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Bézier curves in modeling the shapes of biological objects

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Abstract*: Bézier curves in modeling the shapes of biological objects.* The paper presents characteristic shapes of the Bézier curves used in modeling the biological objects. Notation of polynomial curves of second, third and fourth order in Bézier representation with application of Bernstein basic functions is presented. The use of polynomials of third order in Bézier representation in modeling of biological objects of complex shapes is recommended. For modeling of fruit and vegetable shapes (3*D* solids) a meridional sphere was proposed, with Bézier curves as meridians. For modeling biological objects of shape approximated to solids of revolution there was worked out a method consisted in rotation of generating line, made of the Bézier curve or the B-spline curve consisted of several Bézier curves. Also 2*D* objects can be modeled with the use of Bézier curves.

Key words: Bézier curves, meridional sphere, rotation of generating line, mathematical models.

INTRODUCTION

The polynomial curves were created by Pierre Bézier, French engineer and mathematician. A polynomial notation of curves was worked out by Bézier for the need of modeling the car bodies in Renault business, where Hebisz [2002] was employed. The space has fundamental importance in designing methodics. Prusakowski et al. [2002] maintain, that in designing of objects of complex shapes it is better to apply a curvilinear representation, which allows for shape description with the use of fewer parameters. Kiciak [2005] reported, that Bézier curve is a *p* curve, where every point *p*(*t*) can be designed for a given parameter *t*. Project realization for 3*D* object with the use of CAD systems in digital space can be used [Mieszkalski 2007, 2012a, 2012b, 2013] in statistical and dynamic analyses, from the simplest transformations to complex animations [Januszkiewicz 2012], with application of computer graphic tools [Foley et al. 1995]. The Bézier curve are used in such computer softwares as: CorelDRAW, Adobe Illustrator, Solid Edge, Solid Works, Catia and others [Przybylski and Deja 2007, Jackowski, 2013]. Kawalec and Magdziak [2011] proposed a method for transformation of Bézier curves into B-spline curves for measurements on the free surfaces used in numerically controlled machines. According to Kiciak [2002], application of tensor forms used in theory of curves enables to prove theorem of continuity of B-spline curves used in production engineering together with the Bézier curves. Dworecki et al. [2012] worked out the system enabling to observe the solids in any direction. This system assists in solving the space problems by presenting lines of interpenetration of solids in projection on the selected projection plane. Reclik and Kost [2008] notice necessity of elaboration of a functional system based on the space B-spline curves, as a system module for programing the off-line industrial robots. B-spline curves are used in civil engineering, e.g. in designing of pipeline route and the like [Berkhahn and Sellerhoff 1996]. Dems and Radaszewska [2008] as well as Wiśniewski and Dems [2008] described the shape of fibers (rectilinear and curvilinear) in the polymer composite with Bézier curves enabling to select the shape so, that material parameters optimal in respect of realized heat flow could be achieved. Precision of representation of objects of substantial distortion expands application of spline functions in parametric notation, proper selection of boundary conditions and approximation functions [Lenda 2006, 2008, 2010, Lenda and Mirek 2013]. Kaliniewicz et al. [2012] maintain that the shape of coniferous trees' seeds (pine, black pine, spruce) can be modelled with ellipsoid. Boniecki and Nowakowski [2008] proposed the neural nets for identification of maize grain shape.

This work aims at proposition of own methods for modeling the biological objects (plant objects) with the use of Bézier curves.

DESCRIPTION OF METHODS FOR MODELING THE BIOLOGICAL OBJECTS WITH THE USE OF BÉZIER CURVES

In the systems of computer-aided designing CAD there are used the curves of smooth course that pass through defined points. The polynomial curves of *n*-order in Bézier representation are defined by $(n + 1)$ vertexes $A_0 \ldots A_n$ for $0 \le t \le 1$. The polynomial curves in Bézier representation with application of Bernstein basic functions are written in the following form [Foley et al. 1995, Kiciak 2005, Przybylski and Deja 2007]:

$$P(t) = \sum_{i=0}^{n} A_i \cdot B_i^n(t) \tag{1}$$

where: $B_i^n(t)$ are Bernstein basic functions of *n*-order written in the form:

$$B_i^n(t) = \binom{n}{i}(1-t)^{n-i} t^i \tag{2}$$

Basing on equations (1) and (2), one can write the polynomial curves of second order for $n = 2$ as follows:

$$P_2(t) = A_0\binom{2}{0}(1-t)^{2-0}t^0 + A_1\binom{2}{1}(1-t)^{2-1}t^1 + A_2\binom{2}{2}(1-t)^{2-2}t^2 \tag{3}$$

