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# Determining Kinematic Characteristics of Planar Mechanisms' Driven Member Using Frenet Trihedron

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**Summary.** In lots of flat mechanisms the leading element is a crank, which is connected by a hinge to the driven member. The junction of these units, i.e. the crank, makes a circumference when rotated. The paper suggests to place the vertex of the Frenet trihedron in the junction, direct the principal unit normal vector to the circumference center, combine the unit tangent vector with the crank speed vector, that is, position it as a tangent to the circumference. When rotating the crank, the trihedron will also be rotated, and its principal unit normal vector will all the time coincide with the crank. Thus, the moving trihedron will accompany the circumference – its crank trajectory and the speed of its motion in a circumference will depend on the angular velocity of the crank rotation.

While rotating the crank, the Frenet trihedron will rotate as well, with the driven member in the form of a straight line segment passing across the vertex of the trihedron, and forming a certain angle with the unit tangent vector. The variation law of this angle will depend on the design and purpose of the mechanism. In order to get the kinematic characteristics of the driven member (its position depending on the angle of crank rotation, trajectory, velocity and acceleration of the random point), it is necessary to know the variation law of the angle of rotation of the driven member in the system of the moving trihedron in the function of the guide curve's arc length – the hinge motion trajectory.

The idea of this research lies in determining the kinematic characteristics of the complex motion of the point, when the latter performs relative motion in the moving coordinate system, and the system itself moves at a certain law towards the fixed system. If consider the convected trihedron of the curve as the moving coordinate system, than the law of the trihedron motion becomes known towards a fixed system. Thus, the rotation of the driven member around the vertex of the trihedron and simultaneous movement together with it determine the relative motion of the driven member towards the fixed coordinate system.

The position of the member is in projections on the unit vectors of the trihedron, and is immediately converted to the axis of the fixed system. The absolute trajectory of the member point movement is found in the same way, which in turn allows to define its velocity and acceleration. The resulting dependencies are common to the mechanisms' driven members, which are articulated by a hinge with a crank. For the specific mechanism, the law of rotation of the driven member in the moving trihedron system is the only thing to be known. The article uses examples of finding this law for certain mechanisms. It provides not only the charts of changes in velocity and acceleration of the individual points of the driven member, but also the direction along the member point's trajectory as a vector of the module, proportional to their size. This distribution of velocities and accelerations along the point movement trajectory may be performed with any density.

**Key words:** planar mechanism, crank, the driven member, Frenet trihedron, the relative motion of the point, trajectory, velocity, acceleration.

## INTRODUCTION

The kinematic analysis of the planar mechanisms involves finding the positions of its members, individual points' trajectories, their velocities and accelerations. For a long time these calculations were carried out by graphic and graphoanalytical methods, including graphical differentiation of functions in the form of a curve, and graphical integration. The computer technologies' emergence allowed us to work at a new level, applying the analytical apparatus.

As one of the possible approaches, in this article it is proposed to apply two coordinate systems: a moving convected trihedron of a circumference (trajectory of the crank ending's movement), and a fixed coordinate system. The angle of the trihedron rotation with respect to the fixed coordinate system is known: it is equal to the angle of the crank rotation. The position of the vertex of a trihedron in a fixed system is also known. Thus, it becomes possible to investigate the motion of the driven member, one end of which coincides with the vertex of the trihedron, in the trihedron system itself. In the future, the resulting kinematic characteristics are recalculated in projection on the axes of the fixed coordinate system.

## THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The study of trajectory curves of the mechanisms' member points' motion is of great importance in problems of synthesizing mechanisms. These are the tasks of the mechanisms' formation, which could reproduce the predefined curves. A group of such problems was resolved by academician P. L. Chebyshev, who applied the method of best function approximation, provided that the rod curve is a symmetric curve [1]. Some works on applied geometry are devoted to this topic [2-4]. A monograph [5] is devoted to finding a set of trajectory curves formed with the help of planetary mechanisms. The kinematics of the segment motion in the plane under the given conditions was considered in [6]. The use of the Frenet trihedron to determine the positions of the plane mechanism members is shown in [7]. The fundamental monographs [8, 9] are devoted to the study of the complex material point motion on the technological material particles' example.

The works about the particle moving along a rough surface are devoted to finding the moving point trajectory. The movement of soil particles along the plow's blade is considered in the monograph [10].

The simplest motion of a particle along an inclined plane is considered in [11, 12].

Finding the trajectory of a particle moving along a cylindrical surface under the influence of backup forces is depicted in [13].

A separate group is formed by the articles that target the particle moving along a rough surface under the influence of gravity, that is, on so-called gravitational surfaces [14-16].

A complex motion of a particle along an oscillating plane was investigated in [17, 18]. In [19], the relative motion of a particle along an internal rough surface of a rotational cone with a vertical axis of rotation is considered.

## OBJECTIVE

The purpose of the research is to find the positions of the planar mechanism's driven member, to determine the trajectories, velocities and accelerations of its individual points.

## THE MAIN RESULTS OF THE RESEARCH

When a trihedron moves along a plane curve, it rotates and makes an angle α at the current point with respect to the fixed coordinate system (Fig 1, a).

Its value depends on the curvature of the k curve.

