Spatial contact problems for elastic layer in case of flat areas of contact

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Received January 23.2014: accepted February 14.2014

S u m m a r y . The following spatial contact problems of the theory of elasticity of strip width 2a stamp sphere forcing in elastic layer of finite thickness h: lying without friction on the hard grounds, rigidly connected to holds its ground. In the area of contact between the die and the friction layer. The stamp is pressed into a layer of force *P*, related to the unit of length, and moments - M_x, M_y. The asymptotic expressions for determination of contact normal voltages under the stamp.

K e y w o r d s . Elastic layer elastic half-space, integral transforms, normal contact stress, hard rubber stamp.

INTRODUCTION

Contact mechanics of deformable solid body interaction is currently the most active and growing field of continuum mechanics. Contact problems of the theory of elasticity are finding more and more applications in the engineering calculations.

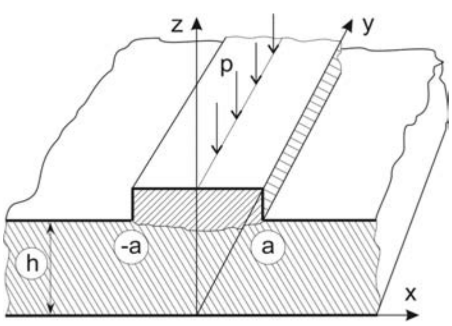

Fig.1. Design scheme

The paper considers the problem of contact interaction of spatial rigid Strip stamp with elastic layer thickness h (Fig. 1).

RESEARCH ANALYSIS

Major publications on the subject are in the works, which contain an overview of the main scientific results on the contact flat static [5, 7, 12, 14, 29], spatial static [1-4, 11], dynamic flat panel [8, 9, 19, 26, 30], spatial dynamic [6, 10, 13, 24, 28], thermo elastic [22, 25, 26, 27, 18], objectives for elastic contact problems of the theory of viscoelasticity [16, 17, 20, 21, 23].

Outlines the mathematical methods of decision of plane and space problems with various boundary conditions at the sites of the contact.

RESEARCH OBJECT

The objective of the proposed work is research of spatial contact problems for elastic layer in case of flat areas of contact and the definition of normal contact stresses under the stamp.

RESULTS OF RESEARCH

1. The task of the integral equation we obtain from these same tasks of integral equation for the free-form contact Ω derived in [1]:

$$\iint_{\Omega} q(\xi,\eta)d\xi d\eta \int_o^{\infty}\int_o^{\infty} \frac{L(u)}{u}\cdot\cos\frac{s}{n}(\xi-x)\cos\frac{t}{n}(\eta-y)dtds =$$

$$= \Delta h\pi^2\delta(x,y)\,,\quad (x,y)\in\Omega\,,\quad u=\sqrt{s^2+t^2}\,. \quad (1.1)$$

Here: $q(x,y)$ – contact pressure distribution function, $\delta(x,y)$ – function of the sediment surface layer points of contact Ω, $\Delta = G(1-\nu)^{-1}$, G,ν – shear modulus of the material layer and Poisson's ratio.

Function $L(u)$ for targets respectively are:

$$1 - \quad L(u)=\frac{ch2u-1}{sh2u+2u}\,, \quad (1.2)$$

$$2 - \quad L(u)=\frac{2k_1 sh2u-4u}{2k_1 ch2u+1+k_1^2+4u^2}\,,$$

$k_1 = 3-4\nu$, $\delta(x,y)=\delta+\alpha x+\beta y - f(x,y)$,

$f(x,y)$ – function of the base surface,

$\delta+\alpha x+\beta y$ – move a stamp under the action of forces *P* and moments M_x, M_y.

After solving the equation (1.1) relationship between *P*, M_x, M_y, the efforts of the stamp, and the values α,β,δ determined from the ratio of:

$$P=\iint_{\Omega} q(\xi,\eta)d\xi d\eta\,,\qquad M_x=\iint_{\Omega}\eta q(\xi,\eta)d\xi d\eta\,,$$

$$M_y=\iint_{\Omega}\xi q(\xi,\eta)d\xi d\eta\,. \quad (1.3)$$

Get the integral equation for contact problems of the contact area in the form of endless bands $(|x|\le a, |y|<\infty)$.

