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Mathematical modelling of hen's egg shape by rotation curve

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Abstract: *Mathematical modelling of hen's egg shape by rotation curve.* The proposed method concerns a mathematical modelling of the hen's eggs. To describe the hen's egg shape there was used the Bézier curve, rotated in relations to *z* axis; the obtained surface was scaled. The Bézier curve described a half of contour of egg's longitudinal section. The developed mathematical model can be used in generating 3*D* solids that are similar to hen's eggs in respect of shape and basic dimensions. The mathematical model allows for the change in egg's length, width and thickness as well as in coordinates of nodal and control points of Bézier curve that describes a half of contour of the egg's longitudinal section.

Key words: hen's egg, shape, mathematical modelling, Bézier curve

INTRODUCTION

Eggs are valuable component of human diet in respect of their nutritional value. They belong to products of animal origin and are valuable raw material in food industry. They contain almost all vitamins and polysaturated fatty acids as well as a balanced composition of amino acids and mineral components [Niewiarowicz et al. 1991, Trziszka 2000, Siepka et al. 2010].

The shape of eggs is considered in designing of machines and equipment used in their production and processing; it is also one of quality parameters assessed by customers [Rashidi and Keshavarzpour 2011]. Geometrical properties of eggs are important in biological investigations and poultry industry [Narushin 2005].

The methods for stock management used in production and trade enterprises [Buliński et al. 2012, 2013], poultry industry and climate controlling in poultry-houses [Lysenko et al. 2011], production robotics [Shvorov et al. 2012], as well as application of the methods for product classification and diagnostics in respect to significantly different properties [Janaszek and Trajer 2010] enable to arrange data and identify these properties, working parameters and design decisions that influence the product quality.

In description of the eggs' shape there can be used the methods that are applied in description of raw material of agricultural origin proposed by many researchers [Kęska and Feder 1997, Mabille and Abecassis 2003, Donev et al. 2004, Frączek and Wróbel 2006, 2009, Mieszkalski 2011]. Information needed for modelling 3*D* objects of various origing are given by computer graphics [Kiciak 2000, Foley et al. 2001].

This work aimed at development of mathematical model describing an external shape of the hen's eggs by rotation of Bézier curve.

MATERIAL AND METHODS

The research material were the hen's eggs originated from a liter keeping system and distributed by P.H. Czachorowski (www.czacharowski.com). Three hen's eggs of different shape were selected and photographed. Their length, width and thickness were measured with a slide caliper with accuracy 0.1 mm; the maximal diameters of eggs were measured along their length every 10, 25, 35, 45 and 55 mm.

To describe the shape of half of contour of the egg's longitudinal section there was used the Bézier curve; by its rotation in respect of *z* axis the rotational surface was obtained. The created rotational surfaces of solids were scaled to obtain their shapes similar to shape of selected hen's eggs. Visualization of mentioned 3*D* solid models was performed with the use of computer program Mathcad.

The hen's egg dimensions are presented in Table 1, exemplary photographs of the hen's eggs are presented in Figure 1.

TABLE 1. Length (a) and diameter dimensions related to sections of selected hen's eggs

Distance between measured diameters of eggs [mm]	Diameters of eggs [mm]		
	I $a = 60$ mm	*II* $a = 57$ mm	*III* $a = 56$ mm
10	35.7	32.9	34
25	46.7	42	41
35	46.6	40.9	37.7
45	40.4	33.2	29.2
55	26.6	14.2	7.9

FIGURE 1. Hen's eggs designated as *I*, *II* and *III* (dimensions in Table 1)

DESCRIPTION OF MODEL

Matrix equations of coordinates *xA*1, *zA*1 of Bézier curve points for egg *I* have the form:

$$xI_t = xI1 \cdot \left[1 - \frac{t}{N}\right]^3 + xI2 \cdot 3\frac{t}{N} \cdot \left[1 - \frac{t}{N}\right]^2 + xI3 \cdot 3 \cdot \left[\frac{t}{N}\right]^2 \cdot \left[1 - \frac{t}{N}\right] + xI4 \cdot \left[\frac{t}{N}\right]^3 \quad (1)$$

$$zI_t = zI1 \cdot \left[1 - \frac{t}{N}\right]^3 + zI2 \cdot 3\frac{t}{N} \cdot \left[1 - \frac{t}{N}\right]^2 + zI3 \cdot 3 \cdot \left[\frac{t}{N}\right]^2 \cdot \left[1 - \frac{t}{N}\right] + zI4 \cdot \left[\frac{t}{N}\right]^3 \quad (2)$$

Matrix equations for eggs *II* and *III* have analogical notation.