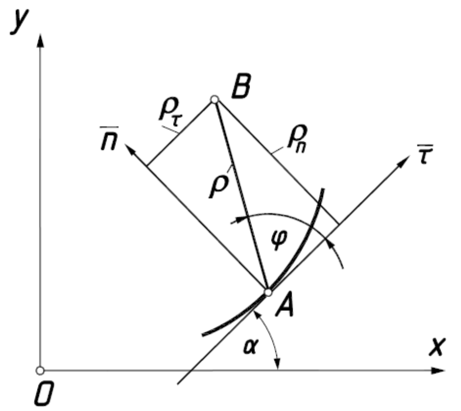
Curvature is a variable and is given by the natural equation $k=k(s)$, where $s$ is the arc length of the curve. The angular size is determined by integrating the expression $\alpha = \int k\, ds$.

For a circumference with a radius $r$ the curvature is the reciprocal of the radius $k=1/r$ – const, and the angle will equal $\alpha=ks$.

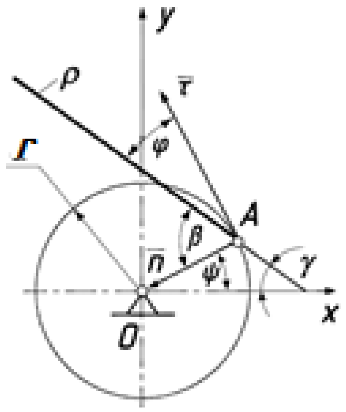
For many mechanisms, the leading member is the crank OA, the *point* A of which circumscribes a circle (Fig. 1, b). In this case, the principal unit normal vector of the trihedron $\bar{n}$ will coincide with the crank OA as it rotates, the unit vector $\bar{\tau}$ will be tangent to the circle, and the crank swing angle ψ will equal $\psi=ks$.

When the crank rotates at a constant angular velocity ω, its point A, which is the vertex of the trihedron, will move with a *constant* velocity $V=\omega r=\omega/k$.

The same point is the beginning of the driven member $\rho$, which forms an angle $\varphi$ with the unit vector $\bar{\tau}$, and an angle $\gamma$ with the $Ox$ axis (Fig.1, b).




a)




b)

**Fig. 1.** Graphic illustrations to the two-link planar mechanism scheme:

a) the position of the AB member in the system of the convected trihedron of the curve,

b) member ρ in the trihedron circular system of the the trajectory of the crank OA's point A.

There is a relationship between $\psi$, $\varphi$, *and* $\gamma$ angles, with which one can find the angle φ:

$$\varphi = 90^0 - (\gamma + ks). \quad (1)$$

The angle change dependency $\gamma = \gamma(s)$ is determined for each specific mechanism.

In [20], the relative motion of a point in the system of the convected trihedron of the curve, given by the natural equation $k=k(s)$, is considered. The coordinates of the point $B$ (Fig. 1, a) can be predefined by the projections $\rho\rho_\tau$ and $\rho_n$ or by the angle $\varphi$ and the distance $\rho$. Taking into account the fact that our guide curve is a circumference and $k=const$, the position of point $B$ in the fixed system will be written as [20]:

$$x_B = \rho\cos(\varphi + ks) + \frac{1}{k}\sin(ks),$$
$$y_B = \rho\sin(\varphi + ks) - \frac{1}{k}\cos(ks). \tag{2}$$

In [20], there are certain expressions for determining velocity and acceleration of the point $B$ in the projections onto the trihedron unit vectors, when both the quantities $\rho$ and $\varphi$ are variables and dependent on the guide curve's arc length $s$. In our case, the distance $\rho$ will be a constant value, indicating at what distance from the hinge A the given member point is located. In this connection, the formulas will be simplified. Expressions for the velocity $V$ in the projections onto the trihedron unit vectors are written as:

$$V_\tau = \frac{\omega}{k}\left[1 - \rho(k + \varphi')\sin\varphi\right],$$
$$V_n = \frac{\omega}{k}\rho(k + \varphi')\cos\varphi. \tag{3}$$

The acceleration projections $W$ look as follows [21]:

$$W_\tau = -\frac{\omega^2}{k^2}\left[\rho\varphi''\sin\varphi + \rho(k + \varphi')^2\cos\varphi\right],$$
$$W_n = \frac{\omega^2}{k^2}\left[\rho\varphi''\cos\varphi - \rho(k + \varphi')^2\sin\varphi + k\right]. \tag{4}$$

The values of both the velocity (3) and the acceleration (4) are found as the square root of the sum of the components' squares, that is, as a vector sum. If it is necessary to know the direction of the velocity or acceleration vector, then it is necessary to switch from the projections (3) and (4) to the projections on the axis of the fixed system, taking into account the known angle between them $\alpha=ks$.

Let us consider some specific examples. Take the de-axial crank-slider mechanism (Fig. 2).

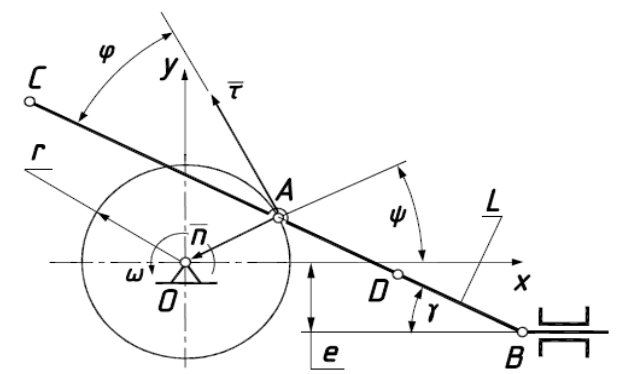



**Fig. 2.** The scheme of the crank-slider mechanism

Let us find the kinematic characteristics of the different points of the connecting rod L. To know the angle change dependency in the angle $\varphi$ (1), it is necessary to find the angle change dependency $\gamma=\gamma(s)$. To do this, let us use the fact that the ordinate of the point $A$ – $AB=L$ – is common for both the crank $OA$ and the connecting rod. For the point $A$ – the end of the crank $OA$, let us write:

$$y_A = r\sin\psi = \frac{1}{k}\sin(ks). \tag{5}$$

For the point $A$ – the end of the connecting rod $AB=L$ let us write:

$$y_A = L\sin\gamma - e. \tag{6}$$

By equating the expressions (5) and (6) and solving for the angle $\gamma$, we get:

$$\gamma = -\arcsin\frac{ek + \cos(ks)}{Lk}. \tag{7}$$

According to (1), the expression for the angle $\varphi$ takes the following form:

$$\varphi = 90^0 - ks + \arcsin\frac{ek + \cos(ks)}{Lk}. \tag{8}$$

To find the velocity and acceleration of a connecting rod's random point, it is necessary to have the first and the second derivatives of the expression (8). The first derivative looks as:

$$\varphi' = \frac{k\cos(ks)}{\sqrt{L^2k^2 - (ek + \sin(ks))^2}} - k. \tag{9}$$

The second derivative is obtained by differentiating the expression (9):

$$\varphi'' = \frac{2k^2\left(1 + e^2k^2 - L^2k^2\right)\sin(ks)}{2\left[L^2k^2 - (ek + \sin(ks))^2\right]^{3/2}} + \frac{ek^3(3 - \cos(2ks))}{2\left[L^2k^2 - (ek + \sin(ks))^2\right]^{3/2}}. \tag{10}$$

The expressions (8), (9) and (10) allow to find all the kinematic characteristics of any point of the connecting rod $L$ for a given distance $\rho$ from the point $A$.

In Fig. 3, the $AB=L$ connecting rod's positions are constructed, with a defined cranking rotation density $OA$ with an angle increment by the amount of $k\Delta s$ within the limits of its incomplete turn.

The trajectory of the connecting rod's starting point (point $A$) was determined by formulas (2) for $\rho=0$. The trajectory of the opposite point $B$ was at $\rho=AB=L$.

The connection of these points by a straight line segment, for a certain value of the parameter $s$, positions the connecting rod as a straight line segment.

Above the positions of the connecting rod *AB*, the trajectories of its individual points are constructed as well in Fig. 3, according to formulas (2).

When $\rho=0$, we get a circumference – the hinge movement trajectory.

When $\rho=AB=L$, we get a straight line – the slider movement trajectory (point *B*).

This proves the reliability of the obtained results.

Fig. 3 shows the trajectory of the point *A* constructed at $\rho=0$, as well as the trajectory of the points: *B* – for $\rho=AB=4m$, *C* for $\rho=-4m$, *D* for $\rho=L/2=2m$. Curvature $k=0{,}5\ m^{-1}$, $e=1m$.

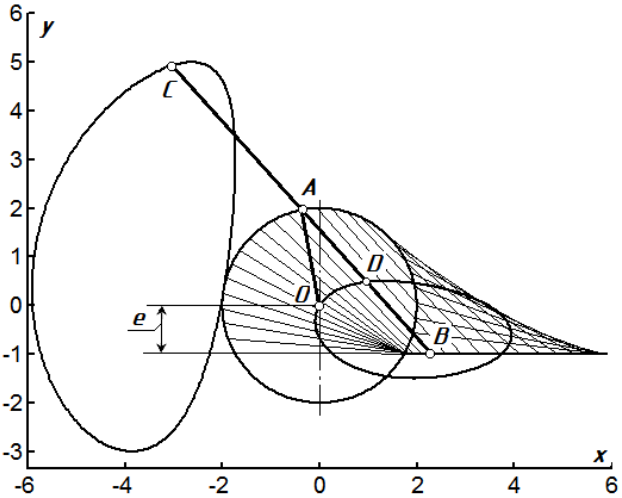



**Fig. 3.** One-parameter set of positions of the connecting rod and the trajectory of its individual points

Let us consider the construction of the connecting rod points' velocity. This will be done in a way that the direction and the value of the velocity along the point motion trajectory can clearly be seen. To do this, let us proceed from the velocity projections on the trihedron's unit vectors (3) to the projections on the fixed coordinate system's axes, rotating them by the angle $\alpha=ks$:

$$\begin{aligned} V_x &= V_\tau \cos(ks) - V_n \sin(ks), \\ V_y &= V_\tau \sin(ks) + V_n \cos(ks). \end{aligned} \quad (11)$$

Depending on the connecting rod's position (variable *s*), the coordinates of a certain point (for example, the point *C* at $\rho=-4$) are found from formulas (2). It is necessary to add the obtained vector (11), previously multiplied by the scale factor *m*, to the coordinates of this point. The end of the velocity vector coordinates are found:

$$\begin{aligned} x_V &= x_C + mV_x\,, \\ y_V &= y_B + mV_y\,. \end{aligned} \quad (12)$$

By connecting the point with the coordinates $x_C$ and $y_C$ on the trajectory with the coordinates of the end of the vector $x_V$ and $y_V$ by a segment, we obtain the velocity vector at a given point of the trajectory.

By increasing the variable *s* by some value $\Delta s$, we can construct vectors along the trajectory with the required density. In Fig. 4, the velocity vectors for the points *C* and *D* are plotted.

However, the visibility deteriorates in the sections of the trajectory close to the straight line, and totally disappears on the straight sections (for instance, for the trajectory of the point *B*).