Coefficients $\binom{n}{k}$ can be easily determined from Pascal's triangle, thus, equation (3) can be written in simplified form:

$$P_2(t) = A_0(1-t)^2 + A_1 2(1-t)\,t + A_2 t^2 \tag{4}$$

For $n = 3$ the polynomial of third order will be obtained:

$$P_3(t) = A_0\binom{3}{0}(1-t)^{3-0}t^0 + A_1\binom{3}{1}(1-t)^{3-1}t^1 + A_2\binom{3}{2}(1-t)^{3-2}t^2 + A_3\binom{3}{3}(1-t)^{3-3}t^3 \quad (5)$$

After rearranging of equation (5) one can obtain:

$$P_3(t) = A_0(1-t)^3 + 3A_1(1-t)^2 t + 3A_2(1-t)\,t^2 + A_3 t^3 \quad (6)$$

For $n = 4$ the polynomial of fourth order will be obtained:

$$P_4(t) = A_0\binom{4}{0}(1-t)^{4-0}t^0 + A_1\binom{4}{1}(1-t)^{4-1}t^1 + A_2\binom{4}{2}(1-t)^{4-2}t^2 + A_3\binom{4}{3}(1-t)^{4-3}t^3 + A_4\binom{4}{4}(1-t)^{4-4}t^4 \quad (7)$$

After rearranging of equation (7) one can obtain:

$$P_4(t) = A_0(1-t)^4 + 4A_1(1-t)^3 t + 6A_2(1-t)^2 t^2 + 4A_3(1-t)\,t^3 + A_4 t^4 \quad (8)$$

Basing on notation of polynomial of fourth order (8), one can write equations of coordinates *x* and *y* of Bézier curve in parametric form:

$$x_4(t) = x_0(1-t)^4 + 4x_1(1-t)^3 t + 6x_2(1-t)^2 t^2 + 4x_3(1-t)\,t^3 + x_4 t^4 \quad (9)$$

$$y_4(t) = y_0(1-t)^4 + 4y_1(1-t)^3 t + 6y_2(1-t)^2 t^2 + 4y_3(1-t)\,t^3 + y_4 t^4 \quad (10)$$

Practically, in respect of calculations economy, for description of curves with sufficient accuracy there are used the polynomials of third order, unequivocally determined by four points. Equations describing the curves in Bézier representation that have two nodal points and two check points on a plane (Fig. 1) are of the following form:

$$x_3(t) = x_0(1-t)^3 + 3x_1(1-t)^2 t + 3x_2(1-t)\,t^2 + x_3 t^3 \quad (11)$$

$$y_3(t) = y_0(1-t)^3 + 3y_1(1-t)^2 t + \\ +3y_2(1-t)\ t^2 + y_3 t^3 \qquad (12)$$

Figure 1 presents examples of characteristic shapes of Bézier curve that can be used in modeling of objects of complex shapes.

The Bézier curves can be used to present meridians in a sphere, named meridional sphere [Mieszkalski 2007]. This method is used for modeling the surface shape of solids of irregular shapes. Several (the number depends on accuracy of object representation) Bézier curves, that are the meridians, are uniformly arranged on a sphere. In pole points of the sphere *A*, *B* (Fig. 2) are nodal points of Bézier

