TEKA. COMMISSION OF MOTORIZATION AND ENERGETICS IN AGRICULTURE – 2016, Vol. 16, No. 4, 49–56

Analytical Method for Determining Porosity in the Layer of Vibro Separated Grain Mixture

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Received October 01.2016; accepted October 03.2016

Summary. The formulas have been entered and approved for the calculation of porosity distribution on the thickness of layer of fine-grained mixture during its separation by the inclined flat or vertical cylinder vibro sieves. It has been attained as a result of approximation of tabular information of the obtained numerical computer integration of the specially worked out nonlinear differential equations of the second order in a dimensionless form. For approximation, the function of degree coefficients and index is used for the degrees which are certain by the Aitken's method. Coefficients of the entered analytical dependence are the vibro sieves related to the parameters obtained by mechanical descriptions of the separated material. Coefficients of the entered analytical dependence are related to the parameters of vibro sieves and mechanical descriptions of the separated material. In the case of cylinder vertical vibro sieve the action of centrifugal force is also taken into account. The method of mixture porosity calculation does not need a computer numerical integration of nonlinear differential equations conducted by other authors for solving this problem. Comparison of numerical results of the proposed analytical method of calculation with the ones described in literature, have confirmed its high accuracy results, for the differences do not exceed one percent. The expounded method is universal enough and simple in use, besides it opens the possibilities of subsequent analytical integration of differential equalizations of motion at the calculation of kinematics descriptions of grain flow. The developed method gives the opportunity to also solve the inverse task when, according to experimental measurements of porosity values of grain mixtures on the thickness of movable separated layer, it is needed to find the value of phenomenological permanent that is included in the expressions of coefficients of initial differential equalization. In this way, the adequacy of the mathematical model is improved. The use of approximation of degree considerably simplifies the method of authentication of differential equalization coefficients. In the article, the examples of grain mixture porosity calculation as well as the examples of phenomenological permanent authentication have been resulted after experimental calculations for both the variants of vibro sieves.

Key words: vibro sieve separation, grain mixture layer, porosity calculation, analytical method, approximation of degree, authentication of phenomenological permanent.

INTRODUCTION

The important stage of post-harvest treatment of grain is cleaning it from admixtures and separation into fractions. This is achieved as a result of separation of grain mixture on vibro sieves. The intensity of separation process depends on the porosity of grain mixture. At large porosity the separation of sifted fraction is accelerated but the mass productivity of sieve diminishes on fraction stair. In the case of small porosity there is an opposite tendency, i.e. the process of segregation is slowed and the percent of sifted faction diminishes. In such cases, the special activators of segregation process are used. Consequently, the efficiency of the use of grain cleaning technique depends on the mixture porosity in connection with how its determination relates to the current scientific and technical tasks.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

At the design of motion separated grain mixture layer, the recent publications have taken into account the effect of porosity or specific mass changes on the thickness of mobile layer. It involves both the separation on flat vibro sieves inclined to horizon [1–6] and the separation in the vertical cylinder sieves of vibro centrifuges [2, 7–9]. Taking into account the effect of porosity or concentration of grains change on the thickness of layer they have summarized the known hydrodynamic models of homogeneous grain flows [10–13]. This applies to the theories set out in the monographs [14-16] reviewed in the relevant literature. For determination of porosity change, the vibrations of sieve and action of force of gravitation or centrifugal force in the sieve of vibro centrifuge were integrated in the mentioned publications by the special nonlinear differential equalization of the second order numerical methods. It is unconnected with the rate of movement of mixture, that is why it can be treated separately, as an autonomous equalization, without integration of differential equalization of motion.

Using approximation of degree, the close analytical decision of the task of porosity distribution in the layer of grain mixture is made here, in dependence of steel degree determination by Aitken's method [17].

OBJECTIVES

The purpose of work was the development and approval of analytical method of close calculation of porosity in the layer of vibro separated grain mixture, which would not need numerical integration of the special nonlinear differential equalization.

For achievement of the put purpose the known tabular information has been used from [18, 19] and the approximation of degree.

Other similar determinations of specific mass distribution of grain mixture in its mobile layer have been described in [20, 21].