Enter into the transformanty Fourier transform functions $\delta(x,y)$, $q(\xi,\eta)$, ratios:

$$\delta(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\delta_\beta(x)e^{-i\beta y}d\beta\,,$$

$$\delta_\beta(x)=\int_{-\infty}^{\infty}\delta(x,y)e^{i\beta y}dy\,. \quad (1.4)$$

$$q(\xi,\eta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}q_\beta(\xi)e^{-i\beta\eta}d\beta\,,$$

$$q(\xi)=\int_{-\infty}^{\infty}q(\xi,\eta)e^{i\beta y}d\eta\,. \quad (1.5)$$

Based on (1.4) and (1.5) integral equation (1.1) takes the form:

$$\int_{-a}^{a} q_\beta(\xi)d\xi\int_o^{\infty}\frac{L(u)}{u}\cos\frac{s}{h}(\xi-x)ds=\pi\Delta\delta_\beta(x),$$

$$|x|\le a\,. \quad (1.6)$$

For the case of rolled stamp (1.3) should be replaced by the following:

$$P(y)=\int_{-a}^{a}q(x,y)dx\,,\quad M(y)=\int_{-a}^{a}xq(x,y)dx\,, \quad (1.7)$$

$P(y)$, $M(y)$ – force and moment acting on the section of the stamp.

2. Properties of the core and the representation of the solution of integral equation contact problems for the layer.

We will specify some of the General properties of the kernel integral equation (1.6) into the form:

$$\int_{-a}^{a}q_\beta(\xi)k(b,s)d\xi=\pi\Delta\delta_\beta(x)\,,|x|\le a\,, \quad (2.1)$$

$$k(b,s)=\int_o^{\infty}\frac{L(u)}{u}\cos(\nu s)d\nu\,, \quad (2.2)$$

here: $b=\beta h$, $s=(x-\xi)h^{-1}$, $u=(\nu^2+b^2)^{1/2}$.

Transform kernel (2.2) as follows:

$$k(b,s)=K_o(b,s)-F(b,s)\,. \quad (2.3)$$

In the expression (2.3) $K_o(b,s)$ - Macdonald function:

$$F(b,s) = \int_o^{\infty} [1 - L(u)] u^{-1} \cos(\nu s) d\nu . \qquad (2.4)$$

Based on the properties of functions $L(u)$, $(L(u) \to 1$ when $u \to \infty$, $L(u) \to Au$ when $u \to 0$), it is easy to show that even on the function $F(b,s)$ is continuous, together with all its derivatives at $-\infty < s < \infty$.

As regards $K_o(b,s)$, then, as it is known, the Macdonald at zero point behaves $\ln|bs|$ like the infinity decays like $|bs|^{-1/2} e^{-|bs|}$. For all other values of the argument is continuous, together with all its derivatives.

Will continue to explore the properties of the function $F(b,s)$.

Laying $\cos(\nu s)$ out under the integral of (2.4) in a number of νs, introduce $F(b,s)$ in the form:

$$F(b,s) = \sum_{i=0}^{\infty} C_i(b) s^{2i} , \qquad (2.5)$$

where:

$$c_i = \frac{(-1)^i}{(2i)!} \int_0^{\infty} \frac{[1 - L(u)]}{u} \nu^{2i} d\nu, (i = 0,1,2,...) . \qquad (2.6)$$

Let us rewrite equation (2.1) based on (2.3) in the form of

$$\int_{-a}^{a} q_\beta(\xi) K_o[\beta(\xi - x] d\xi = \pi \Delta \delta_\beta(x) + \int_{-a}^{a} q_\beta(\xi) F(b,s) d\xi ,$$

$$|x| \le a . \qquad (2.7)$$

We will find the solution to the equation (2.7) in the class $L_p(-a,a)$, $1 < p < 2$. Then based on the properties $F(b,s)$, when $b > 0$, you can easily conclude that function:

$$\varphi(x) = \int_{-a}^{a} q_\beta(\xi) F(b,s) \cdot d\xi , \qquad (2.8)$$

is continuing with all its derivatives on $x \in [-a,a]$.