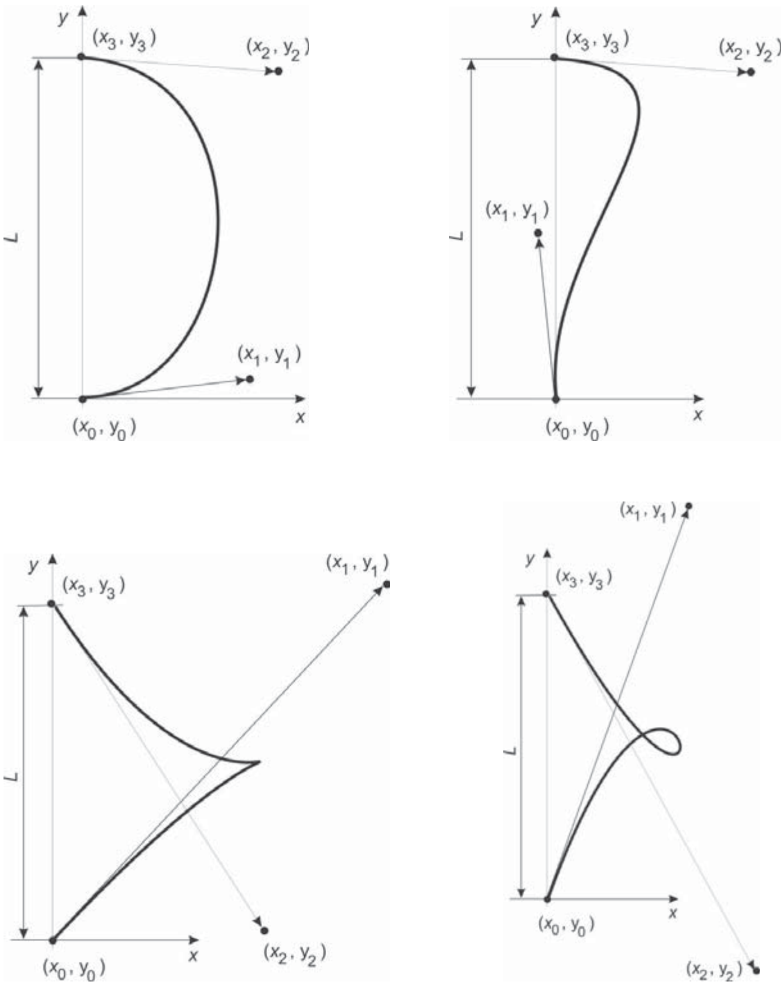

FIGURE 1. Designations of 2*D* Bézier curves of shape: convex, with inflexion point, with acute vertex and with loop

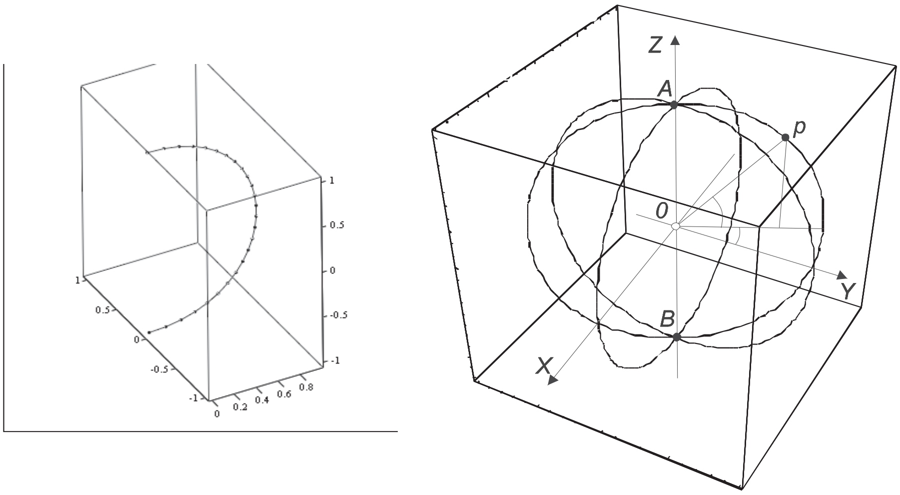

FIGURE 2. Meridian of sphere and layout of poles and meridians' distribution on sphere

curves. Using parametric equations (13), (14) and (15), one can draw one meridian on diagram 3*D* (Fig. 2).

where:
$t = 0 \ldots N$; $N = 25$; $\alpha 1, \alpha 2 \ldots \alpha n = 0, 30^{o} \ldots 360^{o}$; $n = 1, 2 \ldots 13$.

$$xn_t = A_x\left(1-\frac{t}{N}\right)^3 + 3\left(1-\frac{t}{N}\right)^2\frac{t}{N}\left[A1xn\cos\left(\alpha n\frac{\pi}{180}\right)\right] + \\ +3\left(1-\frac{t}{N}\right)\left(\frac{t}{N}\right)^2\left[A2xn\cos\left(\alpha n\frac{\pi}{180}\right)\right] + B_x\left(\frac{t}{N}\right)^3 \tag{13}$$

