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Cooling-times of tungsten filament lamps

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ABSTRACT

During steady-state operation, hot-coiled filaments in incandescent lamps provide luminous flux for illumination, but when switched off, the temperature, as well as the light output drops quite fast. The cooling-time of a lamp is the time required for the hot filament to cool down to ten per cent light output after the circuit is opened. In this paper, the exercise of estimating luminous flux and cooling-times for typical 10, 100, 500 and 1000 W lamps has been undertaken for the first time for the benefit of students. This problem involves three disciplines: electricity, optics and heat. Information drawn from field of Electrical studies allows us to understand the power that quickly heats the filament, followed by that from optics that helps us in determining the light output, while heat studies are responsible for understanding the cooling of the hot filament. This last is largely explained through the Stefan-Boltzmann law. In this paper, we show that the supposition of linear configurations for the filaments neither matches luminous flux nor the cooling-times. Both fall short. H.S. Leff's suggestion of introducing a shadow factor that reduces the exposed surface area, as it so happens in the coiled filaments, successfully explains the measured observations.

***Keywords*:** Tungsten filament lamps, linear and coiled configurations, 10-1000 W, steady-state operation, lumen output, and cooling-time

1. INTRODUCTION

General Electric in its bulletin[1] *Incandescent lamps* quotes "Tungsten filaments operate at temperatures between $4000°$ and $5400°\,F$. These temperatures are higher than any others ordinarily produced by man". This high enough temperature leads to electromagnetic emission in the visible band providing the light for the purpose of illumination. This is achieved in practice in tungsten filament lamps commonly referred to as “large” lamps[2], which operate on electricity for heating to such a high temperature and that’s too quite fast. In contrast, the switching off an incandescent lamp operating in steady-state cools quickly largely through Stefan-Boltzmann radiation law; the filament temperature and consequently the lumen output both drop quite rapidly. The cooling-time[1] of it is defined as the time required for the hot filament to cool down to the point where the light output drops to ten per cent after the circuit has been opened compared to its value when it was closed. The light output and cooling-time data for coiled filament bulbs in the range $6 - 1000\ W$ are reported by General Electric in their catalogue[1]. The objective of this paper is to elucidate these numbers from the point of view of students and teachers of physics demonstrating how the disciplines electricity, optics and heat play roles in accomplishing the objectives; the first three sections are devoted to the roles played by these disciplines. Section 5 will describe Numerical work part. Finally, Section 6 will discuss the conclusions.

2. ELECTRICITY

The electricity part is concerned with the Joule heating of the tungsten filament through the electric power operating the lamp. Suppose the resistance of the cold filament of a bulb at room temperature T_0 is R_0 . As soon as the power is turned on, a maximum in-rush current

$$I_{max} = V/R_0 \qquad \text{…….. (1)}$$

enters into it. Here V is the constant voltage value for the direct current and the root-mean-square value for an alternating current; the inductance effects[3] in coiled filaments are quite small for alternating currents, assuring that the current I and voltage V are approximately in phase and they satisfy the relation (1). Since I_{max} is quite large (typically of the order of 10 A or more) the corresponding Joule heat makes the temperature T of the filament rise very rapidly. The physical properties of the metal constituting the filament are expected to be temperature dependent; the resistance R starts amplifying quickly enough according to the power-law parameterization[3]

$$R = R_0(T/T_0)^{1.214} \qquad \text{……. (2)}$$

The index 1.214 is obtained by making a least-square fit to the available data on the resistivity of tungsten[4] in the range 293–3000 K. Consequently, the Joule heating gets decelerated in view of constant voltage supply. Once the steady-state temperature T_S is reached the filament resistance R_S and the power P will satisfy the relation

$$P = \frac{V^2}{R_S} = \frac{V^2}{R_0(T_S/T_0)^{1.214}} \qquad \text{……. (3)}$$

The above case corresponds to adiabatic heating, that is, no heat leaves the system. However, this is not true in the case of incandescent lamps. At this time the simultaneous phenomenon of black-body radiation from the hot filament depending on the fourth-power of instant temperature cools the filament. This fact modifies the steady-state temperature, slowdowns the rise of temperature and there is a drop in the light output. The steady-state temperature T_S will be governed by the following relation for uncoiled filament

$$P = \frac{V^2}{R_0(T_S/T_0)^{1.214}} = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot (T_S^4 - T_W^4) \qquad \text{......... (4)}$$

