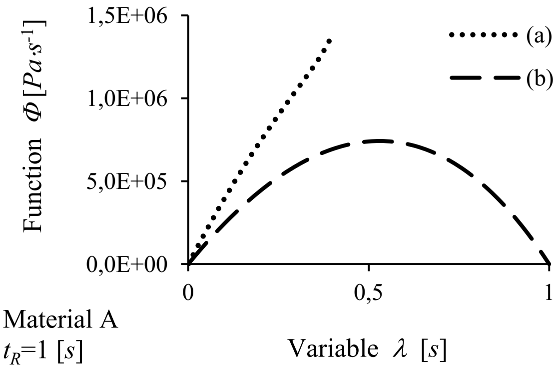

Fig. 3. The function $\Phi(\lambda, t, t_R)$ defined by equation (6) for material (3), $t_R = 1$ [s]: (a) $t = \frac{2}{5}t_R$, (b) $t = 2t_R$, $0 \leq \lambda \leq t$

We now wish to find the formula for relaxation modulus identification for the time interval $t > t_R/4$. To do this, note that on the basis of equations (1) and (2) in the second constant strain phase of the relaxation test, i.e. for $t > t_R$ the stress is given by the expression:

$$\sigma(t) = a\int_0^{t_R} G(t-\lambda)\lambda(\lambda - t_R)d\lambda, \qquad (8)$$

or in equivalent form:

$$\sigma(t) = \int_0^{t_R} \Phi(\lambda, t, t_R)d\lambda, \qquad (9)$$

where the integrand $\Phi(\lambda, t, t_R)$ is given by (6). Now, for an arbitrary $t > t_R$ also for the upper limit of integration in (9) we have $\Phi(t_R, t, t_R) = 0$. The function $\Phi(\lambda, t, t_R)$ is continuous and non-negative definite for any $\lambda \leq t_R$ and, as it may be easily verified on the basis of the stationary point condition $\dot{G}(t-\lambda)\lambda(\lambda - t_R) = G(t-\lambda)(2\lambda - t_R)$ achieves the maximum for λ_{max} such that $t_R/2 < \lambda_{max} < t_R$. An example of the integrand function $\Phi(\lambda, t, t_R)$ for material (3), the time instant $t = 2t_R$ as a function of variable $0 \leq \lambda \leq t$ is illustrated in Figure 3(b). The paraboidal nature of this function makes reasonable the choice of Simpson's rule to evaluate the integral (8). We use both the simple three-point and the generalized five-point rule. Applying simple Simpson's rule we obtain the formula:

$$\sigma(t) \approx \frac{4}{3}aG\left(t - \frac{t_R}{2}\right)\frac{t_R}{2}\left(\frac{t_R}{2} - t_R\right)\frac{t_R}{2},$$

and whence:

$$\sigma(t) \approx \varepsilon_0 G\left(t - \frac{t_R}{2}\right), \qquad (10)$$

which is the classical Zapas-Phillips formula. By dividing the finite time interval $[0, t_R]$ into four equal subintervals and applying the generalized Simpson's rule, after simple algebraic manipulations we have:

$$\sigma(t) \approx -\frac{a}{48}{t_R}^3\left[3G\left(t - \frac{t_R}{4}\right) + 3G\left(t - \frac{3t_R}{4}\right) + \right.$$
$$\left. + 2G\left(t - \frac{t_R}{2}\right)\right],$$

whence, in view of the definition of the parameter a, we finally obtain:

$$\sigma(t) \approx \frac{1}{8}\varepsilon_0\left[3G\left(t - \frac{t_R}{4}\right) + 3G\left(t - \frac{3t_R}{4}\right) + \right.$$
$$\left. + 2G\left(t - \frac{t_R}{2}\right)\right]. \qquad (11)$$

Similar to (11), we have:

$$\sigma\left(t + \frac{t_R}{4}\right) \approx \frac{1}{8}\varepsilon_0\left[3G(t) + 3G\left(t - \frac{t_R}{2}\right) + \right.$$
$$\left. + 2G\left(t - \frac{t_R}{4}\right)\right]. \qquad (12)$$

Combining the expressions (10), (11) and (12) treated as the equalities and applying the next equations, which follows from (10):

$$\sigma\left(t + \frac{t_R}{2}\right) = \varepsilon_0 G(t),$$

$$\sigma\left(t + \frac{t_R}{4}\right) = \varepsilon_0 G\left(t - \frac{t_R}{4}\right),$$

after simple algebraic manipulations we obtain:

$$G^{(NM)}\left(t - \frac{3t_R}{4}\right) = \frac{8}{3\varepsilon_0}\sigma(t) - \frac{7}{3\varepsilon_0}\sigma\left(t + \frac{t_R}{4}\right) +$$
$$+ \frac{2}{3\varepsilon_0}\sigma\left(t + \frac{t_R}{2}\right), \qquad (13)$$

which is the desired result. Thus we have achieved the formula for relaxation modulus approximate identification for $t > t_R/4$.

