ISSN 1644-0633
www.acta.media.pl

Acta Sci. Pol. Architectura 15 (1) 2016, 27–39

DISTRIBUTION OF TEMPERATURE IN MULTICOMPONENT FUNCTIONALLY GRADED MULTILAYERED COMPOSITES

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Abstract. The object of analysis is a heat conduction problem within the frames of tolerance modelling in multicomponent, multilayered composites with functional gradation of effective material properties. The equations of proposed model for considered composites are partial differential equations with slowly-varying coefficients. The one-dimensional, stationary problem of heat conduction in direction perpendicular to layers will be analysed.

Kay words: heat conduction, tolerance modelling, functional gradation

INTRODUCTION

The object of the investigation is a heat conduction problem in a multicomponent, multilayered structure with functional gradation of effective material properties. The problem of heat conduction in multilayered two-component periodic composite and composite with functional gradation of effective material properties (FGM) is well known in the literature. We can mention here some papers in which was used a concept of asymptotic methods, nonstandard analysis, tolerance modelling and G-convergence: Bensoussan et al. [1978], Sanchez-Palencia [1980], Bakhvalov and Panasenko [1984], Woźniak [1987a, b], Wągrowska [1988], Briane [1990], Matysiak [1991], Jikov et al. [1994], Suresh and Mortensen [1998], Nagórko and Zieliński [1999], Lewiński and Telega [2000], Woźniak and Wierzbicki [2000], Wierzbicki and Siedlecka [2004], Nagórko and Piwowarski [2006], Łaciński and Woźniak [2006], Michalak and Woźniak [2006], Michalak et al. [2007], Michalak and Ostrowski [2007], Woźniak and Nagórko [2007], Ostrowski [2009a, b], Jędrysiak and Radzikowska [2007, 2011, 2012],Woźniak et al. (ed.) [2008,

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2010], Jędrysiak [2010], Szlachetka and Wągrowska [2010, 2011], Michalak [2011], Nagórko and Woźniak [2011], Ostrowski and Michalak [2011, 2015, 2016], Szlachetka [2012], Szlachetka et al. [2012], Woźniak et al. [2012, 2015].

The heat conduction in multicomponent periodic composites based on the tolerance modelling procedure was presented by Woźniak [2012, 2013] and applied for example by Wągrowska and Woźniak [2014], by Szlachetka and Wągrowska [2014a, b, 2015] and Wągrowska and Szlachetka [2016]. The process of modelling of heat conduction problems for composites with functional gradation of effective material properties will be discussed within the frames of the tolerance modelling. Example will be narrowed down to the one-dimensional, stationary problem.

OBJECT OF ANALYSIS

The object of analysis is a rigid heat conductor which occupies a region $\Omega \equiv (0, L_1)\times\Xi, \Xi \equiv (0, L_2)\times(0, L_3)$, in the physical space parameterized by an orthogonal Cartesian coordinate system $Ox_1x_2x_3$. It is assumed that conductor is multilayered in the Ox_1 direction. The composite is made of N thin layers with constant thickness λ, where $\lambda = L_1/N$. It has to be emphasized that λ has to be much smaller than the characteristic dimension of the composite L, where $L = \max(L_1; L_2; L_3)$. Every layer (with constant thickness λ) of the composite is assumed to be made of M different orthotropic, homogeneous components with known mass densities, specific heats and thermal conductivities. These components are sublayers of the layer and number of them is P, $P \geq M$.

Let us introduce a midplane of the n-th, $n = 1, ..., N$, layer which is defined as $x_1 = x_1^n \equiv \frac{\lambda}{2} + (n-1)\lambda,\ n = 1, ..., N$ in $Ox_1x_2x_3$ space.

Moreover let us define continuous functions $\varphi_p(\cdot)$, $p = 1, ..., P$, such that $\varphi_1(x_1) + ... + \varphi_p(x_1) = 1$ for every $x_1 \in (0, L_1)$. The thickness of the p-th ($p = 1, ..., P$) sublayer in the n-th, $n = 1, ..., N$ layer is equal to $\lambda_p^n = \varphi_p\left(x_1^n\right)\lambda$. The value of this function on midplane assigned to the layer will be interpreted as the fraction of the -th sublayer in this layer. The scheme of the one of the layer is presented in Figure 1.