Here σ is the Stefan-Boltzmann constant, A is the surface area, $\varepsilon_{Overall}$ is the average emissivity of the metal tungsten and T_W is the temperature of the walls of the enclosure; the term T_W^4 will not be considered for making the calculation simple. Thus, there are following two options for fixing the operating temperature T_S

- $\frac{V^2}{R_0(T_S/T_0)^{1.214}} = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot T_S^4$ .... (5)
- $P = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot T_S^4$ .... (6)

Both of these methods were attempted. The problem faced with the first method was that usually the T_S estimate's were not consistent with the power of the bulb. Therefore the second expression was adopted.

It will be worth devoting one paragraph on the coiling of the filament[3] for the reason that light-bulb filaments do not appear in the linear configurations. Rather, they are coiled once or twice to reduce their effective radiating surface areas; this favors the portable size of light-bulbs. The coiling enforces some of the surface area to radiate inward[3] to the region bounded by the coil resulting in higher filament temperatures. For example, a tightly wound, singly coiled filament would radiate outward only from about half of its surface area. Indeed, the temperature dependence of the resistivity of tungsten wire is not affected by its shape[4] but as the coiling leads to higher temperatures, the resistance of the wire gets amplified (vide 2) which further enhances Joule heating; thus, coiling leads to higher operating temperature[3] and thereby a larger lumen output. The reality that effective radiating surface is reduced due to coiling this feature can be achieved mathematically[3] by incorporating a correction factor δ in the surface area of uncoiled filament. This is called shadow factor[5] having a value unity for linear configuration and a value which is less than one for coiled filaments. Suppose, at room temperature T_0 the uncoiled length of the filament is L_0 and diameter D_0 are known. Then the expression for the area of uncoiled filament would be

$$A = 2\pi r_0 L_0; \; radius\ of\ the\ filament\ r_0 = D_0/2 \qquad \text{............ (7)}$$

As per suggestion of Leff[3] the effective surface area due to coiling would be

$$A_{Coil} = 2\pi r_0 L_0 \cdot \delta \qquad \text{............. (8)}$$

For numerical illustration, first of all a linear configuration for the tungsten filament will be considered and it will be shown that both the luminous fluxes as well as the cooling-times fall short of the measured values. This hints that coiling factor has to be brought in. For simplicity the heat losses such as gas losses, end losses and bulb and base losses will be ignored. Next section will take up the optics part to estimate light output from hot filaments.

3. OPTICS

Once the steady-state temperature of the filament is known we proceed to estimate the light output from such a source during its operation. The Planck's law for the electromagnetic radiation from an object at temperature T says

$$I(\lambda, T)d\lambda = \frac{\varepsilon(\lambda,T)\cdot A\cdot 2\pi hc^2\cdot d\lambda}{\lambda^5[exp(hc/\lambda kT)-1]} \text{ W.} \qquad \ldots\ldots\ldots (9)$$

Here $I(\lambda, T)d\lambda$ is the power radiated between the wavelengths (in meters) λ and $\lambda + d\lambda$ from its surface having area A square meters and emissivity ε ; h and k are Planck's constant and Boltzmann's constant, respectively. The expression for the luminous flux[6,7] Q can be written as

$$Q(\lambda_i \to \lambda_f) = \int_{\lambda_f}^{\lambda_i} \frac{683\cdot V(\lambda)\cdot \varepsilon_{Visible}\cdot A\cdot 2\pi hc^2\cdot d\lambda}{\lambda^5[exp(\mathrm{hc/\lambda kT})-1]}. \qquad \ldots\ldots\ldots (10)$$

The factor $V(\lambda)$ takes care of the fact that the electromagnetic waves in the wavelengths region $\lambda_i = 380$ nm to $\lambda_f = 760$ nm are perceptible to our eyes; it is best at $\lambda_m = 555$ nm and becomes vanishingly small outside this interval. This fact is represented by

$$V(\lambda) \cong \exp(-\mathrm{a}z^2 + \mathrm{b}z^3); z \equiv \lambda/\lambda_\mathrm{m}; \lambda_\mathrm{m} = 555 \text{ nm} \qquad \ldots\ldots\ldots (11)$$
$$a = 88.90, b = 112.95$$