$$yn_t = A_y\left(1-\frac{t}{N}\right)^3 + 3\left(1-\frac{t}{N}\right)^2\frac{t}{N}\left[A1yn\sin\left(\alpha n\frac{\pi}{180}\right)\right] + \\ +3\left(1-\frac{t}{N}\right)\left(\frac{t}{N}\right)^2\left[A2yn\sin\left(\alpha n\frac{\pi}{180}\right)\right] + B_y\left(\frac{t}{N}\right)^3 \tag{14}$$

$$zn_t = A_z\left(1-\frac{t}{N}\right)^3 + 3A1_z\left(1-\frac{t}{N}\right)^2\frac{t}{N} + \\ +3B1_z\left(1-\frac{t}{N}\right)\left(\frac{t}{N}\right)^2 + B_z\left(\frac{t}{N}\right)^3 \tag{15}$$

For the sphere of radius equal to 1, the coordinate values of nodal points of Bézier curve are written down in matrix (16):

$$\begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \tag{16}$$

However, coordinate values of the check points of Bézier curves are written down in matrixes (17) and (18):

$$\begin{bmatrix} A1z \\ B1z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad (17)$$

$$\begin{bmatrix} A1xn & A2xn \\ A1yn & A2yn \end{bmatrix} = \begin{bmatrix} 1.3355 & 1.3355 \\ 1.3355 & 1.3355 \end{bmatrix} \qquad (18)$$

The values 1.3355 in matrix (18) enable to obtain the sphere of radius 1 on the basis of equations (13), (14) and (15). 3*D* diagrams of meridional sphere with Bézier curves and meridional sphere as a surface diagram are presented in Figure 3. Surface diagram of meridional sphere is created by connection, with horizontal straight line sections, of points on Bézier curve that are the sphere meridians.

Bézier curves lying on the sphere surface are the basis and starting point for modeling of solids of irregular shape. By the change of pole coordinate values, where the nodal points (*Ax*, *Ay*, *Az*, *Bx*, *By*, *Bz*) common for all Bézier curve are accommodated, one can obtain a pencil of special curves lying on the surface of modeled solid of the shape other than the sphere. Changing values of check point coordinates (*A*1*xn*, *A*2*xn*, *A*1*yn*, *A*2*yn*, *A*1*z*, *B*1*z*) assigned to particular curve lying on the solid surface and making the net of its surface one can obtain a change of modeled solid shape. In development of mathematical model for the shape of modeled object and its visualization one can use computer software Mathcad. A view of exemplary modeled solid of the Jonagored variety apple, made with the use of computer software Mathcad, is presented in Figure 4. Modification

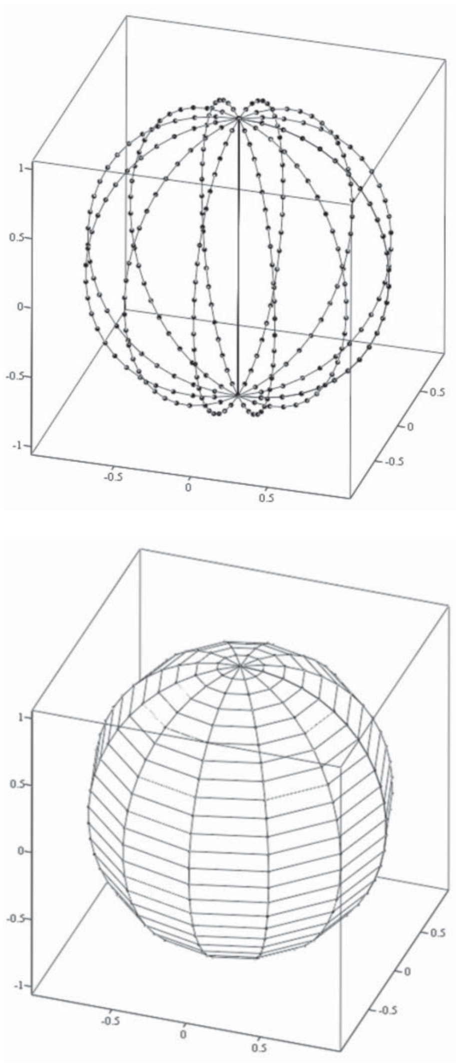

FIGURE 3. Bézier curves (meridional sphere) and surface diagram of sphere

of meridional sphere net enables to obtain the models for various types of solid shapes. Using this method one can model the solids that approximate, among other things, the shapes of seeds, fruits, vegetables and the like.