The graph shows that the velocity of the point *A* is constant, and the velocity of the point *B* at a certain moment equals zero (in the extreme positions of the slider).

In the same sequence, we construct acceleration vectors using the expressions (4). Fig. 6 shows the visual distribution of the acceleration vectors along the *A*, *C* and *D* points' trajectory, and Fig. 7 – their value change graphs.

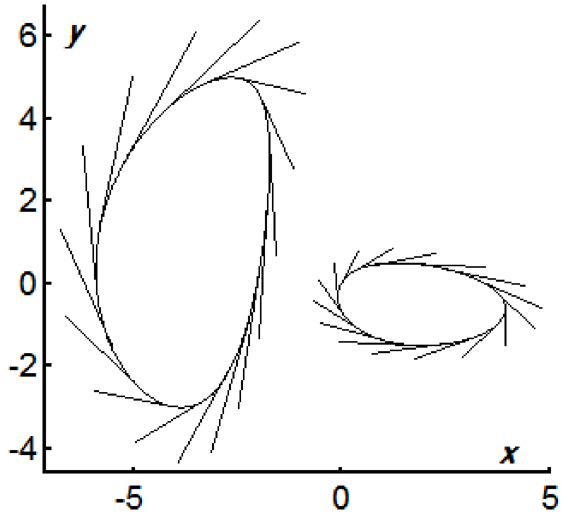



**Fig. 4.** Distribution of the velocity vectors along the trajectories of the points *C* and *D*

In such case, it is possible to construct a graph of the velocity value change. In Fig. 5 such graph is constructed, covering the four points of the connecting rod, which are indicated in Fig. 3.

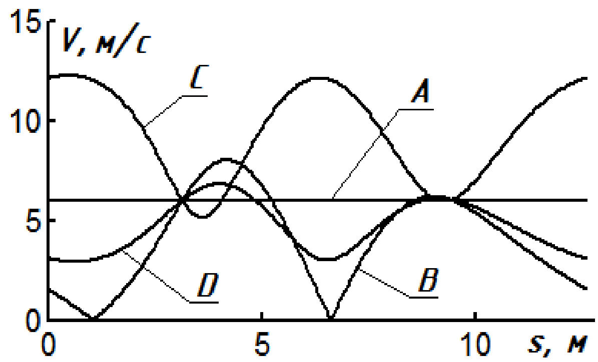



**Fig. 5.** A graph of the velocity value change for the connecting rod's points

To enable the mechanism work, the necessary relationships between the design parameters of the mechanism must be observed. This follows from the expression (7), in which the fraction in its absolute value should not exceed the unity. Fig. 8 shows some positions of the connecting rod and the individual points' trajectory for the following boundary values of the design parameters: $k=0{,}5\ m^{-1}$, $e=2\ m$, $L=4\ m$. In the extreme position of the slider, the connecting rod coincides with the crank along the vertical line.

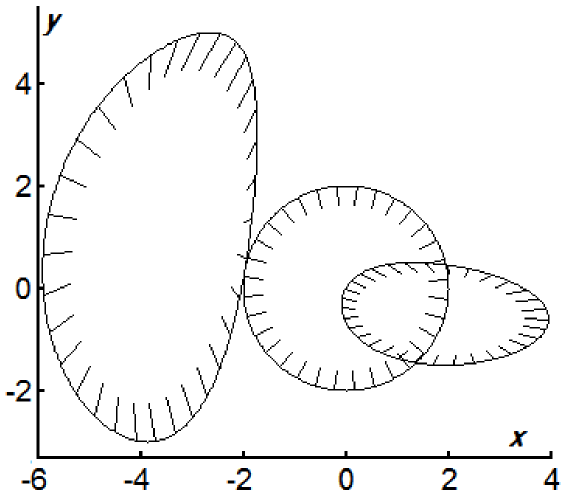



**Fig. 6.** Distribution of the acceleration vectors along the *A, C* and *D* points' trajectories.

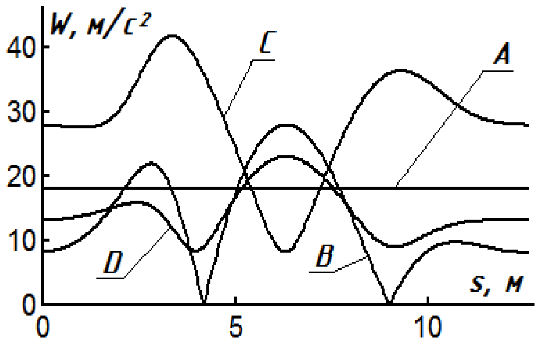



**Fig. 7.** A graph of the acceleration value change for the connecting rod's points

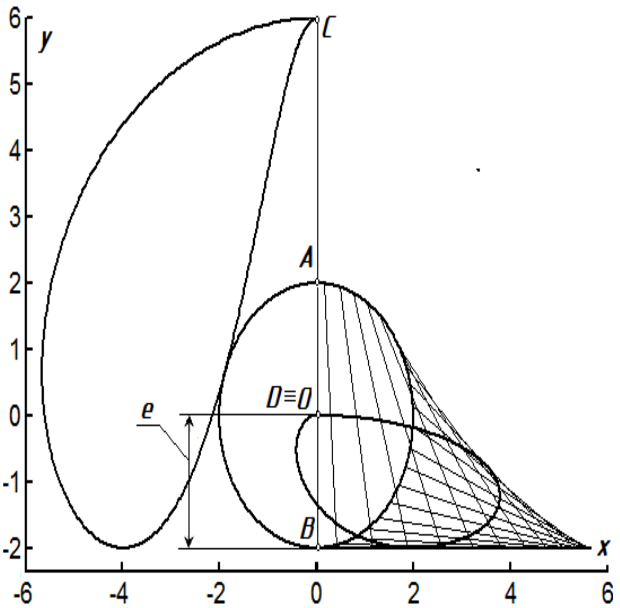



**Fig. 8.** One-parameter set of positions of the connecting rod and the trajectory of its individual points

In Fig. 9, the acceleration vectors are constructed along the *A*, *C* and *D* points' trajectories. With the analytical expressions for constructing velocity and acceleration vectors, it is very simple and quick to obtain their visual distribution along the trajectories, when the design parameters and the location of the point on the connecting rod change.