The factor 683 occurs for the reason that at $\lambda_m = 555$ nm the electromagnetic radiation of one watt provides a luminous flux of 683 lumens. The expression for the effective surface area of the coil (vide 8) modifies the relation (10) to be

$$Q(\lambda_i \to \lambda_f) = \int_{\lambda_f}^{\lambda_i} \frac{683V(\lambda)\cdot \varepsilon_{Visible}\cdot 2\pi\cdot r_0\cdot L_0\cdot \delta\cdot 2\pi hc^2\cdot d\lambda}{\lambda^5\cdot[exp(\mathrm{hc/\lambda kT})-1]} \qquad \ldots\ldots\ldots (12)$$

The above integral will provide the luminous flux emitted by a bulb during its steady-state operation to be evaluated through Simpson rule; for simplicity thermal expansion of filament has been ignored. It is well known that emissivity of tungsten is substantially large in the visible wavelength region[8] and decreases with rise in the temperature of the filament. In contrast, the average value of emissivity over the entire wavelength spectrum is rather small and increases with the rise of temperature of the filament[4]; in view of the pedagogic nature of this article $\varepsilon_{Visible} = 0.44$ (vide ref. 8) and $\varepsilon_{Overall} = 0.0000689T^{1.0748}$ would be adopted; $\varepsilon_{Overall}$ has been obtained through a least-square fit from the data in the temperature range 293-3000 K reported by Jones and Langmuir[4].

The estimated operating temperatures vide (6) and the corresponding luminous fluxes vide (12) for couple of typical lamps viz. 10, 100, 500, and 1000 W are listed in Table IIa for filaments believed to have linear configurations implying $\delta = 1$; luminous fluxes fall short of the analogous reported values by General Electric. This suggests that the coiling factor has to be brought in. For this purpose, δ was decreased in the steps of 0.01 until the estimated flux matched with the observed value; these delta values and the particular temperatures of the hot filaments are reported in Table IIb. The last sentence needs to be further elaborated. Once the shadow factor δ is reduced the area of the filament also gets condensed through (8), the steady-state temperature rises (vide 6), and there is enhancement of lumen output via the integral (12).

Having successfully reproduced the light outputs of lamps, the steady-state temperature as well as the shadow factor for each lamp is known to us. Next, we move to the calculations of their cooling-times; as per definition of the cooling-time the cold temperature T_{Cold} at which the light output falls to 10% with respect to closed circuit assessment has to be ascertained; in other words while the temperature falls from T_S to T_{Cold} the associated lumen falls from 100% to 10% during the cooling process. For this reason, a program was developed in GW-BASIC to evaluate the integral (12) for the luminous flux by Simpson rule for each one degree Kelvin fall of temperature of the filament starting from T_S. This process was carried out for each lamp. The temperature T_{Cold} of the filament was recorded both for linear and coiled configurations and these are mentioned in Tables IIa and IIb, respectively. The next section will be devoted to evaluation of cooling-times for the same couple of typical light-bulbs.

4. HEAT

Estimating the cooling-time as per definition

The Stefan-Boltzmann law states that the thermal power radiated from a body having thermal energy $H\,Joules$ at uniform hot temperature $T\,Kelvin$ surface area $A\,square\,meters$, and averaged emissivity over the entire spectrum $\varepsilon_{Overall}$ is proportional to the fourth power of the absolute temperature and is represented as

$$dH/dt = -\sigma \cdot A \cdot \varepsilon_{Overall} \cdot (T^4 - T_W^4) \qquad \text{.........} \ (13)$$

where minus sign represents cooling phenomenon. Putting the expression for H one gets

$$MC\frac{dT}{dt} = -\sigma \cdot A \cdot \varepsilon_{Overall} \cdot T^4 \qquad \text{..............} \ (14)$$