THE MAIN RESULTS OF THE RESEARCH

1. Heterogeneous layer of grain mixture was placed on a flat vibro sieve, inclined to horizon. The calculation scheme of the vibro sieve is presented in Fig. 1.

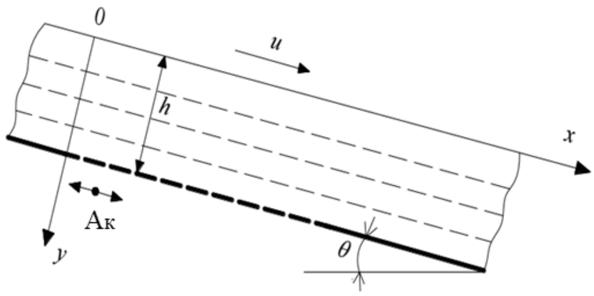

Fig. 1. Calculation scheme of flat vibro sieve

Here $A_к$ - amplitude of longitudinal vibrations of sieve with frequency ω; h - thickness of mobile layer of mixture; θ - angle of slope of sieve.

According to [1–3], change of concentration of caryopsides on a coordinate in y ($v = v(y)$) was described by differential equation:

$$\frac{d}{dy}\left[\alpha\psi\left(\frac{dv}{dy}\right)^2\right] - \gamma g \cos\theta v = 0\,, \quad (1)$$

at initial conditions:

$$v(0) = v_0\,;\ \left.\frac{dv}{dy}\right|_{y=0} = 0\,. \quad (2)$$

In (1), (2): $\psi = \frac{\sqrt{1+f^2} - f}{f}$,

$$f = \frac{f_0}{2}\left(1 + e^{-B}\right);\ B = \frac{A_k\omega^2}{g}\,,$$

f_0 - coefficient of internal dry friction in the mixture in a state of rest; g - free fall acceleration; γ - specific mass of material of grain; α - phenomenological constant the value of which is associated with the state of the surface of sieves (presence of ribs, riffles etc.).

Passing to dimensionless variables in [18] Cauchy's problem (1), (2) a formula is given:

$$\frac{d^2\zeta}{d\eta^2} = \frac{\zeta}{\frac{d\zeta}{d\eta}}\,, \quad (3)$$

$$\zeta(0) = 1\,;\ \left.\frac{d\zeta}{d\eta}\right|_{\eta=0} = 0\,,$$

where: $\zeta = \frac{v}{v_0}$; $\eta = \sqrt[3]{\frac{\beta}{v_0}}y$; $\beta = \frac{\gamma g \cos\theta}{2\alpha\psi}$.

By numerical integration (3) the table of ζ values was made for different η [18]. Approximation was obtained by tabular findings by the function of degree results by Aitken's method [17] in the analytical dependence:

$$v = v(y) = v_0\left(1 + 1{,}028\eta^{1{,}579}\right). \quad (4)$$

To find out the errors of approximation, calculate $v(y)$ at v_0=0,325 and other η. The results of calculations v on formula (4) are written in the numerators of Table 1, and in denominators, for comparison, v is indicated, obtained with the tabular method in [18].

Table 1. Values v, got two methods

η	0	0,116	0,233	0,349	0,465
v	$\frac{0{,}325}{0{,}325}$	$\frac{0{,}336}{0{,}337}$	$\frac{0{,}358}{0{,}360}$	$\frac{0{,}388}{0{,}389}$	$\frac{0{,}425}{0{,}425}$

Divergences between numerators and denominators are unimportant here, which confirms the efficiency of analytical method of calculation.

Let us consider further, how to use formula (4) for authentication of value α. At the thickness of mixture layer h, the concentration of caryopses on a free topside is evened v_0, and on a lower surface near a sieve evened v_*. Then $\zeta_* = {v_*}/{v_0}$ and by formula (4):

$$1 + 1{,}028\eta_*^{1{,}579} = \zeta_*$$

or:

$$\eta_* = \exp\left(\frac{1}{1{,}579}\ln\frac{\zeta_* - 1}{1{,}028}\right) = \sqrt[3]{\frac{\beta}{v_0}}h.$$