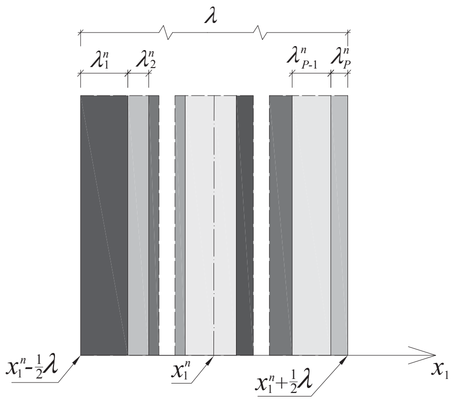

Fig. 1. The scheme of one of the layer in multicomponent multilayered composite with functional gradation of effective material properties

The heat conduction problem in the discussed composite will be described by the Fourier law and the heat balance equation in the forms:

$$q^{\alpha}\left(\boldsymbol{x},t\right)=-k^{\alpha\beta}\left(\boldsymbol{x}\right)\partial_{\beta}\theta\left(\boldsymbol{x},t\right) \tag{1}$$

$$c\left(\boldsymbol{x}\right)\partial_{t}\theta\left(\boldsymbol{x},t\right)-\partial_{\alpha}\left(k^{\alpha\beta}\left(\boldsymbol{x}\right)\partial_{\beta}\theta\left(\boldsymbol{x},t\right)\right)=0 \tag{2}$$

where: $\boldsymbol{x}\equiv\left(x_1,x_2,x_3\right)\in\Omega$; $\theta(\cdot,\cdot)$ – temperature in the region of Ω for every $t\in\left[0,t_*\right)$, $k^{\alpha\beta}\left(\cdot\right)$ – components of the thermal conductivity tensor (for orthotropic materials $k^{\alpha\beta}(x)$ for $\alpha\neq\beta$ and $k^{\alpha\alpha}(x)\equiv k^{\alpha}(x)$), $c(\cdot)$ – specific heat, subscripts and superscripts α and β are equal to 1, 2 and 3 (summation convention holds), $\partial_{\alpha}\left(\cdot\right)\equiv\frac{\partial}{\partial x^{\alpha}}$, $\partial_{\beta}\left(\cdot\right)\equiv\frac{\partial}{\partial x^{\beta}}$, $\partial_t\equiv\frac{\partial}{\partial t}$.

The equation (2) which holds for all points of region Ω and $\forall t\in\left(0,t_*\right)$, is a partial differential equation with discontinuous and highly oscillating coefficients $k^{\alpha\beta}(\cdot)$, $c(\cdot)$ which depend only on the x_1 coordinate. The solution of the heat conduction problem for multicomponent, multilayered composites will be considered within the frames of the tolerance modelling method, Woźniak [2012, 2013]. The most important feature in the process of tolerance modelling is fact that the discontinuous coefficients in equation (2) can be replaced by the slowly-varying coefficients.

MODELLING CONCEPTS

The process of tolerance modelling for composites with functional gradation of effective material properties is based on some basic concepts i.e. slowly-varying function, tolerance averaging approximation, local layer, local oscillating micro-shape function and global micro-shape function.

Slowly varying functions [Woźniak et al. (ed.) 2008, 2010]. Two classes of slowly varying functions will be used in the process of tolerance modelling: weakly slowly varying function (WSV) and slowly varying function (SV) [Woźniak et al. (ed.) 2010].

Let us defineas an arbitrary convex set in the space R^m, and let $f\in C^1\left(\Pi\right)$ be an arbitrary real-valued function. Moreover let us define the tolerance parameter $d\equiv(\lambda,\delta_0,\delta_1)$ as a triplet of real positive numbers and use the notation $\partial_j\equiv\frac{\partial}{\partial x_j}$, $j=1,\dots,m$.

Function $f\in C^1\left(\Pi\right)$ is weakly slowly varying function ($f\in \mathrm{WSV}_d^1\left(\Pi\right)\subset C^1\left(\Pi\right)$) if the condition $\left\|\mathbf{x}-\mathbf{y}\right\|\leq\lambda$ implies the conditions $\left|f\left(\mathbf{x}\right)-f\left(\mathbf{y}\right)\right|\leq\delta_0$ and $\left|\partial_j f\left(\mathbf{x}\right)-\partial_j f\left(\mathbf{y}\right)\right|\leq\delta_1$ for $j=1,\dots,m$ and for each $\left(\mathbf{x},\mathbf{y}\right)\in\Pi^2$.