Coordinates of nodal and control points that occur in the equations (1, 2) for Bézier curve of eggs *I*, *II* and *III* are written down in matrixes 3, 4 and 5:

$$\begin{bmatrix} xI1 & zI11 \\ xI2 & zI12 \\ xI3 & zI13 \\ xI4 & zI14 \end{bmatrix} = \begin{bmatrix} 0 & 60 \\ 32.1 & 60 \\ 28.6 & 0 \\ 0 & 0 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} xII1 & zII11 \\ xII2 & zII12 \\ xII3 & zII13 \\ xII4 & zII14 \end{bmatrix} = \begin{bmatrix} 0 & 57 \\ 23.9 & 57 \\ 29.7 & 0 \\ 0 & 0 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} xIII1 & zIII11 \\ xIII2 & zIII12 \\ xIII3 & zIII13 \\ xIII4 & zIII14 \end{bmatrix} = \begin{bmatrix} 0 & 56 \\ 21.7 & 56 \\ 39 & 0 \\ 0 & 0 \end{bmatrix} \quad (5)$$

To obtain rotational surface that represents the egg *I* one should rotate Bézier curve in relations to *z* axis with the use of equations:

$$XI1_{t,j} = xI_t \cdot \sin(\alpha_j) \quad (6)$$

$$YI1_{t,j} = xI_t \cdot \cos(\alpha_j) \quad (7)$$

$$ZI1_{t,j} = zI_t \quad (8)$$

where:

$$\alpha_j = \frac{2 \cdot \pi \cdot j}{N} \quad (9)$$

$$j = 0 \ldots \mathrm{N} \quad (10)$$

$$t = 0 \ldots \mathrm{N} - 1 \quad (11)$$

where:
N – matrix size (it was assumed $N = 81$);
t – number of rows;
j – number of columns.

Similarly, one should rotate Bézier curves for the eggs *II* and *III*.

The length – *a*, width – *b* and thickness – *c* of real measurements on the eggs *I*, *II* and *III* are included in matrix 12.

$$\begin{bmatrix} aI & aII & aIII \\ bI & bII & bIII \\ cI & cII & cIII \end{bmatrix} = \begin{bmatrix} 60 & 57 & 56 \\ 47 & 42 & 41 \\ 46 & 41 & 39 \end{bmatrix} \quad (12)$$

To obtain the given dimensions (*a*, *b*, *c*) of the hen's egg model one should scale equations 7, 8 and 9. The scaled matrix equations of coordinates *XI*, *YI* and *ZI* of nodal points of the net of surface that describes the shape of egg *I* have the following form:

$$XI = \frac{bI}{\max(XI1) - \min(XI1)} \cdot XI1 \quad (13)$$

$$YI = \frac{cI}{\max(YI1) - \min(YI1)} \cdot YI1 \quad (14)$$

$$ZI = \frac{aA}{\max(ZI1) - \min(ZI1)} \cdot ZI1 \quad (15)$$

Similar calculations should be made for eggs II and III.

The 3*D* models for eggs *I*, *II* and *III* are presented in Figure 2.

VERIFICATION OF MODEL

The developed mathematical model that describes the shape of hen's eggs was initially verified by comparison of overlapped projections on *XZ* plane of the eggs' models and images (Fig. 3).

Comparing the overlapped projections of eggs *I*, *II* and *III* models and images one can find, that the shapes of solids represented by the eggs' models (Fig. 2) are similar to real eggs presented in Figure 1. Basic dimensions – length, width and thickness – are identical for the models and images (result of scaling).