When $\lambda_1 = ha^{-\infty}$ the function (2.8) takes the form:

$$\varphi(x) = c_o(b) P_\beta , \quad P_\beta = \int_{-a}^{a} q_\beta(\xi) d\xi . \qquad (2.9)$$

If the relative thickness of the layer is so great that b – it is quite large, then given the asymptotic estimate for the numbers $c_i(b) \sim O(b^{i+3/2} e^{-2b})$ for large values b, can conclude $\varphi(x) = 0$.

Therefore, when $\lambda_1 = \infty$, if that fits the occasion of elastic half-space, the surface integral equation for strip stamp (2.7) will take the form of:

$$\int_{-a}^{a} q_\beta(\beta) K_o[\beta(\xi - x)] d\xi = \pi \Delta \delta_\beta(x) , \quad |x| \le a . \qquad (2.10)$$

Note now that because of the properties of the function $\varphi(x)$ at all λ_1 and b the nature and characteristics of the integral equation (2.7) is defined by equation (2.10).

3. Solution of contact problems with large values of the parameter $\lambda_1 = ha^{-1}$.

Asymptotic at the big λ_1 challenges for the Strip to stamp can only be obtained based on integral equation (2.7), which, taking into account the (2.5) will be in the form:

$$\int_{-a}^{a} q_\beta(\beta) K_o[\beta(\xi - x)] d\xi = \pi \Delta \delta_\beta(x) +$$

$$+ \sum_{i=0}^{\infty} \frac{c_i(b)}{h^{2i}} \cdot \int_{-a}^{a} q_\beta(\xi)(\xi - x)^{2i} d\xi , \ |x| \le a . \qquad (3.1)$$

As the formula (2.5) occurs when $|s| = |(x - \xi)h^{-1}| < 2$, $\max|x - \xi| = 2a$, while equation (3.1) and all the results to be obtained from it will make sense, at least when $\lambda_1 > 1$.

Solution of integral equation (3.1) will search in the form of the following asymptotic number of degrees h^{-1}:

$$q_\beta(\xi) = \sum_{\hat{e}=0}^{\infty} q_{\beta\hat{e}}(\xi) \cdot \frac{1}{h^{2\hat{e}}} . \qquad (3.2)$$

Substituting the expression (3.2) in equation (3.1) and equating members under the same degrees, h^{-1} we get an infinite system of integral equations for sequential definitions $q_{\beta\hat{e}}(\xi)$:

a) $$\int_{-a}^{a} q_{\beta 0}(\xi) K_o[\beta(\xi - x)] d\xi = \pi\Delta\delta_\beta(x) + $$

$$+ c_o(b)\int_{-a}^{a} q_{\beta 0}(\xi) d\xi ,$$

b) $$\int_{-a}^{a} q_{\beta 1}(\xi) K_o[\beta(\xi - x)] = c_1(b)\int_{-a}^{a} q_{\beta 0}(\xi)(\xi - x)^2 d\xi +$$

$$+ c_0(b)\int_{-a}^{a} q_{\beta 1}(\xi) d\xi ,$$

c) $$\int_{-a}^{a} q_{\beta 2}(\xi) K_o[\beta(\xi - x)] d\xi = c_2(b)\int_{-a}^{a} q_{\beta 0}(\xi) \times$$

$$\times (\xi - x)^4 d\xi + + c_1(b)\int_{-a}^{a} q_{\beta 1}(\xi)(\xi - x)^2 d\xi +$$

$$+ c_0(b)\int_{-a}^{a} q_{\beta 2}(\xi) d\xi , |x| \le a , \qquad (3.3)$$

etc.

Obviously, the equation a) system of equations (3.3) coincides with an integral equation, the occasion of very large relative layer thicknesses.

Found the approximate solution of integral equation of the contact problem for elastic half-space [1], which is different from the equation a) (3.3) the last element on the right side. Since this term has a functionality, the corresponding approximate solution of integral equation a) and can be considered notable.