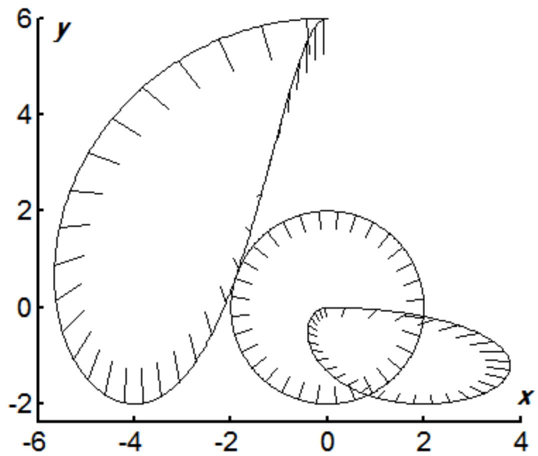



**Fig. 9.** Distribution of the acceleration vectors along the trajectories of the points *A*, *C* and *D*

Another mechanism with the boundary values of the design parameters: $k=0{,}5\ m^{-1}$, $e=1\ m$, $L=3\ m$ is shown in Fig. 10. It also demonstrates some of the connecting rod positions and points *A*, *B*, *C* and *D* trajectories. As in the previous case, in slider extreme position the connecting rod coincides with the crank along the vertical line.

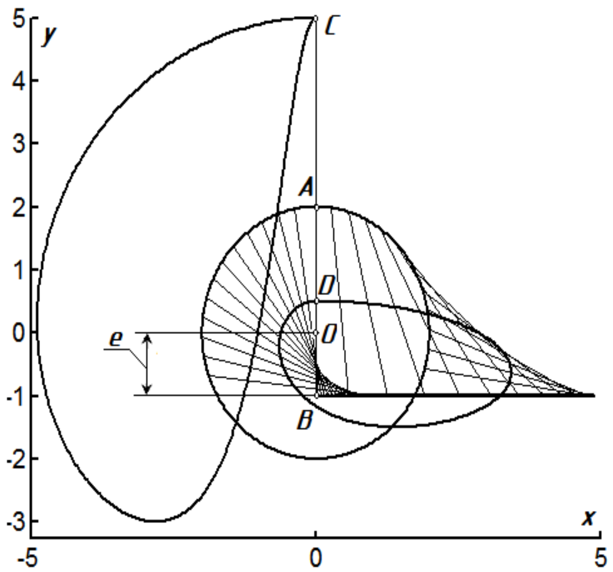



**Fig. 10.** One-parameter set of positions of the connecting rod and the trajectory of its individual points

In Fig. 11, the *A*, *C* and *D* points' trajectories are constructed with acceleration vectors for the mechanism (10).

One can judge about the point's velocity by the density of the vectors' arrangement along the trajectory.

For instance, for the point *C* at the top of the trajectory, the density of the vectors is lower, so the point's velocity will be greater.

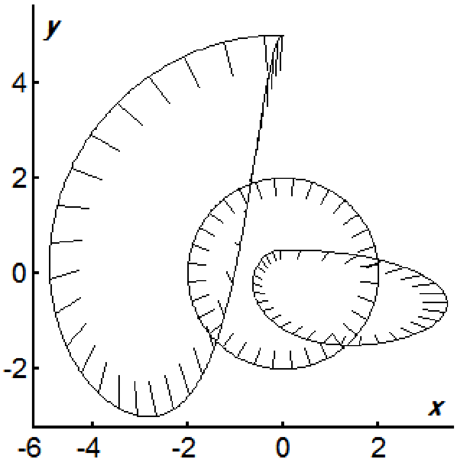



**Fig. 11.** Distribution of the acceleration vectors along the trajectories of the points *A*, *C* and *D*

Let us consider one more mechanism – a crank-rocker with points *A, C, D* on the rocker arm (Fig. 12). Its characteristic feature is that point *A* on the rocker arm moves along the circumference, and point *B* is fixed. This is ensured by sliding the rocker in a rocking or rotating stone, fixed at point *B*. To find the dependency of the angle $\varphi$ (1) changing, it is necessary to know the expression for the angle $\gamma$. The guide vector of the rocker arm is found as a segment connecting point *A* with the coordinates $\{\cos(ks)/k,\ \sin(ks)/k\}$ with the fixed point with the coordinates $\{0, d\}$.