In modeling the shape of solids similar in shape to solids of revolution (e.g.

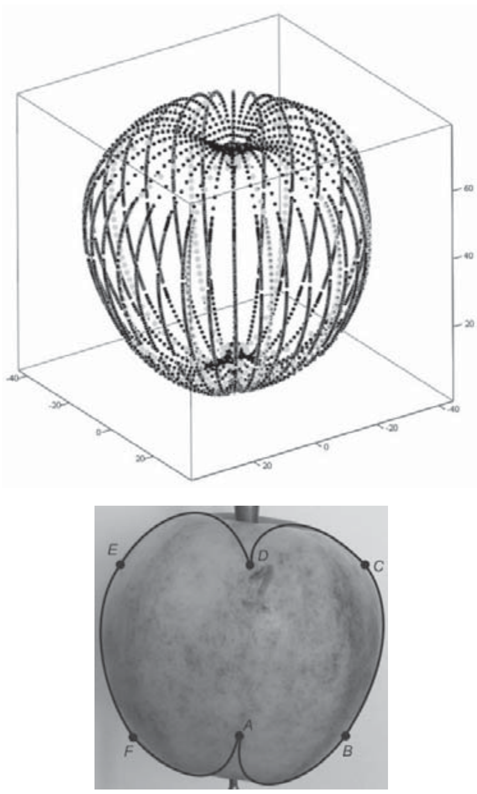

FIGURE 4. Graphical model of Jonagored variety apple and real apple with plotted Bézier curves

carrot roots, bird's eggs) one can use the method consisting in rotating about axis *Z* of coordinate system of Bézier curves that make generating line of the solid. To obtain surface of revolution of the modeled object the following equations are used:

$$Xka_{i,j} = xk_i \cdot \sin(s_j) \tag{19}$$

$$Yka_{i,j} = xk_i \cdot \cos(s_j) \tag{20}$$

$$Zka_{i,j} = zk_i \tag{21}$$

Number of row vectors and columns of matrix is determined with index variables:

$$i = j = 0 ... N \tag{22}$$

Abgle of Bézier curve rotation is described with equation given below:

$$s_j = \frac{2 \cdot \pi \cdot j}{N} \tag{23}$$

The view of exemplary modeled solid of carrot root, made with the use of computer program Mathcad, is presented in Figure 5 [Mieszkalski 2013], while the model for hen's egg in Figure 6.

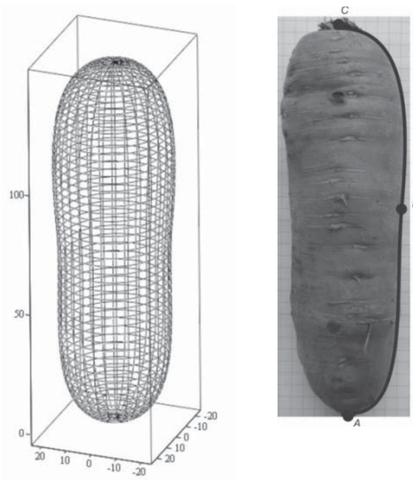

FIGURE 5. Carrot root model and carrot root with Bézier curve as generating line of solid

The Bézier curves are used in modeling of 2*D* objects (Fig. 7), e.g. layouts of flowers, inscriptions and the like.

For instance, to model a rose flower shape there were used 24 Bézier curves described with 24 nodal points and 24 check points; in total with 192 coordinates (Fig. 7). Modeling the inscriptions called for utilization of 43 Bézier curves described with 43 nodal points and 43 check points; in total with 344 coordinates. The model of frame consisted of 4 Bézier curves described with 4 nodal