3.1. Solution of integral equations (3.3) by the method of successive approximations.

We will write the system (3.3) in another form:

a) $$\int_{-a}^{a} q_{\beta 0}(\xi)\{K_o[\beta(\xi - x)] - c_o(b)\} d\xi = \pi\Delta\delta_\beta(x) ,$$

b) $$\int_{-a}^{a} q_{\beta 1}(\xi)\{K_o[\beta(\xi - x)] - c_o(b)\} d\xi =$$

$$= c_1(b)\int_{-a}^{a} q_{\beta 0}(\xi)(\xi - x)^2 d\xi , \qquad (3.4)$$

c) $$\int_{-a}^{a} q_{\beta 2}(\xi)\{K_o[\beta(\xi - x)] - c_o(b)\} d\xi =$$

$$= c_2(b)\int_{-a}^{a} q_{\beta 0}(\xi)(\xi - x)^4 d\xi + c_1(b)\int_{-a}^{a} q_{\beta 1}(\xi)(\xi - x)^2 d\xi ,$$

$$|x| \le a .$$

Further, consider the case:

$$\delta_\beta(x) = \delta_\beta = const .$$

Let's move into the equation a) system (3.4) to have variables $\xi = \xi' a$, $x = x' a$, $\delta_\beta = \delta_\beta \cdot a^{-1}$, $\varphi_o(\beta) = q_{\beta 0}(\xi' a)$ and putting down the finishing touches, we get:

$$\int_{-1}^{1} \varphi_0(\xi) k\left(b, \frac{\xi - x}{\lambda}\right) d\xi = \pi\Delta\delta_\beta , \ |x| \le 1 , \ \lambda = (a\beta)^{-1}, \qquad (3.5)$$

$$k(b,t) = \int_0^\infty \frac{\cos tu}{\sqrt{u^2 + 1}} du + c_0(b) . \qquad (3.6)$$

Using asymptotic representation for Macdonald $K_0(t)$ at small t, get the idea for the kernel $k(b,t)$ in the form:

$$k(b,t) = -\ln|t| + \sum_{K=0}^{\infty} a_k t^{2k} + \ln|t| \sum_{k=1}^{\infty} b_k t^{2k} - c_o(b) . \qquad (3.7)$$

Write some first coefficients of decomposition (3.7):

$$a_0 = 0.1159 + c_o(b) ,$$

$$a_1 = 0.2790, \qquad b_1 = -0.2500 ,$$

$$a_2 = 0.2525, \qquad b_2 = -0.01563 .$$

Substituting (3.7) in (3.5) and drawing the logarithmic part, here is the equation (3.5) equivalent $L_p(-1,1)$, $1 < p < 2$ to the integral equation of the second kind [2]:

$$\varphi_0(x)=\frac{1}{\pi\sqrt{1-x^2}}\left[P_{\beta 0}-\int_{-1}^{1}\frac{\psi'(\tau)\sqrt{1-\tau^2}}{\tau-x}d\tau\right], \quad (3.8)$$

$$P_{\beta 0}=\int_{-1}^{1}\varphi_0(\xi)d\beta=\int_{-1}^{1}\frac{\psi(\tau)d\tau}{\sqrt{1-\tau^2}} \quad . \qquad (3.9)$$

Here:

$$\psi(x)=\Delta\delta_\beta-\frac{1}{\pi}\int_{-1}^{1}\varphi_0(\xi)\left[\sum_{K=0}^{\infty}\lambda^{-2k}(\xi-x)^{2k}+\right.$$

$$+(\ln|\xi-x|-\ln\lambda)\times$$

$$\left.\times\sum_{K=1}^{\infty}b_k\lambda^{-2k}(\xi-x)^{2k}-c_0(b)\right]d\xi . \qquad (3.10)$$

The solution to the equation (3.8) will be looking for in the form of [3]:

$$\varphi_0(x)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\varphi_{ij}(x)\lambda^{-2}\ln^j\lambda . \qquad (3.11)$$

Substituting the expression (3.11) in (3.8) and equating members of the left and right parts with identical grades $\lambda^{-2i}\ln^j\lambda$ will have a number of relationships, of which consistently define $\varphi_{ij}(x)$. Then, from equations (3.9) to find the value $P_{\beta 0}$.