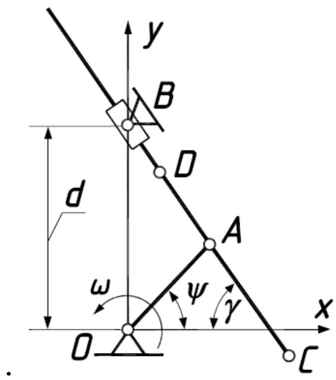



**Fig. 12.** The scheme of the crank-rocker mechanism

The vector coordinates will be:{cos*(ks)/k*, *d*-sin*(ks)/k*}.The $\gamma$ angle between the rocker arm's vector and the *Ox* axis is defined from the following expression:

$$\gamma = \arccos\frac{\cos(ks)}{\sqrt{\cos^2 ks + (kd - \sin ks)^2}} = \arccos\frac{\cos(ks)}{\sqrt{1 + k^2 d^2 - 2kd\sin ks}}. \quad (13)$$

According to (1), the expression for angle $\varphi$ is written as:

$$\varphi = 90^0 - ks - \arccos\frac{\cos(ks)}{\sqrt{1 + k^2 d^2 - 2kd\sin ks}}. \quad (14)$$

Let us find the first and the second derivatives of the expression (14):

$$\varphi' = -\frac{k\left(2 + d^2k^2 - 3dk\sin ks\right)}{1 + k^2d^2 - 2kd\sin ks}. \quad (15)$$

$$\varphi'' = \frac{dk^3\left(d^2k^2 - 1\right)\cos ks}{\left(1 + k^2d^2 - 2kd\sin ks\right)^2}. \quad (16)$$

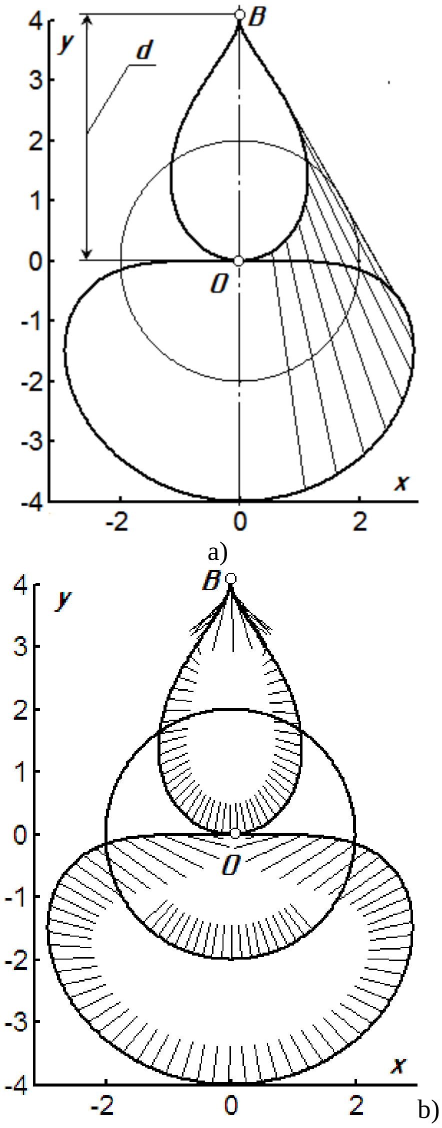



**Fig. 13.** Representation of the crank-rocker mechanism's kinematic elements:

a) points' trajectories and some rocker arm positions,

b) acceleration vectors' distribution along the trajectories.

Expressions (14), (15), (16) are sufficient to construct all the kinematic characteristics of the rocker arm points. Some rocker arm positions and the *A*, *C* and *D* points' trajectory are built in Fig. 13,a for $k=0{,}5\ m^{-1}$, $d=4\ m$, and the distance $\rho=\pm 4\ \ m$ from points $C$ and $D$ to point $A$. Fig. 13,b presents a graphic representation of the acceleration vectors of these points along the trajectories of their motion, and Fig. 14 is a graph of the change in their values. It follows from the graph that at a certain moment the acceleration of one of the rocker arm points is zero.

It can be visually determined from Fig. 13,b that this point belongs to the lower trajectory (the motion of the point $C$), when it coincides with the point $O$.

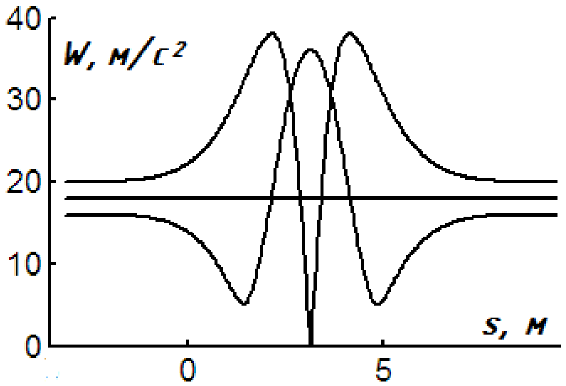



**Fig. 14.** A graph of the acceleration value change for the rocker arm's points

While analyzing expression (16) it can be seen that in the case $1/k=r=d$, the angular acceleration of the rocker arm in the trihedron system will equal zero, i.e. the angular velocity of its rotation will be constant. The family of rocker arm positions of such mechanism is shown in Fig. 15.

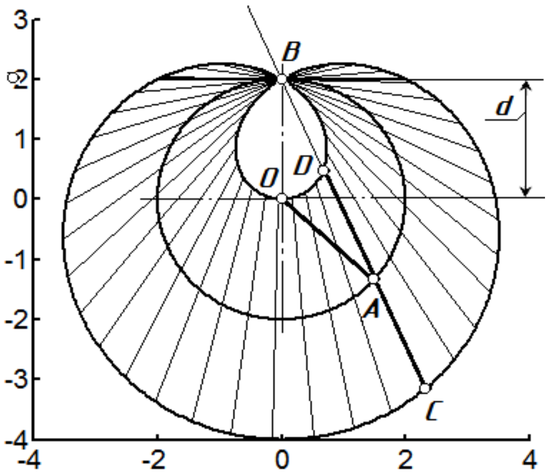



**Fig. 15.** Trajectories of the points and the position of the driven member in a special case of the crank-rocker mechanism

Its characteristic feature is that in the absence of an $AC$ segment the pattern would not change.