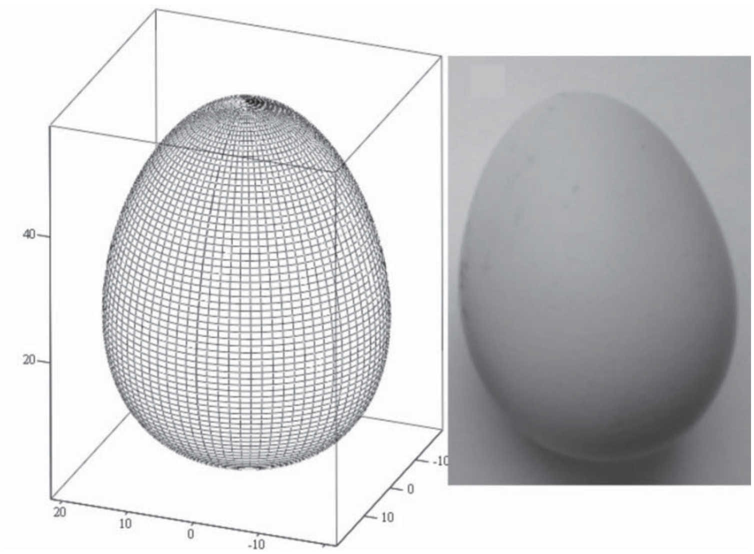

FIGURE 6. Model for hen's egg and its real view

points and 4 check points; in total with 32 coordinates. The entire casual object 2*D* consists of 71 Bézier curves described with 71 nodal points and 71 check points, in total with 568 coordinates.

TABLE 1. Values of coordinates of nodal and check points of Bézier curves for casual object

Number of Bézier curve	Coordinates of points of Bézier curve segments							
	nodal		check		check		nodal	
	x_0	y_0	x_1	y_1	x_2	y_2	x_3	y_3
1	2	3	4	5	6	7	8	9
1	68.6	180.7	66.6	179.2	68.3	181.7	68.6	180.7
2	60.7	179.9	58.7	182.9	73.4	186.5	72.6	180.3
3	68.0	179.2	71.3	178.3	70.3	182.1	71.2	182.3
4	68.0	179.4	66.3	178.9	63.6	180.3	63.6	181.8
5	60.6	179.9	58.9	182.8	73.5	186.4	72.5	180.4
6	57.9	188.5	57.2	193	74.7	192.4	72.9	191
7	57.6	191.1	54.4	197.3	66.4	193.6	71.6	192.7
8	77	193.6	81.8	187.5	89.7	185.5	88.4	180.2
9	89.9	175.2	91.2	178.9	97.3	181.6	89.7	184.3
10	87.1	161.6	87.7	163.3	114	177.6	96.4	178.8
11	67.5	147.4	78.9	144.5	81.6	157.6	87.1	161.6
12	73.3	131.1	75.7	144.9	85.2	147.7	91.8	144.1
13	66.5	137.6	66.5	146.9	75.9	155.9	78.1	153.2

Table 1 cont.

1	2	3	4	5	6	7	8	9
14	60	178.9	63.7	177.5	83.4	176.1	82.5	182.6
15	67	165.8	71.8	175.2	87.4	177.1	92.3	179.7
16	46.1	163.9	47.5	154.9	92.1	171.9	91.2	166.6
17	45.7	163.7	51.2	146.5	75.8	167.8	91.2	166.7
18	41.6	143.6	48.7	147.4	56.8	146.3	58.9	140.9
19	63.6	135.9	63.5	156.4	40.8	152	39	162.9
20	39	162.9	36.4	172.2	23.1	174	40.8	183.3
21	51.6	164.8	40.6	171.3	36.5	199.8	60.5	186.2
22	44.4	179.1	48.5	188.7	46	168.9	75.9	166.6
23	66.8	168.7	53.4	176.6	38.4	192.7	60.2	186
24	57.5	177.7	53.2	182.1	58.2	188.7	66.6	184.5
25	14.1	97.2	7.7	91.2	16.7	124.9	13.6	126.1
26	14.1	97.2	15.4	97.9	20.8	104.4	19.8	105.7
27	22.7	96.4	22.1	97.5	19.3	107.2	18.1	106.4
28	22.7	96.4	24.7	94.1	20	124.4	25.5	125.6
29	29.2	103.5	26.6	110.5	34.5	110.1	33.2	107.4
30	30.2	97.7	39.3	91.6	32.7	104.9	29.2	103.5
31	45.4	96.5	23.8	93.5	59.9	113.3	36.8	108.3
32	54.1	96.9	42.5	91.9	49.7	110.5	47.9	109.2
33	51.2	85	59	82.9	49.9	115.4	53.8	108.4
34	56.9	103.8	54.4	111.1	62.8	109.9	61.6	107.5
35	58.3	98.3	68.6	90.7	60.6	105.4	56.9	103.8
36	72.4	96.7	60.4	91.8	69.1	128	66.2	125.3
37	72.5	115.9	70.9	114.1	61.7	119.1	61.4	116.9
38	75.2	96.3	72.9	94.2	76.7	127.5	76.1	125.8
39	83.6	97.9	78.9	92	78.2	103.4	74.9	102.5
40	75.6	103.1	74.6	108	84.3	110.5	82.9	108.3
41	89.5	96.8	82.1	92.7	88.4	111.5	86.8	108.8
42	88.2	113.0	91.3	112.3	88.1	116.2	86.8	114.9
43	96.3	96.3	88.7	94.4	89.4	108.4	91.2	108.9
44	90.4	102.6	91.1	100.8	107.9	110.6	91.4	108.7
45	103.4	84.8	111.2	83.3	106.4	115.3	108.9	108.2
46	108.2	96.3	92.0	97.3	107.9	115.5	107.9	107.0
47	110.6	104.4	124.4	78.5	123.4	122.4	110.8	105.1
48	11.1	51.5	17.4	35.9	15.8	84.5	16.4	82.4
49	19.1	48.8	17.6	49.6	18.4	86	17.4	82.2