where M is the mass of the filament in kilogram, C is the specific heat in Joule per kilogram per degree Kelvin and t is the time variable in seconds; as already pointed out in the beginning the term T_W^4 has been neglected for making the calculation simple. The specific heat of tungsten metal as reported by metallurgists[9] in the range $0 - 3000\,°C$ has the following expression

$$C = 3R_g(1 - \theta_D^2/20\,T^2) + 2aT + 4bT^3\ J\,kg^{-1}K^{-1}. \qquad \text{............} \ (15)$$

where T is in Kelvin, $R_g = 45.2268\, J\, kg^{-1}K^{-1}$ is gas constant for tungsten, $\theta_D = 310\, K$ is a constant called the Debye temperature for tungsten at room temperature, $a = 4.5549 \cdot 10^{-3}\, J\, kg^{-1}K^{-2}$ and $b = 5.77874 \cdot 10^{-10}$ $J\, kg^{-1}K^{-4}$. Mass of the filament can be written to be

$$M = Volune\ of\ filament \cdot Density\ of\ tungsten$$
$$= \pi \cdot r_0^2 \cdot L_0 \cdot 1.93 \cdot 10^4\ Kg. \qquad \text{...... (16)}$$

The heat equation (14) can be rearranged as

$$dt = -\frac{\pi \cdot r_0^2 \cdot L_0 \cdot 1.93 \cdot 10^4 \cdot \left[3R_g\left(1 - \theta_D^2/20T^2\right) + 2aT + 4bT^3\right]}{\sigma \cdot 2\pi \cdot r_0 \cdot L_0 \cdot \delta \cdot 0.0000689 \cdot T^{1.0748} \cdot T^4} \cdot dT. \qquad \text{........(17)}$$

Integration of this and taking the limits from T_S to T_{Cold} yields the cooling-time

$$t_{Cooling} = -\frac{r_0 \cdot 1.93 \cdot 10^4}{\sigma \cdot 2 \cdot \delta \cdot 0.0000689}\left\{\frac{3R_g}{(-4.0748)}\left(\frac{1}{T_S^{4.0748}} - \frac{1}{T_{Cold}^{4.0748}}\right) - \frac{3R_g \cdot \theta_D^2}{20 \cdot (-6.0748)}\left(\frac{1}{T_S^{6.0748}} - \frac{1}{T_{Cold}^{6.0748}}\right) + \frac{2a}{(-3.0748)}\left(\frac{1}{T_S^{3.0748}} - \frac{1}{T_{Cold}^{3.0748}}\right) + \frac{4b}{(-1.0748)}\left(\frac{1}{T_S^{1.0748}} - \frac{1}{T_{Cold}^{1.0748}}\right)\right\}. \qquad \text{........(18)}$$

This expression has been used to estimate cooling-times for 10, 100, 500, and 1000 W lamps and the corresponding values are listed in Table IIa for linear configuration and in Table IIb for coiled filaments.

5. NUMERICAL WORK

It will be worth recapitulating the numerical part described at various stages so far. Based on expression (6) the steady-state temperatures for typical lamps of 10, 100, 500, and 1000 W operational on 120 V were estimated followed by their light outputs values vide (12) for filaments supposed to have linear configurations. The lumen values so determined fell short of the reported ones (Table IIa).

This guided us to take up the shadow factor suggested by HS Leff. The shadow factor δ, which is unity for linear configuration and a value less than one for coiled shape, was decreased in the steps of 0.01 and through a program in GW-BASIC the integral (12) was evaluated by Simpson rule for luminous flux; this program took care of the fact that once the shadow factor δ is reduced the area of the filament also gets condensed through (8), the steady-state temperature rises (vide 6), and there is an enhancement of lumen output via the integral (12).

This process was carried out for each lamp until the lumen output matched with the consequent measured ones (Tables IIb). This provided us the steady-state temperature T_S and shadow factor δ which had reproduced the observed light output. Our next concern is the cooling-time for each lamp; for this purpose, we should have prior knowledge of the cold temperature T_{Cold}.