When lifting up, point $D$, moving along the internal curve, after passing point $B$ starts to move along the outer curve, and eventually takes the place of point $C$.

The segment of the rocker arm $AD$ alternately occupies the inner and the outer spaces, delineated by the circumference – the trajectory of point $A$.

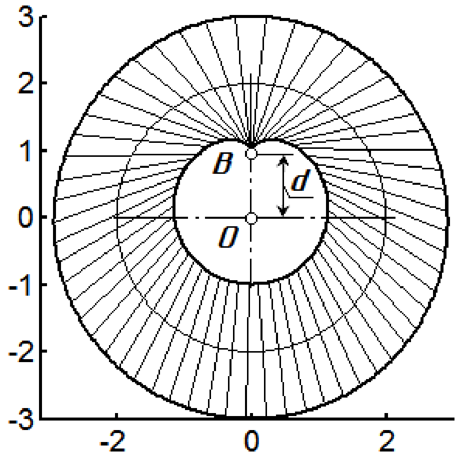



**Fig. 16.** Trajectories of the points and the position of the rocker arm mechanism when $k=0{,}5$ and $d=1$

If the stone rocks while the mechanism shown in Fig. 12 works, provided that $d>r$, then at $d=r$ (Fig. 15) it already rotates. Another illustration of the mechanism with a rotating stone for $d<r$ is shown in Fig. 16.

## CONCLUSIONS

1. In certain planar mechanisms the leading member is a crank, which rotates with the permanent angular velocity. The trajectory of the crank's ending movement is a circumference, which is to be taken as a guide curve for the convected Frenet trihedron. The trihedron moves along the circumference in such a way that its main unit normal vector coincides with the crank.
2. The motion of the driven member is described analytically in the trihedron system. This allows to receive general relationships for determining all the necessary kinematic characteristics of the driven member: the family of its positions, the trajectories of the individual points' motion, their velocities and accelerations. For this, it is necessary to find the law of rotation of the driven member in the trihedron system for each mechanism.
3. The developed approach makes it possible to construct a visual representation of the velocity and acceleration vectors' distribution of the driven member's points along their curvilinear trajectory with the required density.

## REFERENCES

1. **Levitskaya O. N., Levitskiy N. I. 1985.** The course of the theory of mechanisms and machines. Moscow. High school. 279. (in Russian).
2. **Berger E. G. 1982.** Method of geometrical and mechanical education of rational curves 3-d and 4-th order / E. G. Berger, V. P. Tabacco // Applied geometry and engineering graphics. Kiev. Budivelnik. Vol. 33. 88-89. (in Russian).
3. **Petisco A. V. 1971.** Playing dumb coils / A. V. Petisco, S. Kobezsky // Applied geometry and engineering graphics. Kiev. Budivelnik. Vol. 13. 84-85. (in Ukrainian).

4. **Subasinghe G. P. 2007.** Geometric methods, kinematic analysis of plane lever mechanisms of high classes / Subasinghe G. P., A. G. Korchenko, T. V. Popkova, N. G. Makarenko, V. P. Shcherbina // Applied geometry and engineering graphics. Kiev. Knuca. Vol. 77. 80-84. (in Ukrainian).
5. **Rosoha S. V. 2007.** Geometric modeling of volume of the working chambers of rotary-planetary Trojan cars / S. V. Rosoha, L. M. Kutsenko. Kharkiv. WCSU. 176. (in Ukrainian).
6. **Pylypaka S. F. 2007.** Kinematics of the segment, the ends of which describe the given lines in the plane / S. F. Pilipuk, V. M. Babka, T. S. Pilipaka // Applied geometry and engineering graphics. Kiev. Knuca, Vol. 77. 36-42. (in Ukrainian).
7. **Capini A. V. 2012.** Determination of the positions of links of the planar mechanism with a system of triangular Fresnes / Capini A. V., Dragonfly V. M. // Applied geometry and engineering graphics. Kiev. Knuca. Vol. 90. 20-26. (in Ukrainian).
8. **Vasilenko P. M. 1960.** Theory of the motion of particles on rough surfaces agricultural machines / P. M. Vasilenko. Kiev. WASHN, 283. (in Russian).
9. **Zaika P. M. 1992.** Special problems in agricultural mechanics / Zaika P. M. Kyiv. Publishing house UAA. 507. (in Russian).
10. **Gachev L. V. 1961.** Theory of share-moldboard surface / L. V. Gyachev. Zernograd, 317. (in Russian).
11. **Pylypaka S. F. 2011.** Trajectory of the particle on a rough inclined plane at their lateral flow / S. F. Pylypaka, Nesvidomyn A. V. // Applied geometry and engineering graphics. Kiev. KNUCA. Vol. 87. 36-42. (in Ukrainian).
12. **Pylypaka S. F. 2011.** Automation of modeling of the motion of the particle in the gravitational surfaces, for example inclined plane in the system Maple / S. F. Pylypaka, Nesvidomyn A. V. // Applied geometry and engineering graphics. Kiev. KNUCA. Vol. 86. 64-69. (in Ukrainian).
13. **Voytyuk D. G. 1999.** Identification of trajectories of movement of soil particles along the cylindrical surface of the working bodies of tillers / D. G. Voytyuk, S. F. Pilipaka // Mechanization of agricultural production. Volume 5. Kiev. NAU. 242-251. (in Ukrainian).
14. **Voytyuk D. G. 2002.** Determination of trajectory of material particles in the gravity bar with horizontal surfaces creative / D. G. Voytyuk, S. F. Pilipaka // Mechanization of agricultural production. Vol. 12. Kiev. NAU. 58-69. (in Ukrainian).
15. **Voytyuk D. G. 2003.** Finding the trajectory of a material point in the gravitational ressorti surface for example razgromnogo helix / D. G. Voytyuk, S. F. Pilipaka // Mechanization and power engineering of agriculture. IV international scientific-technical conference MOTROL-2003. Kiev. NAU. Volume 6. 113-126. (in Ukrainian).
16. **Voytyuk D. G. 2003.** Peculiarities of motion of material particle in gravitational ruled surfaces / D. G. Voytyuk, S. F. Pilipaka // Bulletin of Kharkiv state technical University of agriculture. Vol. 21. Kharkov. 75-88. (in Ukrainian).
17. **Pylypaka S. F. 2013.** Interaction of an inclined plane, all points of which the sustained oscillation describe ellipses, with the particles of the material / S. F. Pylypaka, M. D. Klendiy // Mechanization and electrification of agriculture. Glevaha. Vol. 98. P. 1. 574-587. (in Ukrainian).
18. **Klendiy M. 2014.** Study of the complex motion of a particle on an inclined plane, oscillates. V. V. Adamchuk, V. M. Bulgakov, S. F. Pylypaka // Bulletin of Zhytomyr national agroecological University. Zhitomir: INEU. № 2 (45), vol. 4, part 1. 54-63. (in Ukrainian).
19. **Klendiy M. 2015.** The particle motion on the internal rough surface of a rotating cone with vertical axis / S. Pylypaka // MOTROL. Commission of motorization and energetics in agriculture. Vol 17. Lublin-Preszow. No. 3. 73-83.
20. **Pylypaka S. F. 2006.** Theory of complex motion of a material point on the plane. Part one. Absolute speed and trajectory / S. F. Pylypaka // Electrical and mechanical engineering. No. 1. 84-94. (in Ukrainian).
21. **Pylypaka S. F. 2006.** Theory of complex motion of a material point on the plane. Part two. Absolute acceleration. Tasks for momentum points / S. F. Pylypaka // Electrical and mechanical engineering. Kiev. No. 2. 88-100. (in Ukrainian).