MONOTONICITY OF THE MODEL

It is assumed here that relaxation modulus is monotonically decreasing function. The monotonicity of the relaxation modulus model obtained by the proposed method is resolved by the two consecutive properties.

Property 1. If the Assumption is satisfied, the stress measurements are noise-free and for any $0 < t \leq t_R/2$ the following inequality holds:

$$2\dot{G}(t) + \ddot{G}(t)t \leq 0, \qquad (14)$$

then the relaxation modulus model $G^{(NM)}(t)$ is monotonically decreasing function in the time interval $0 < t \leq \frac{1}{4}t_R$.

Proof. On the basis of (7) we have:

$$\dot{G}^{(NM)}(t) = \frac{{t_R}^3}{12\varepsilon_0}\frac{2\dot{\sigma}(2t)(t_R-t)t-\sigma(2t)(2t_R-3t)}{(t_R-t)^2t^3}. \qquad (15)$$

In order to examine the monotonicity of the model $G^{(NM)}(t)$ (7) it is enough to check the sign of the numerator of right-hand side of (15), i.e. of the expression:

$$\Psi(t) = 2\dot{\sigma}(2t)(t_R - t)t - \sigma(2t)(2t_R - 3t). \quad (16)$$

Taking into account (5) and (6), after suitable change of variables, we obtain:

$$\sigma(t) = a\int_0^t G(w)(t-w)(t-w-t_R)dw. \quad (17)$$

Whence, on the basis of known Leibnitz theorem concerning the differentiation of the integral with the limits depending on the variable we have:

$$\dot{\sigma}(t) = a\int_0^t G(w)(2t-2w-t_R)dw. \qquad (18)$$

Using (17) and (18) we can rewrite the function $\Psi(t)$ (16) as follows:

$$\Psi(t) = a(4t^3 + 2tt_R^2 - 4t^2t_R)\psi_1(t) + a(7tt_R - 8t^2 - 2t_R^2)\psi_2(t) + a(3t - 2t_R)\psi_3(t), \qquad (19)$$

where: $\psi_1(t) = \int_0^{2t} G(w)dw$, $\psi_2(t) = \int_0^{2t} G(w)wdw$ and $\psi_3(t) = \int_0^{2t} G(w)w^2dw$.

By convexity of the function $G(t)$ we have for an arbitrary t and w:

$$G(w) \geq G(t) + \dot{G}(t)(w - t).$$

Hence, the next inequality follows:

$$\psi_1(t) \geq 2tG(t) + \dot{G}(t)\int_0^{2t}(w-t)dw = 2tG(t). \quad (20)$$

In order to estimate the integral $\psi_2(t)$ it is enough to note that under the Assumption the function $G(w)w$ is concave. Thus, for an arbitrary t and w such that $w \leq 2t \leq t_R/2$ the following inequality holds:

$$G(w)w \leq G(t)t + \left[G(t) + \dot{G}(t)t\right](w - t), \quad (21)$$

on the basis of which, we obtain the upper bound:

$$\psi_2(t) \leq 2t^2G(t) + \left[G(t) + \dot{G}(t)t\right]\int_0^{2t}(w-t)dw = 2t^2G(t)$$

and therefore, taking into account the inequality (20), we have:

$$\psi_2(t) \leq t\psi_1(t). \qquad (22)$$

In a similar fashion, using once more the inequality (21), it may be proved that the next inequality holds:

$$\psi_3(t) \leq \tfrac{4}{3}t^2\psi_1(t). \qquad (23)$$

Since the polynomial $(7tt_R - 8t^2 - 2t_R^2)$ of variable t is negative definite for any $0 < t \leq t_R/4$ and the expression $4t^3 + 2tt_R^2 - 4t^2t_R = 2t[t^2 + (t - t_R)^2]$ is positive definite for any $t > 0$, taking into account that the parameter $a < 0$, by combining (19), (22) and (23) we obtain for an arbitrary $0 < t \leq t_R/4$ the following estimation:

$$\Psi(t) \leq \tfrac{1}{3}at^2t_R\psi_1(t) < 0.$$

Which finally concludes the proof.

Remark 1. It is easy to check that for KWW model (3) the condition (14) takes the form $(t/\tau)^\beta - 1 \leq 1/\beta$, therefore is satisfied for every $0 < t \leq t_R/2$, whenever $(t_R/2\tau)^\beta \leq 1/\beta + 1$. In particular, if $\beta = 0{,}5$ (see Example), then the condition (14) means that $t_R \leq 18\tau$, and is not difficult to satisfy.