As mentioned above when the bulb is switched off the temperature and consequently light output falls and we have to locate the temperature where light has dropped to ten percent from the steady-state magnitude. Once again the program mentioned above was run to evaluate the integral (12) for the luminous flux by Simpson rule for each one degree Kelvin fall of temperature of the filament starting from the steady-state value until the objective is achieved (Table IIb). Finally, the expression (18) was employed to estimate cooling-times for each lamp consequent upon the temperature fall in the range $T_S \rightarrow T_{Cold}$ both for linear configuration and actual coiled case. Next, we focus on the conclusions.

6. DISCUSSIONS AND CONCLUSIONS

The incandescent coiled tungsten filament lamps are in the process of being phased out because of being poor efficient, nevertheless, they will continue to be source of illumination to the minds of physics students as evident from a large number of publications on this subject in the last three decades; the topics covered therein are the temperature and colour of the filament[10], efficiency and efficacy of the lamp[6], mortality statistics and life of the bulb[11,12], exponent – rules[13] and so on. On the other hand, no attention was paid to the topic cooling-times of these lamps and this has been accomplished here.

Of course, there are couple of methods to arrive at the operating temperature of a coiled tungsten filament lamp which consumes electric power for heating the filament to incandescence moreover quickly. For our purpose the temperature which yields the power of the bulb through Stefan-Boltzmann radiation is perfect. Under the supposition of linear configuration the light output during steady-state operation do not match for typical bulbs of 10, 100, 500, and 1000 W but rather fall short of the quoted values for each lamp. This finding supports the inclusion of coiling factor suggested by HS Leff[3].

The coiling not only reduces the radiating surface area, it enforces some of the area to radiate inward to the region bounded by the coil as well, resulting in higher filament temperatures. This aspect is achieved by multiplying the area of uncoiled filament by the shadow factor which is equal to one for linear configuration and a value less than one for the coiled case.

The shadow factor was decreased in the steps of 0.01 until the estimated luminous flux matched with consequent observed ones; a fall in the shadow factor results in higher coil temperature as well as the associated light output. In the next step the luminous flux output for one degree fall in temperature of cooling filament was evaluated starting from the steady-state temperature.

This ascertained the cold temperature at which light output had dropped to 10% from the steady state operation. Lastly the expression for cooling-time from temperature T_S to T_{Cold}, when the lamp is switched off, was derived. Based on this cooling-times were estimated for both the configurations viz. linear and coiled one. The salient findings may be concluded as follows.

- The supposition of linear configuration yields the luminous flux falling short of reported ones for typical lamps viz. 10, 100, 500, and 1000 W. As far as cooling-times are concerned its values for 10 and 100 watts bulbs are satisfactory but for 500 and 1000 W they fall short of reported ones.

- These findings favoured the role of coiling which reduces the exposed surface area so that the steady-state temperature as well as light outputs are having higher values as required by us.
- The introduction of shadow factor not only reproduced the luminous fluxes for each lamp the estimated cooling-times matched with those reported ones for 10, 100, and 500 W lamps. For 1000 W lamp the estimate falls marginally short; this requires correction in the reported luminous flux value by General Electric. According to Jones and Langmuir[4] one percent light is absorbed by the glass lamp bulb, in other words lumens inside the bulb would be one percent higher; this fact will bring maximum change in the reported value of lumens for 1000 W lamp compared to corrections in lower watts lamps. When this change was taken care of the cooling-time value was reproduced.
- It will be a good exercise for students having interest in theoretical physics to repeat the calculations for the lamps of 2000, 5000, and 10000 W whose characteristics data[1] are mentioned in Table III for luminous fluxes only; cooling-times data are not available.
- Those inclined towards experiments should design experiments for measuring the cooling-times of the above mentioned higher watts lamps for validating the theory.

References

[1] Incandescent Lamps, Pamphlet TP-110R2 (Nela Park, OH: General Electric Company, 1984) pp. 35, 24

[2] The incandescent lamps discussed in the above “bulletin are those that are commonly referred to as “large” lamps. This designation does not necessarily imply large physical size, but refers, usually, to lamps that are operated on standard-voltage circuits, universally found in residential, commercial, and industrial use.” p. 2 in the ref. 1

[3] Harvey S. Leff., Illuminating physics with light bulbs, *Phys. Teach* 28, 30-35 (1990).