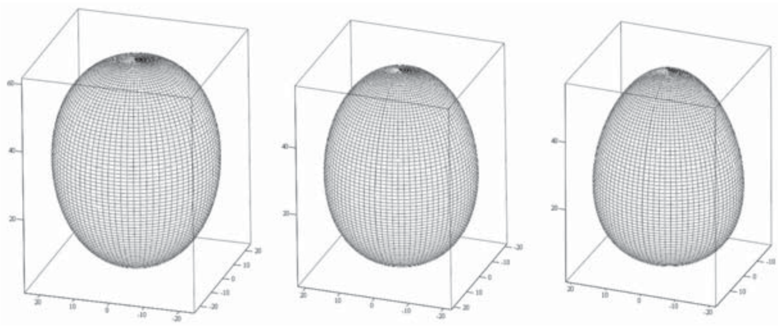

FIGURE 2. 3*D* models for eggs *I*, *II* and *III*

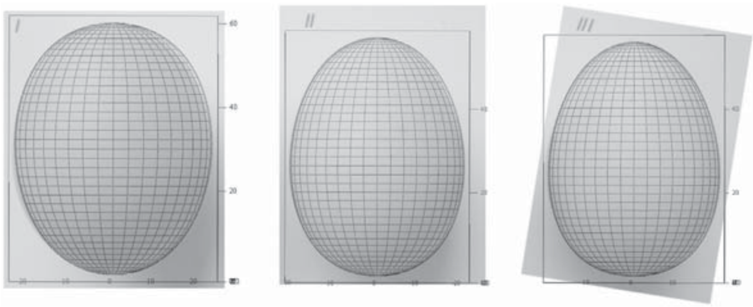

FIGURE 3. Overlapped projections on *XZ* plane of models and images of eggs *I*, *II* and *III*

Small discrepancy may occur in the remaining sections (Table 2). Imaging inaccuracy occurred within the section at distance 10 mm (egg *II* with relative error –5.7%, egg *III* with relative error –5.8%) and within the section at distance 55 mm (egg *II* with relative error –6.7%, egg *III* with relative error –5.7%). The proposed mathematical models can be used in generating the 3*D* solids similar to hen's eggs in respect of their shape and basic dimensions and can be applied in agricultural and food engineering, where there is no need for very high imaging accuracy.

TABLE 2. Difference in diameter measurements for model and egg and relative error

Distance between measured diameters of eggs [mm]	Difference in diameter measurement for model and egg [mm]			Relative error [%]		
	I	*II*	*III*	*I*	*II*	*III*
1	2	3	4	5	6	7
10	–0.5	–1.9	–2	–1.4	–5.7	–5.8
25	0.6	0.05	0.2	1.3	0.1	0.6
35	0.08	0.8	0.9	–0.2	2	2.4
45	–1.4	–0.2	0.5	–3.4	–0.6	1.8
55	–1.3	–0.9	–0.5	–4.8	–6.7	–5.7

CONCLUSIONS

1. The developed model represented with surface obtained by rotation of Bézier curve after scaling can be used in generating 3*D* solids similar to the hen's eggs in respect of their shape and basic dimensions.
2. In the proposed mathematical model the parameters that control the shape are basic dimensions of egg (length, width, thickness) and coordinates of the nodal and control points of Bézier curve that describes the half of longitudinal contour of the egg section.

REFERENCES

BULIŃSKI J., WASZKIEWICZ C., BURACZEWSKI P. 2012: Stock management as an element of enterprise strategy. Annals of Warsaw University of Life Sciences – SGGW, Agriculture 60: 137–148.

BULIŃSKI J., WASZKIEWICZ C., BURACZEWSKI P. 2013: Utylization of ABC/ /XYZ analysis in stock planning in the enterprise. Annals of Warsaw University of Life Sciences – SGGW, Agriculture 61: 89–96.

DONEV A. et al. 2004: Improving the density of jammed disordered packings using ellipsoids. Science 303: 990–993.