From where we get:

$$\beta = v_0\left(\frac{\eta_*}{h}\right)^3 = \frac{\gamma g\cos\theta}{2\alpha\psi},$$

$$\alpha = \frac{\gamma g\cos\theta}{2v_0\psi}\left(\frac{h}{\eta_*}\right)^3.$$

As an example, let us calculate α, when: γ=1350 kg/m^3; θ=6^0; h=0,01 m; v_0 =0,325; η_* =0,465; f_0 =0,47; A_k =0,0075 m; ω =41,86 s^{-1}. For these initial data:

B=1,340; f =0,297; ψ =2,512. The substitution of the obtained values of constants in formula (5) gives:

$$\alpha = \frac{1350\cdot 9{,}81\cdot 0{,}995}{2\cdot 0{,}325\cdot 2{,}512}\left(\frac{0{,}01}{0{,}465}\right)^3 \approx 0{,}08H\cdot$$

Consequently, after experimental determination $\zeta_* = v_*/v_0$, authentication α is taken to the simple calculations.

2. For a heterogeneous layer of grain mixture in the vibro sieve cylinder, the calculation scheme of vibro sieve is presented in Fig. 2.

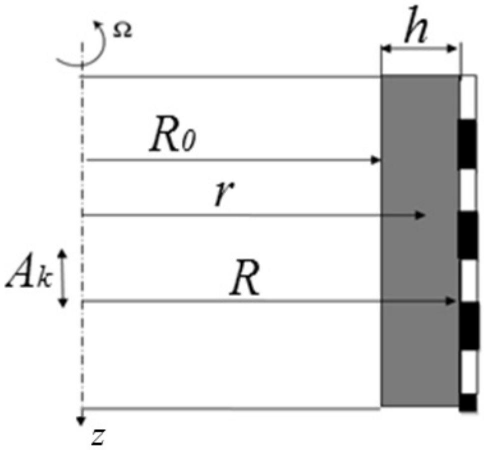

Fig. 2. Design scheme of vertical cylindrical sieve with separated mixture

In the case of vertical cylinder vibro sieve, the distribution of caryopses concentration on the thickness of mobile layer of mixture (coordinate *r*) is described by differential equation [2, 7, 8]:

$$\frac{d}{dr}\left[\alpha(\Phi+2)\left(\frac{dv}{dr}\right)^2\right] + \frac{2}{r}\alpha\left(\frac{dv}{dr}\right)^2 - \gamma\Omega^2 rv = 0, \quad (6)$$

at initial conditions:

$$v(R_0) = v_0; \left.\frac{dv}{dr}\right|_{r=R_0} = 0.$$

The size of Φ is determined by the formula:

$$\Phi = \left(\sqrt{1+f^2} - f\right)/f,$$

$$\text{де } f = \frac{f_0}{2}\left(1 + e^{-G}\right); \; G = \frac{A_k \omega^2}{R\Omega^2},$$

and permanent α, as well as before, depends on the state of sieve surface.

As equation (6) is nonlinear with variable coefficients, it will integrated by numerical methods on a computer [2, 7, 8].

But due to the small thickness of mixture layer, in comparison to the radius of vibro sieve, equation (6) possible for simplification, replacing variable coefficients by averaged values on the interval of integration $r \in [R_0, R]$. After such simplification, the Cauchy's problem is given expressions [19]:

$$\frac{d^2\zeta}{d\eta^2} + a\frac{d\zeta}{d\eta} - \frac{\zeta}{\frac{d\zeta}{d\eta}} = 0, \qquad (7)$$

$$\zeta(0) = 1; \; \left.\frac{d\zeta}{d\eta}\right|_{\eta=0} = 0.$$

Тут

$$\zeta = \frac{v}{v_0}; \; \alpha = \frac{1}{\lambda(\Phi + 2)(1 + 0{,}5h/R_0)};$$

$$\lambda = R_0 \sqrt[3]{\frac{\gamma\Omega^2 (R_0 + 0{,}5h)}{2\alpha v_0 (\Phi + 2)}};$$

$$\eta = \lambda\left(\frac{r}{R_0} - 1\right).$$

The numerical integration of equation (7) shows that the function $\zeta = \zeta(\eta, a)$ very poorly depends on a, because of the real terms of separating coefficient a<<1. Therefore, the results of integration actually taken to the table are also the approximated expression (4). But now η will be calculated by another formula (8), using sizes λ and r.