Table 1 cont.

1	2	3	4	5	6	7	8	9
50	20.2	50.2	20.3	47.8	20.9	88.9	26.4	79.6
51	33.2	50.7	13.8	38.1	28.9	81.9	32.9	57.0
52	37.2	50.2	30.6	42.3	35	65.3	33.2	64.5
53	35.4	38.2	41.2	30.5	36.7	61.5	38.7	64.9
54	38.9	68.1	42.1	67.7	38.9	71.3	37.8	70.2
55	47.5	49.8	39	40.7	45.8	85.6	42.8	81.6
56	53.7	49.2	46.4	47.2	43.7	64.7	48.7	64.9
57	46.7	56	52.4	56	66	65.2	48.8	64.8
58	56.2	38.2	62.1	30.4	57.7	61.6	59.6	65
59	59.3	56.5	77.9	28	68.1	82.2	59.3	59.3
60	72.5	57.8	69.8	64.6	77.1	66.2	76.7	61.9
61	73.8	51.4	83.4	41.1	76.9	60.3	72.6	57.6
62	88.9	49	67.1	45.8	102.3	69.5	79.3	64.2
63	96.2	48.9	89	46.9	90.1	62.6	92.1	64.1
64	90.8	55.4	93.6	55.8	109.2	65.3	92.1	64.3
65	106.2	38	114	36.2	109.1	71.8	111.5	64.5
66	110.6	48.6	94	49.8	107.3	72.6	110.9	61.9
67	114.2	57	133.1	27.7	123.7	81.8	113.8	59.2
68	1.1	3.2	3.0	23.6	118.3	–18.6	142.8	21.9
69	5.3	0.4	17.9	1.9	–10.7	197.3	17.0	210.3
70	0.4	193.5	–1.8	229.9	142	173.1	137.8	197.2
71	131.1	0	150.1	–6.3	113.4	173.2	127.2	204.0

FIGURE 7. Model for casual object

SUMMARY

In modeling the shapes of biological solids of complex shapes it is convenient to use the polynomials of third order in Bézier representation. The meridional sphere with Bézier curves as meridians can be used in modeling shapes of fruits and vegetables' shapes. In modeling of biological objects, that are similar in shape to solids of revolution, one can use the method that consists in rotating of generating line, made of Bézier curve or B-spline curve built-up of Bézier curves.

Modeling of 2*D* objects calls for big number of Bezier curves, that depends on the object shape.

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Streszczenie: *Krzywe Béziera w modelowaniu kształtu obiektów biologicznych.* W pracy zamieszczono charakterystyczne kształty krzywych Béziera stosowanych w modelowaniu obiektów biologicznych. Przedstawiono zapis krzywych wielomianowych drugiego, trzeciego i czwartego stopnia w reprezentacji Béziera z zastosowaniem funkcji bazowych Bernsteina. Obiekty biologiczne o złożonym kształcie zaleca się modelować, używając wielomianów trzeciego stopnia w reprezentacji Béziera. Do modelowania kształtu (bryły 3*D*) owoców i warzyw zaproponowano kulę południkową, w której południkami są krzywe Béziera. Do modelowania obiektów biologicznych zbliżonych kształtem do brył obrotowych opracowano metodę polegającą na obrocie tworzącej, którą stanowią krzywa Béziera lub złożona z kilku krzywych Béziera krzywa B-sklejana. Za pomocą krzywych Beziera mogą być również modelowane obiekty 2*D*.

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