FOLEY J.D., van DAM A., FEINER, S.K., HUGHES J.F., PHILLIPS R.L. 2001: Wprowadzenie do grafiki komputerowej. WNT, Warszawa.

FRĄCZEK J., WRÓBEL M. 2006: Metodyczne aspekty oceny kształtu nasion. Inżynieria Rolnicza 12: 155–163.

FRĄCZEK J., WRÓBEL M. 2009: Zastosowanie grafiki komputerowej w rekonstrukcji 3D nasion. Inżynieria Rolnicza 6 (115): 87–94.

JANASZEK M., TRAJER J. 2010: Classification system as diagnostics tool for the plant products. Annals of Warsaw University of Life Sciences – SGGW, Agriculture 56: 51–56.

KĘSKA W., FEDER S. 1997: Trójwymiarowa rekonstrukcja kształtu elementów roślinnych z dwuwymiarowych obrazów rastrowych. Prace Przemysłowego Instytutu Maszyn Rolniczych: 15–17.

KICIAK P. 2000: Podstawy modelowania krzywych i powierzchni. Zastosowanie w grafice komputerowej. WNT, Warszawa.

LYSENKO V., GOLOVINSKIY B., RESHETYUK V., GOLUB B., SHCHERBATYUK V. 2011: Dynamics of quality indexes of laying hens keeping process due to fluctuations of temperature disturbances in an industrial poultry house. Annals of Warsaw University of Life Sciences – SGGW, Agriculture 57: 79–92.

MABILLE F., ABECASSIS J. 2003: Parametric modelling of wheat grain morphology: a new perspective. Journal of Cereal Science 37: 43–53.

MIESZKALSKI L. 2011: Metoda matematycznego modelowania kształtu bryły ziarna pszenicy za pomocą parametrycznej krzywej przestrzennej i czterowęzłowej siatki. Postępy Techniki Przetwórstwa Spożywczego 1, 21/38: 41–45.

NARUSHIN V.G. 2005: Production, Modeling, and Education. Egg Geometry Calculation Using the Measurements of Length and Breadth. Poultry Science 84: 482–484.

NIEWIAROWICZ A. 1991: Budowa, skład chemiczny, właściwości fizykochemiczne i funkcjonalne oraz wartość odżywcza jaj. W: Technologia jaj – praca zbiorowa. WNT, Warszawa: 18.

RASHIDI M., KESHAVARZPOUR F. 2011: Classification of Egg Size and Shape Based on Mass and Outer Dimensions Analysis. Libyan Agriculture Research Center Journal International 2 (5): 221–223.

SHVOROV S., RESHETYUK V., BOLBOT I., SHTEPA V., CHIRCHENKO D. 2012: Theoretical issues costruction and operation of agricultural mission robotic system. Annals of Warsaw University of Life Sciences – SGGW, Agriculture 60: 97–102.

SIEPKA E., BOBAK Ł., TRZISZKA T. 2010: Frakcjonowanie żółtka w celu pozyskiwania preparatów wzbogaconych w substancje biologicznie aktywne. Żywność. Nauka. Technologia. Jakość 6 (73): 158–167.

TRZISZKA T. (red.) 2000: Jajczarstwo. Nauka, technologia, praktyka. Wyd. AR we Wrocławiu, Wrocław.

Streszczenie: *Matematyczne modelowanie kształtu jaja kurzego przez obrót krzywej.* Proponowana metoda dotyczy matematycznego modelowania jaj kurzych. W modelu matematycznym do opisu kształtu jaja kurzego zastosowano krzywą Béziera, którą obrócono względem osi *z*, a uzyskaną powierzchnię skalowano. Opracowany model matematyczny może być stosowany do generowania brył 3*D* podobnych pod względem podstawowych wymiarów do jaj kurzych. W modelu matematycznym istnieje możliwość zmiany wartości długości, szerokości, grubości jaja oraz współrzędnych punktów węzłowych i kontrolnych krzywej Béziera opisującej linię połowy konturu wzdłużnego przekroju jaja.

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