With the purpose of verification of adequacy of formulas (4) and (8) calculation is conducted $v(r)$ at $v_0 = v(R_0) = 0{,}367$ and different η. The obtained results are written in numerators in Table. 2.

Table 2. Values $v(r^*)$, where $r^* = (r - R_0)/h$

r^*	0	0,25	0,5	0,75	1
η	0	0,085	0,170	0,255	0,34
v	$\frac{0{,}367}{0{,}367}$	$\frac{0{,}375}{0{,}376}$	$\frac{0{,}390}{0{,}391}$	$\frac{0{,}411}{0{,}412}$	$\frac{0{,}436}{0{,}437}$

It is indicated $v(r^*)$ in denominators, calculated by a tabular method in [19]. The coordination of results is good, which confirms the adequacy of approximation (4) and (8).

Let us further check as approximate formulas (4) and (8), the results are given in continuous graphs in Fig. 3. This Fig. is printed in [8] (Fig. 3).

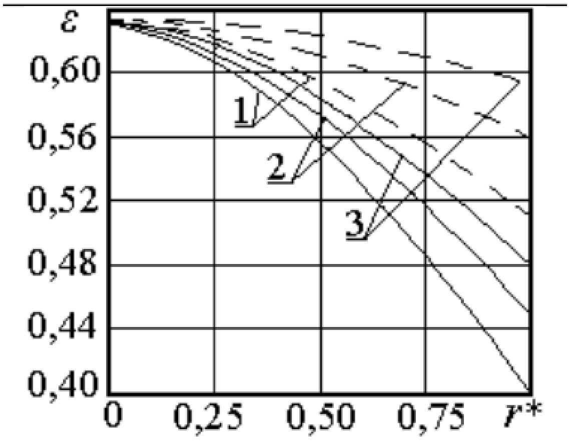

Fig. 3. Dependences of grain mixture porosity ε on the layer depth r^*, loading q and constructional parameters of ribs: 1 – serial sieve; 2 – attitude of diameter is toward length of rib 0,024; 3 – attitude of diameter is toward length of rib 0,07; – – – q=110 kg/h·dm^2; ——— – q=180 kg/h·dm^2

Porosity of mixture $\varepsilon = \varepsilon(r)$ is calculated by formula:

$$\varepsilon(r) = 1 - v(r).$$

In accordance with the expounded method:

$$v(r^*) = v_0\left\{1 + 1{,}028\left[r^* \exp\left(\frac{1}{1{,}579}\ln\frac{v_*/v_0 - 1}{1{,}028}\right)\right]^{1{,}579}\right\},$$

thus:

$$\nu_0 = \nu(R_0);\ \nu_* = \nu(R);\ r^* = \frac{r - R_0}{h}.$$

As can be seen, for the calculation of distribution $\nu_* = \nu(R)$ and $\varepsilon = \varepsilon(r^*)$ it is needed only to set the relation ν_*/ν_0. This is made: for graph 1 - ν_*/ν_0 =1,635; for graph 2 - ν_*/ν_0 =1,507; for graph 3 - ν_*/ν_0 =1,417, and ν_0 =0,367. The obtained $\nu(r^*)$ and $\varepsilon(r^*)$ for separate r^* are indicated in Table 3.

Table 3. Approximated values of porosity ε, compared to the continuous graphs

r^*	graph 1		graph 2		graph 3	
	$\nu(r^*)$	$\varepsilon(r^*)$	$\nu(r^*)$	$\varepsilon(r^*)$	$\nu(r^*)$	$\varepsilon(r^*)$
0	0,367	0,633	0,367	0,633	0,367	0,633
0,25	0,393	0,607	0,388	0,612	0,384	0,616
0,5	0,445	0,555	0,439	0,571	0,418	0,582
0,75	0,515	0,485	0,485	0,515	0,464	0,536
1	0,6	0,4	0,553	0,447	0,520	0,480

The results indicated in this table, as separate points, deviate little from continuous graphs in Fig. 3.