Remark 2. For Maxwell model $G(t) = G_0e^{-t/\tau}$ we have $\beta = 1$. Thus, the condition (14) is satisfied, whenever $t_R \leq 4\tau$.

Having known even rough estimation of the relaxation time of the material we can choose without difficulties the ramp-time t_R so as to satisfy the condition (14).

Property 2. If the Assumption is satisfied and the stress measurements are noise-free, then the relaxation modulus model $G^{(NM)}(t)$ is monotonically decreasing function in the time interval $\frac{1}{4}t_R < t \leq T - \frac{5}{4}t_R$.

Proof. Let us first examine the stress derivative $\dot{\sigma}(t)$ for $t > t_R$. On differentiating formula (8) we arrive at the following expression:

$$\dot{\sigma}(t) = a\int_0^{t_R}\dot{G}(t-\lambda)\lambda(\lambda - t_R)d\lambda$$

and then, on differentiating equation (8) with respect to t twice, we obtain:

$$\ddot{\sigma}(t) = a\int_0^{t_R}\ddot{G}(t-\lambda)\lambda(\lambda - t_R)d\lambda.$$

Since the parameter $a < 0$ and the ramp-time $t_R > 0$, in view of the Assumption, the above implies the following inequalities: $\dot{\sigma}(t) < 0$ and $\ddot{\sigma}(t) > 0$. Thus, for any $t > t_R$ the stress derivative $\dot{\sigma}(t)$ is negative definite monotonically increasing function. Since on the basis of (13):

$$\dot{G}^{(NM)}\left(t-\frac{3t_R}{4}\right)=\frac{8}{3\varepsilon_0}\dot{\sigma}(t)-\frac{7}{3\varepsilon_0}\dot{\sigma}\left(t+\frac{t_R}{4}\right)+$$
$$+\frac{2}{3\varepsilon_0}\dot{\sigma}\left(t+\frac{t_R}{2}\right),$$

the above implies:

$$\dot{G}^{(NM)}\left(t-\frac{3t_R}{4}\right)<\frac{8}{3\varepsilon_0}\dot{\sigma}\left(t+\frac{t_R}{4}\right)-\frac{7}{3\varepsilon_0}\dot{\sigma}\left(t+\frac{t_R}{4}\right)+$$
$$+\frac{2}{3\varepsilon_0}\dot{\sigma}\left(t+\frac{t_R}{2}\right)<0,$$

and completes the proof.

FINAL REMARKS

1. Based on the mathematical properties of the considered problem of the relaxation modulus determination using the stress measurements from non-ideal ramp test, new method for approximate identification of relaxation modulus is derived for the case when the time-variable strain in the loading phase of the relaxation test is described by the time polynomial of the third order.
2. It is proved, under quite typical assumptions concerning both the viscoelastic material and the experiment, that for noise-free stress measurements determined model of the relaxation modulus is monotonically decreasing time-function with at most one discontinuity point.
3. For discrete-time experiment the proposed method can be used to develop a new fast identification scheme, in which the approximations of the relaxation modulus in successive time instants are computed using at most three measurements of the stress in appropriately chosen sampling points. Numerical studies of the algorithm, in particular the model error analysis for ideal and noise corrupted stress measurements, is the subject of the forthcoming paper [25].

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IDENTYFIKACJA MODUŁU RELAKSACJI MATERIAŁÓW LEPKOSPRĘŻYSTYCH NA PODSTAWIE RZECZYWISTEGO TESTU RELAKSACJI NAPRĘŻEŃ O NIELINIOWYM ODKSZTAŁCENIU WSTĘPNYM. PROBLEM I METODA

Streszczenie. W pracy rozważa sie problem wyznaczania modułu relaksacji materiałów liniowo lepkosprężystych na podstawie pomiarów naprężenia zgromadzonych w rzeczywistym teście relaksacji naprężeń o nieliniowym odkształcaniu próbki w fazie wstępnej testu opisanym wielomianem czasu trzeciego stopnia. Celem pracy jest opracowanie metody przybliżonej identyfikacji modułu relaksacji na podstawie danych z takiego testu. Bazując na regule punktu środkowego oraz uogólnionej formule Simpsona opracowano metodę identyfikacji, w której przybliżenie modułu relaksacji w dowolnej chwili czasu wyznaczane jest na podstawie pomiarów naprężenia w co najwyżej trzech wybranych punktach jego próbkowania. Zbadano, przy bardzo ogólnych założeniach dotyczących modułu relaksacji badanego materiału oraz eksperymentu, własności wyznaczonego modelu. Opracowana metoda jest punktem wyjścia dla syntezy szybkiego algorytmu identyfikacji modułu relaksacji podczas trwania eksperymentu.

Słowa kluczowe: test relaksacji naprężeń, skończony czas odkształcania, moduł relaksacji, algorytm identyfikacji.