Thus, the asymptotic solution of integral equation (3.5) can be represented in the form:

$$\varphi_0(x)=\frac{P_{\beta o}}{\pi\sqrt{1-x^2}}\left[1+A_1\lambda^{-2}+A_2\lambda^{-4}-\right.$$

$$-(2A_1\lambda^{-2}+A_3\lambda^{-4})x^2+A_4\lambda^{-4}x^4+O(\lambda^{-6}\ln^3\lambda),$$

$$P_{\beta 0}=\pi\Delta\delta_\beta\left[a_0-c_0(b)+\ln 2\lambda+\right.$$

$$+C_1\lambda^{-2}+C_3\lambda^{-4}+O(\lambda^{-6}\ln^3\lambda),$$

$$A_1=\left(a_1+\frac{3}{2}b_1-b_1\ln 2\lambda\right), \qquad (3.12)$$

$$A_2=\frac{7}{2}a_2+\frac{103}{24}b_2+\frac{7}{2}b_2\ln 2\lambda-\frac{5}{12}b_1A_1,$$

$$A_3=4(a_2+\frac{7}{12}b_2-b_2\ln 2\lambda)-\frac{4}{3}b_1A_1,$$

$$A_4=-4(a_2+\frac{25}{12}b_2-b_2\ln 2\lambda)-\frac{2}{3}b_1A_1,$$

$$C_1=a_1+b_1-b_1\ln 2\lambda,$$

$$C_3=-\frac{1}{4}\left[A_1^2-9(a_2+\frac{7}{6}b_2-b_2\ln 2\lambda\right].$$

Turning to have variables in the integral equations b) and c) system (3.4) and calculating the right parts, here is the integral equations in dimensionless variables:

$$\int_{-1}^{1}\varphi_1(\xi)k(b,\frac{\xi-x}{\lambda})d\xi=\pi(\delta+A_0x^2),\ |x|\le 1, \quad (3.13)$$

$$\int_{-1}^{1}\varphi_2(\xi)k(b,\frac{\xi-x}{\lambda})d\xi=\pi(\gamma_1+\gamma_2x^2+\gamma_3x^4),$$

$$|x|\le 1 . \qquad (3.14)$$

Here:

$$\varphi_1(\xi)=q_{\beta 1}(\xi'a),\ \delta=c_1(b)a^2\pi^{-1}B_2P_{\beta 0},$$

$$A_0=c_1(b)a^2\pi_{-1}P_{\beta 0},\ B_2=1/2-\frac{1}{4}A_1\lambda^{-2}+B_1\lambda^{-4},$$

$$B_1=-(a_2+\frac{4}{3}b_2-b_2\ln 2\lambda)+\frac{1}{12}b_1A_1,$$

$$\varphi_2(\xi)=q_{\beta 2}(\xi'a),$$

$$\gamma_1=a^2[c_2(b)a^2S_3P_{\beta 0}+c_1(b)(P_{\beta 1}B_2+A_o\pi S_6)]\pi^{-1},$$

$$\gamma_2=-\frac{33}{32}\left(a_2+\frac{179}{132}b_2-b_2\ln 2\lambda\right)+\frac{5}{64}b_1A_1,$$

$$S_2=-6(a_2+\frac{4}{3}b_2-b_2\ln 2\lambda)+\frac{1}{2}b_1A_1,$$

$$S_3=\frac{3}{8}-\frac{1}{4}A_1\lambda^{-2}+S_1\lambda^{-4},\ S_4=3-\frac{3}{2}A_1\lambda^{-2}+S_2\lambda^{-4},$$

$$S_5=-\frac{3}{8}a_2+\frac{39}{64}b_2+\frac{11}{384}b_1^2+\frac{3}{8}b_2\ln 2\lambda,$$

$$S_6=\frac{1}{4}-\frac{1}{12}b_1\lambda^{-2}+S_5\lambda^{-4}.$$

Applying to solve integral equations (3.13) and (3.14) the same method to the equation (3.5) we their solutions in the form of:

$$\varphi_1(x)=\frac{P_{\beta_1}}{\pi\sqrt{1-x^2}}[1-A_1\lambda^{-2}+A_2\lambda^{-4}-(2A_1\lambda^{-2}+A_3\lambda^{-4})x^2+$$
$$+A_4\lambda^{-4}x^4+O(\lambda^{-6}\ln^3\lambda)+\frac{c_1(b)a^2P_{\beta 0}}{\pi\sqrt{1-x^2}}\left\{-1+\frac{5}{12}b_1\lambda^{-2}+\right.$$
$$+A_5\lambda^{-4}+(2-\frac{4}{3}b_1\lambda^{-2}+A_6\lambda^{-4})x^2+$$
$$+\left[\frac{2}{3}b_1\lambda^{-2}-\frac{11}{5}(b_2+\frac{1}{6}b_1^2\lambda^{-4}\right]x^4+$$
$$+\frac{2}{5}(b_2+\frac{1}{6}b_1^2)\lambda^{-4}x^6+O(\lambda^{-6}\ln^3\lambda)\,, \qquad (3.15)$$