Function $f\in \mathrm{WSV}_d^1\left(\Pi\right)$ is slowly varying function ($f\in \mathrm{SV}_d^1\left(\Pi\right)$) if conditions $\lambda\left|\partial_j f\left(\mathbf{x}\right)\right|\leq\delta_0$ hold for $j=1,\dots,m$ and for every $\mathbf{x}\in\Pi$.

Obviously, $\mathrm{WSV}_d^1\left(\Pi\right)\supset \mathrm{SV}_d^1\left(\Pi\right)$.

Tolerance averaging approximation. Let us define $\Delta \equiv \left(-\frac{\lambda}{2}, \frac{\lambda}{2}\right)$ and local interval $\Delta(x) \equiv \left(x-\frac{\lambda}{2}, x+\frac{\lambda}{2}\right)$ for every $x \in \left[\frac{\lambda}{2}, L-\frac{\lambda}{2}\right]$. Let $f_x \in L^2\left((0, L)\right)$ then:

$$\langle f\rangle(x) \equiv \frac{1}{\lambda} \int_{\Delta(x)} f_x(z)dz \tag{5}$$

Let $f_x \in L^2\left(\Delta(x)\right)$ and $F \in \mathrm{WSV}_d^1\left((0, L)\right)$. The tolerance averaging approximation of the product of functions $f_x(\cdot)\, F(\cdot)$ and functions $f_x(\cdot)\partial_1 F(\cdot)$ at point x will be defined as [Woźniak et al. (ed.) 2010]:

$$\begin{aligned} \langle fF\rangle_T(\boldsymbol{x}) &\equiv \langle f\rangle(\boldsymbol{x})F(\boldsymbol{x}) \\ \langle f\partial_1 F\rangle_T(\boldsymbol{x}) &\equiv \langle f\rangle(\boldsymbol{x})\partial_1 F(\boldsymbol{x}) \end{aligned} \tag{6}$$

Local layer. The local layer $LL(x_1)$ with the midplane x_1 = const is a Cartesian product of local interval $\Delta_{\mathrm{loc}}(x_1) \equiv \left(x_1-\frac{\lambda}{2}, x_1+\frac{\lambda}{2}\right)$ for every $x_1 \in \left[\frac{\lambda}{2}, L-\frac{\lambda}{2}\right]$ and region $\boldsymbol{\Xi}$ on Ox_2x_3 plane $(\boldsymbol{\Xi} \equiv (0, L_2)\times(0, L_3))$: $LL(x_1) \equiv \Delta_{\mathrm{loc}}(x_1)\times\boldsymbol{\Xi}$. Let us introduce a local coordinate y, $y \in \Delta_{\mathrm{loc}}(x_1)$, for each local layer. The local coordinate is perpendicular to the layers. The fragment of cross-section through a local layer is presented in Figure 2.

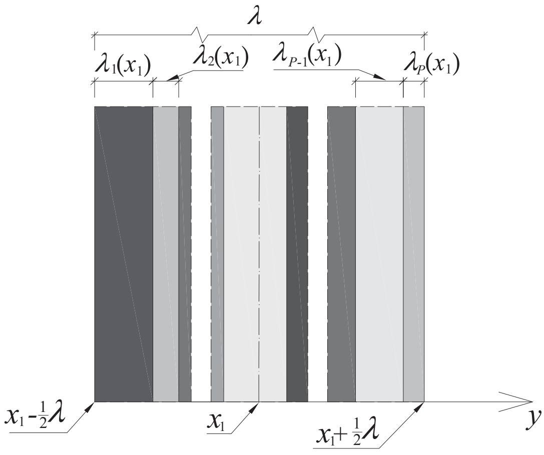

Fig. 2. The cross-section through a local layer

Local oscillating micro-shape function [Woźniak 2012]. The local oscillating micro-shape shape function $\gamma^{x_1}(\cdot)$ referred to the local interval $\Delta_{\mathrm{loc}}(x_1) \equiv \left(x_1-\frac{\lambda}{2}, x_1+\frac{\lambda}{2}\right)$

for arbitrary but fixed $x_1 \in \left[\frac{\lambda}{2}, L-\frac{\lambda}{2}\right]$ is function which depends only on *y* coordinate. This function is piecewise linear and takes the following values on the interfaces between sublayers:

$$\gamma_p^{x_1} = \gamma_{p-1}^{x_1} + \lambda\varphi_p(x_1)\left(\frac{K^0(x_1)}{K_p(x_1)} - 1\right), \; p = 1, 2, ..., P \tag{7}$$

where: $K^0(x_1) \equiv \left(\frac{\varphi_1(x_1)}{K_1} + ... \frac{\varphi_P(x_1)}{K_P}\right)^{-1}$; $K_m \equiv k_m^{11} = k_m^1$.