## ОПРЕДЕЛЕНИЕ КИНЕМАТИЧЕСКИХ ХАРАКТЕРИСТИК ВЕДОМОГО ЗВЕНА ПЛОСКИХ МЕХАНИЗМОВ С ПОМОЩЬЮ ТРЕХГРАННИКА ФРЕНЕ

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**Аннотация.** У многих плоских механизмов ведущим звеном является кривошип, который посредством шарнира соединен с ведомым звеном. Точка соединения этих звеньев, то есть кривошип, при вращении описывает окружность. В статье предлагается в точку соединения звеньев поместить вершину трехгранника Френе, орт главной нормали направить к центру окружности, орт касательной совместить с вектором скорости кривошипа, то есть расположить по касательной к окружности. При вращении кривошипа трехгранник тоже будет вращаться, причем его главная нормаль все время будет совпадать с кривошипом. Таким образом, подвижный трехгранник будет сопровождающим для окружности – траектории движения кривошипа и скорость его движения по окружности будет зависеть от угловой скорости вращения кривошипа.

При вращении кривошипа вместе с ним будет вращаться трехгранник Френе, при этом ведомое звено в виде прямолинейного отрезка будет проходить через вершину трехгранника и образовывать с ортом касательной определенный угол. Закон изменения этого угла будет зависеть от конструкции и назначения механизма. Чтобы получить кинематические характеристики ведомого

звена (его положение в зависимости угла поворота кривошипа, траекторию, скорость и ускорение произвольной точки), необходимо знать закон изменения угла поворота ведомого звена в системе подвижного трехгранника в функции длины дуги направляющей кривой – траектории движения шарнира.

Идея работы состоит в нахождении кинематических характеристик сложного движения точки, когда она совершает относительное движение в подвижной системе координат, а сама подвижная система по определенному закону движется по отношению к неподвижной системе. Если за подвижную систему координат взять сопровождающий трехгранник кривой, то закон движения трехгранника становится известным по отношению к неподвижной системе. Таким образом поворот ведомого звена вокруг вершины трехгранника и одновременное движение вместе с ним определяет относительное движение ведомого звена по отношению к неподвижной системе координат.

Положение звена находится в проекциях на орты трехгранника и сразу же пересчитывается на оси неподвижной системы. Таким же образом находится абсолютная траектория движения точки звена, что в свою очередь позволяет найти ее скорость и ускорение. Найденные зависимости являются общими для ведомых звеньев механизмов, которые сочленены посредством шарнира с кривошипом. Для конкретного механизма нужно знать только закон поворота ведомого звена в системе подвижного трехгранника. В статье наведены примеры нахождения этого закона для некоторых механизмов. Построены не только графики изменения величины скорости и ускорения отдельных точек ведомого звена, но и их направление вдоль траектории точки звена в виде вектора с модулем, пропорциональным их величине. Такое распределение векторов скоростей и ускорений вдоль траектории движения точки может быть выполнено с любой плотностью.

**Ключевые слова:** плоский механизм, кривошип, ведомое звено, подвижный трехгранник Френе, относительное движение точки, траектория, скорость, ускорение.