$$P_{\beta 1}=c_1(b)a^2P_{\beta 0}\left\{B_2+\frac{1}{2}+c_0(b)+C_1\lambda^{-2}+\right.$$
$$+\frac{1}{4}\left[\frac{1}{3}b_1A_1+9A_7\right]\times\left[a_0-c_0(b)+\ln 2\lambda+c_1\lambda^{-2}-\right.$$
$$\left.-\frac{1}{4}(A_1^2-9A_7)\lambda^{-4}\right]^{-1},$$

$$A_5=\frac{3}{2}a_2+\frac{89}{40}b_2-\frac{3}{20}b_1^2-\frac{3}{2}b_2\ln 2\lambda\,,$$

$$A_6=-\left(3a_2+\frac{61}{20}b_2-\frac{8}{15}b_1^2-3b_2\ln 2\lambda\right),$$

$$A_7=a_2+\frac{7}{6}b_2-b_2\ln 2\lambda\,.$$

$$\varphi_2(x)=\frac{P_{\beta 2}}{\pi\sqrt{1-x^2}}\left[1+A_1\lambda^{-2}+A_2\lambda^{-4}-(2A_1\lambda^{-2}+A_3\lambda^{-4})x^2+\right.$$
$$\left.+A_4\lambda^{-4}x^4+O(\lambda^{-6}\ln^3\lambda)\right]+\frac{1}{\sqrt{1-x^2}}\left\{-\gamma_2-\frac{1}{2}\gamma_3+\frac{1}{4}b_1\times\right.$$
$$\times\left(\frac{5}{3}\gamma_2+\frac{7}{3}\gamma_3\right)\lambda^{-2}+D_1\lambda^{-4}+\left[2(\gamma_2-\gamma_3)-4b_1(\frac{1}{3}\gamma_2+\frac{1}{5}\gamma_3)\times\right.$$
$$\left.\times\lambda^{-2}+D_2\lambda^{-4}\ \right]x^2+\left[4\gamma_3+\frac{1}{3}b_1(2\gamma_2-\frac{3}{5}\gamma_3)\lambda^{-2}-\right.$$
$$\left.-\frac{1}{5}\left(\frac{1}{6}b_1^2+b_2\right)\left(11\gamma_2+\frac{53}{7}\gamma_3\right)\lambda^{-4}\right]x^4+\left[\frac{2}{5}b_1\gamma_3\lambda^{-2}+\right.$$
$$\left.+\frac{1}{5}\left(\frac{1}{3}b_1^2+2b_2\right)\left(\gamma_2-\frac{1}{7}\gamma_3\right)\lambda^{-4}\right]x^6+\frac{2}{35}\left(\frac{1}{3}b_1^2+2b_2\right)\times$$
$$\left.\times\gamma_3\lambda^{-4}x^8+O(\lambda^{-6}\ln^3\lambda)\right\},$$

$$P_{\beta 2}=\pi\left\{\gamma_1+\frac{1}{2}\gamma_2+\frac{3}{8}\gamma_3-\frac{1}{4}A_1(\gamma_2+\gamma_3)\lambda^{-2}+\left[\frac{3}{32}b_1^2\gamma_3+\right.\right.$$
$$+\frac{1}{4}b_1A_1\cdot(\frac{1}{3}\gamma_2+\frac{1}{16}\gamma_3)-A_7(\gamma_2+\frac{33}{32}\gamma_3)-\frac{1}{2}b_2\times$$
$$\left.\left(\frac{1}{3}\gamma_2+\frac{25}{64}\gamma_3\right)\lambda^{-4}\right\}\times\left[a_o-C_o(b)-\ln 2\lambda+C_1\lambda^{-4}-\right.$$
$$\left.-\frac{1}{4}(A_1^2-9A_7)\right]^{-1},$$