Let us consider that approximation (4), (8) suitable also for the authentication of rheological permanent α. Let us calculate it, when: $A_k = 0{,}006$ m; $\omega = 96$ s^{-1}; R=0,3075 m; Ω=11,77 s^{-1}; $f_0 = 0{,}47$; h=0,014 m; γ=1350 kg/m^3; $\nu_0 = 0{,}367$; $\nu_* = 0{,}497$. Assume that $\nu_0 = \nu(R_0)$ and $\nu_* = \nu(R)$ are certain experimentally. For them, according to (4):

$$\zeta_* = 1 + 1{,}028\eta_*^{1{,}579} = \frac{\zeta_*}{\zeta_0} = 1{,}354,$$

where:

$$\eta_* = \exp\left(\frac{1}{1{,}579}\ln\frac{1{,}354 - 1}{1{,}028}\right) = 0{,}509.$$

Then, using formula (8), we get:

$$\eta_* = \lambda\frac{h}{R_0} = h\sqrt[3]{\frac{\gamma\Omega^2(R_0 + 0{,}5h)}{2\alpha\nu_0(\Phi + 2)}}.$$

Deciding this equation relatively to α, we find that:

$$\alpha = \frac{\gamma\Omega^2(R_0 + 0{,}5h)}{2\nu_0(\Phi + 2)}\left(\frac{h}{\eta_*}\right)^3. \qquad (9)$$

For the accepted numeric data: $G = 1{,}298$; $f = 0{,}299$; $\Phi = 2{,}491$. The substitution of the proper numbers in formula (9) gives:

$$\alpha = \frac{1350 \cdot 11{,}77^2 0{,}3005}{2 \cdot 0{,}367 \cdot 4{,}491}\left(\frac{0{,}014}{0{,}509}\right)^3 = 0{,}355H.$$

The obtained result well conforms to that in [19], where α =0,353 H.

CONCLUSIONS

The conducted calculations and comparative analysis of numerical results have proved that the solution offered is close to the analytical method of calculation of porosity in the mobile layer of separated mixture and is universal enough, has a high accuracy level and is comfortable to use. It also enables to find the value of phenomenological permanent in the initial differential equation as a result of the experimental measuring of values of porosity in two points of grain mixture layer.

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Аналитический способ определения пористости в слое вибросепарированной зерносмеси.

Аннотация. Выведено и апробировано приближенные формулы для расчета распределения пористости по толщине слоя мелкозернистой смеси, при ее сепарировании наклонным плоским или вертикальным цилиндрическим виброрешетами. Это достигнуто в результате аппроксимации табличных данных, полученных численным компьютерным интегрированием специально составленного дифференциального уравнения второго порядка в безразмерной форме. Для аппроксимации использована степенная функция, коэффициенты которой и показатель степени определены методом Эйткена. Коэффициенты введенной аналитической зависимости связаны с параметрами виброрешета и механическими характеристиками сепарированного материала. В случае цилиндрического вертикального виброрешета учтено также действие центробежной силы. Разработанный метод расчета пористости смеси не требует численного компьютерного интегрирования нелинейных дифференциальных уравнений, что приводили другие авторы при решении этой задачи. Сравнение численных результатов, к которым приводит предложенный аналитический способ расчета, с опубликованными в литературе, подтвердило его высокую точность, ибо расходимость результатов не превышает одного процента. Изложенный способ достаточно универсален и прост в использовании, к тому же он открывает возможность дальнейшего аналитического интегрирования дифференциальных уравнений движения при вычислении кинематических характеристик зернопотока. Разработанный способ дает возможность также решать обратную задачу, когда по данным экспериментальных измерений значений пористости зерносмеси по толщине движущегося сепарируемого слоя, нужно найти значение феноменологической постоянной, которая входит в выражения коэффициентов исходного дифференциального уравнения. Таким путем достигается улучшение адекватности математической модели. Использование степенной аппроксимации значительно упрощает проведение идентификации коэффициентов дифференциального уравнения. В статье приведены примеры расчета пористости зерносмеси и примеры идентификации феноменологической постоянной по экспериментальным результатам, для обоих вариантов виброрешет.

Ключевые слова: виброрешетное сепарирование, слой зерносмеси, расчет пористости, аналитический способ, степенная аппроксимация, идентификация феноменологической постоянной.