[4] H. A. Jones and Irving Langmuir, The characteristics of tungsten filaments as functions of temperature: Part II, *Gen. Electr. Rev.* 30,354-361 (July 1937)

[5] M. R. Vukcevich, The Science of Incandescence, (Advanced Technology Department, GE Lighting, Nela Park, Cleveland, 1992), Chapter 9.

[6] D. C. Agrawal, H. S. Leff, and V. J. Menon, Efficiency and efficacy of incandescent lamps, *Am. J. Phys.* 64, 649- 654 (1996)

[7] D. C. Agrawal, Moon, Super-Moon, Planets of the Solar System and Star Vega: Brightness and Size. *J Phys Astron*. 5, 1-20 (2017).

[8] R. D. Larrabee, The spectral emissivity and optical properties of tungsten. Technical Report 328, (Research Laboratory of Electronics, MIT, Cambridge, Mass, May 21, 1957)

[9] W. H. Stephen and Chun T. Wang Tungsten Sources, Metallurgy, Properties and Applications (Plenum, New York, 1979) chapter 6

[10] W. S. Wagner, Temperature and color of incandescent lamps, *Phys. Teach* 29, 176-177 (1991).

[11] V. J. Menon, and D. C. Agrawal, Lifetimes of incandescent bulbs, *Phys. Teach* 41, 100-101 (2003).

[12] V. J. Menon, and D. C. Agrawal, A theory for the mortality curve of filament lamps, *Jour. Mater. Engg. Perf.* 16, 1-6 (2007).

[13] D. C. Agrawal,and V. J. Menon, Light bulb exponent-rules for the classroom, *IEEE Trans. Educ.* 43, 262-265 (2000).

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Table I. Operating data[1] for standard lamps designed for 120 V at room temperature 293 K.

Power of Lamp P $watts$	Length L_0 $meters$	Diameter D_0 $meters$	Gas Loss P_{Gas} %	End Loss P_{End} %	Bulb & Base Loss P_{Bulb} %
10*	0.43180	0.001626	–	1.5	5.0
100**	0.47498	0.006096	11.5	1.3	5.2
500**	0.87376	0.018034	8.8	1.8	7.1
1000**	1.016	0.02794	6.0	1.9	4.7

*Vacuum-single coiled **Gas filled-coiled-coil

Table IIa. Estimates of operating temperature and luminous flux, observed luminous flux[1], estimate of temperature of cold filament when the luminous flux has dropped to 10%, and estimated and observed cooling-times for filaments having linear configurations

Power of Lamp P $watts$	Steady-state temperature T_S in Kelvin	Estimated luminous flux in lumens $(\delta = 1)$	Observed luminous flux in lumen	Temperature at which luminous flux falls to 10% $(\delta = 1)$	Cooling time for range $T_S \rightarrow T_{Cold}$ for $(\delta = 1)$	Observed cooling time
10	2305	74	80	1898 K	0.020 s	0.02 s
100	2744	1736	1920	2190 K	0.052 s	0.06 s
500	2699	8528	10850	2160 K	0.159 s	0.19 s
1000	2755	17638	23740	2197 K	0.235 s	0.30 s

Table IIb. Estimated operating temperature, shadow factor δ, observed luminous flux, estimate of cold temperature while cooling when the luminous flux has dropped to 10%, calculated cooling-time and the observed cooling-time

Power of Lamp P $watts$	Steady-state temperature T_S in Kelvin	Estimated shadow factor δ which reproduces luminous flux of column 4	Observed luminous flux in lumen	Temperature at which luminous flux falls to 10%	Cooling time for range $T_S \to T_{Cold}$ in seconds vide (18)	Observed cooling time in seconds
10	2338	0.93	80	1920 K	0.022 s	0.02 s
100	2814	0.88	1920	2235 K	0.056 s	0.06 s
500	2903	0.68	10850	2292 K	0.199 s	0.19 s
1000	2990	0.66	23740	2389 K	0.267 s	0.30 s

Table III. Primary data for gas filled single coiled higher power lamps designed for 120 V at room temperature 293 K.

Watts	Length of filament in meters	Diameter of filament in meters	Approximate initial Lumens
2000	1.41478	0.0004572	58000
5000	1.12775	0.0007366	145000
10000	1.3843	0.0011684	335000