$$D_1=\left[-\frac{3}{20}b_1^2+\frac{1}{2}\left(\frac{111}{30}b_2+3A_8\right)\right]\gamma_2+$$
$$+\left[-\frac{61}{448}b_1^2+\frac{1}{2}\left(\frac{433}{112}b_2+3A_8)]\gamma_3\right)\right],$$
$$D_2=\left[\frac{8}{15}b_1^2-\left(\frac{23}{10}b_2+3A_8\right)\right]\gamma_2+$$
$$+\left[-\frac{16}{35}b_1^2-\left(\frac{193}{70}b_2+3A_8)]\gamma_3\right)\right],$$

$$A_8=a_2+\frac{1}{4}b_2-b_2\ln 2\lambda\,.$$

Given the equation (3.2), (3.12) - (3.16) and turning to the dimensional variables. Get the contact problems with accuracy to members $O\left(\lambda_1^{-6}\right)$:

$$q_\beta(x)=q_{\beta 0}(x)+h^{-2}q_{\beta 1}(x)+h^{-4}q_{\beta 2}(x)+O(\lambda^{-6}). \qquad (3.17)$$

Note that the parameters $\lambda_1=ha^{-1}$ and $\lambda=(a\beta)^{-1}$ are related by $\lambda_1=\lambda b$.

4. Solution of contact problems for small values of the parameter $\lambda_1=ha^{-1}$.

To construct asymptotic solutions integral equation (2.1) the scheme is applicable for small values of the method of work [4].

Function $L(u)$ depending on the required accuracy will approximate expressions of the form (2.3) - (2.5), in which instead of the constant B, C, D, V, F and G will be coefficients B(b), C(b), D(b), V(b), F(b) and G(b) depend on the b.

Consider together with equation (2.1) the auxiliary equation:

$$\int_{-\infty}^{a}q_\beta-(\xi)k(b,s)d\xi=\pi\Delta\delta_\beta(x)\,,-\infty<x\le a\,, \qquad (4.1)$$

$$\int_a^\infty q_\beta(\xi)k(b,s)d\xi = \pi\Delta\delta_\beta(x)\,,\quad -a \le x < \infty\,, \quad (4.2)$$

$$\int_{-\infty}^\infty \nu_\beta(\xi)k(b,s)d\xi = \pi\Delta\delta_\beta(x)\,,\quad |x| < \infty\ . \quad (4.3)$$

Solution of equations (4.1) and (4.2) can be produced by Wiener-Hopf, equation (4.3) - the use of theorems on the packages for the Fourier transform.

Omitting the intermediate calculations we obtain for the case $\delta_\beta(x) = \delta_\beta = const$ under (1.9) [4], the Chief member of the asymptotic of solutions of the equation (2.1).

$$q_\beta(x) = \varphi\left(b, \frac{a+x}{h}\right) + \varphi\left(b, \frac{a-x}{h}\right) - \frac{\Delta\delta_\beta}{hA(b)}\,. \quad (4.4)$$

Here:

$$\varphi(b,t) = \frac{\Delta\delta_\beta}{h\sqrt{A(b)}}\left[A^{-1/2}(b)erf\sqrt{D(b)t} + \frac{e^{-D(b)t}}{\sqrt{\pi t}}\right]. \quad (4.5)$$

Substituting (4.5) in (4.4) we get:

$$q_\beta(x) = \frac{\Delta\delta_\beta}{hA(b)}\left\{erf\sqrt{B(b)\left(\frac{1+x}{\lambda_1}\right)} + \frac{\sqrt{A(b)e^{-B(b)\left(\frac{1+x}{\lambda_1}\right)}}}{\sqrt{\pi\left(\frac{1+x}{\lambda_1}\right)}} + \right.$$

$$\left. + erf\sqrt{B(b)\left(\frac{1-x}{\lambda_1}\right)} + \frac{\sqrt{A(b)e^{-B(b)\left(\frac{1-x}{\lambda_1}\right)}}}{\sqrt{\pi\left(\frac{1-x}{\lambda_1}\right)}} - 1\right\}. \quad (4.6)$$

Using (3.5) defines P_β [4]:

$$P_\beta = \Delta\delta_\beta\left[\gamma A^{-1}(b) - \tilde{S}_1 + \tilde{S}_2 e^{-B(b)\gamma}\right],\ \gamma = \frac{2}{\lambda_1}\,, \quad (4.7)$$

$$\tilde{S}_1 = \frac{\sqrt{C(b)}\left[2B(b) - \sqrt{C(b)}\right]}{B^2(b)},\quad \tilde{S}_2 = \frac{\left[B(b) - \sqrt{C(b)}\right]}{B^2(b)}.$$

In formulas (4.5)-(4.7) approximation has been used:

$$L(u) = \frac{\sqrt{u^2 + B^2(b)}}{u^2 + C(b)}u\,, \qquad \frac{B(b)}{C(b)} = A(b)\,. \quad (4.8)$$

Decomposing the kernel function $L(u)$ in the ranks of the small $\nu(u = \sqrt{\nu^2 + b^2})$ and equating coefficients under identical degrees ν to find the coefficients of the regression function in the kernel.

In the case of the layer, the core without friction on the hard ground will have:

$$B(b) = \left[-A^2(b) + \sqrt{A^4(b) + A(b)B_1(b)}\right]B_1^{-1}(b)\,,$$

$$C(b) = B(b)A^{-1}(b)\,,$$

$$A(b) = \frac{ch2b - 1}{b(sh2b + 2b)}\,,$$

$$B_1(b) = \frac{4b^2 sh2b - (ch2b - 1)(sh2b + 2b)}{b^3(sh2b + 2b)^2}\,. \quad (4.9)$$

You can find these coefficients is much easier:

$$B^2(b) = b^2 + B\,,\quad C(b) = b^2 + C\,,\quad D(b) = b^2 + D\,,$$

$$V(b) = b^2 + V,\ F(b) = b^2 + F,\ G(b) = b^2 + G. \quad (4.10)$$

Here B, C, D, V, F, G-ratios approximating functions of flat tasks found in [4].

Table coefficients $A(b)$, $C(b)$, $B(b)$ for approximating functions (4.8) with different values of the parameter b formulas (4.9) and (4.10). Calculation formulas (4.10) B = 1, C = 2.

Table. Computation of coefficients of approximate function

b	A(b)		B(b)		C(b)	
	(4.9)	(4.10)	(4.9)	(4.10)	(4.9)	(4.10)
0.25	0.5000	0.5000	1.0066	1.0308	2.0625	2.0114
0.5	0.4993	0.4969	1.0230	1.1180	2.2500	2.1487
0.75	0.4969	0.4878	1.0539	1.1500	2.5625	2.1811
1.00	0.4909	0.4714	1.2007	1.4142	3.0000	2.2423
1.25	0.4802	0.4493	1.1667	1.6008	3.5625	2.4292
1.50	0.4644	0.4242	1.2547	1.8028	4.2500	2.7018
1.75	0.4440	0.3981	1.3673	2.0156	5.0625	3.0795
2.00	0.4204	0.3727	1.5061	2.2361	6.0000	3.5825

CONCLUSIONS

1. Received two-dimensional integral equations that describes the contact problems.
2. Investigated the properties of kernels of integral equations.
3. Found simple asymptotic expression for determination of contact normal stress for the entire range of parameter $\lambda_1 = h/a$.

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ПРОСТРАНСТВЕННЫЕ КОНТАКТНЫЕ ЗАДАЧИ ДЛЯ УПРУГОГО СЛОЯ В СЛУЧАЕ ПОЛОСОВОЙ ОБЛАСТИ КОНТАКТА

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Аннотация. В работе рассматриваются пространственные контактные задачи теории упругости о вдавливании полосового штампа ширины 2a в упругий слой конечной толщины h: лежащий без трения на жёстком основании и жёстко соединённый с недеформируемым основанием. Форма основания штампа является функцией двух переменных x и y, причём $|y| < \infty$. Штамп вдавливается в слой силой P, отнесённой к единице длины, и моментами M_x и M_y. Получены асимптотические выражения для определения контактных нормальных напряжений под штампом.

Ключевые слова: упругий слой, упругое полупространство, интегральное преобразование, нормальные контактные напряжения, жёсткий штамп.