Moreover the local oscillating micro-shape function satisfies the condition $\langle \rho\gamma^{x_1} \rangle (x_1) = 0$.

Global micro-shape function [Woźniak 2012]. Global micro-shape function γ(·) which is defined for all points x_1, $x_1 \in [0, L]$ satisfies the following conditions:

$$\gamma_p^n = \gamma_{p-1}^n + \lambda\varphi_p\left(x_1^n\right)\left(\frac{K^0\left(x_1^n\right)}{K_p\left(x_1^n\right)} - 1\right), \; p = 1, 2, ..., P, \;\; n = 1, 2, ..., N$$

$\gamma(\cdot)$ is piecewise linear between interfaces

$\langle \rho\gamma \rangle = 0$ (8)

where: γ_p^n, $p = 1, 2, ..., P$, are values of function γ(·) on the interfaces between sublayers in the *n*-th layer, and

$$K^0\left(x_1^n\right) \equiv \left(\frac{\varphi_1\left(x_1^n\right)}{K_1} + ... \frac{\varphi_P\left(x_1^n\right)}{K_P}\right)^{-1}; \; K_m \equiv k_m^{11} = k_m^1.$$

MODELLING PROCEDURE AND MODELLING EQUATIONS

Let temperature assigned to the local layer $LL(x_1)$ with the midplane x_1 = const for all values of $x_1 \in [0, L]$ and time $t \in (0, t_*)$ be denoted as $\theta_{x1}(y, x_2, x_3, t)$, where $y \in \Delta_{loc}(x_1)$, $(x_2, x_3) \in \Xi$, $x_1 \in (0, L)$, $t \in (0, t_*)$. The process of tolerance modelling is based on two assumptions.

The first assumption called micro-macro decomposition. The temperature field $\theta_{x1}(y, x_2, x_3, t)$ can be approximated by the field $\tilde{\theta}_{x_1}(y_1, x_2, x_3, t)$ [Woźniak et al. (ed.) 2010]:

$$\tilde{\theta}_{x_1}(y, x_2, x_3, t) = \vartheta(y, x_2, x_3, t) + \gamma^{x_1}(y)\psi(y, x_2, x_3, t) \tag{9}$$

where: $\vartheta(\cdot, x_2, x_3, t)$ and $\psi(\cdot, x_2, x_3, t)$, called macro-temperature and amplitude fluctuation of temperature which are arbitrary weakly slowly varying functions of argument x_1 for all $(x_2, x_3) \in \boldsymbol{\Xi}$ and $t \in (0, t_*)$.

Before introducing the second assumption define the residual field of $\tilde{\theta}_{x1}(\cdot)$ in the region Ω for $t \in [0, t_*]$ [Woźniak et al. 2015]:

$$r_{x1}(\cdot) \equiv \partial_1\left(k_{x1}^1 \partial_1 \tilde{\theta}_{x1}\right)(\cdot) + k_{x1}^2 \partial_2^2 \tilde{\theta}_{x1}(\cdot) + k_{x1}^3 \partial_3^2 \tilde{\theta}_{x1}(\cdot) - c_{x1} \partial_t \tilde{\theta}_{x1}(\cdot) \tag{10}$$

The second assumption. The second assumption is:

$$\begin{aligned} &\langle r_{x1} \rangle_T = 0 \\ &\langle \gamma r_{x1} \rangle_T = 0 \end{aligned} \tag{11}$$

where $\langle \cdot \rangle_T$ is defined by equations (6).

After implementation both assumptions the system of equations for unknown functions $\vartheta(\cdot)$, $\psi(\cdot)$ ($\vartheta(\cdot, x_2, x_3), \psi(\cdot, x_2, x_3) \in \mathrm{WSV}_d^1((0, L_1))$ takes the form:

$$\begin{aligned} &\partial_1\left(\left\langle k^1\right\rangle(x_1)\partial_1\vartheta(\boldsymbol{x}, t)\right) + \left\langle k^2\right\rangle(x_1)\partial_2^2\vartheta(\boldsymbol{x}, t) + \\ &+\left\langle k^3\right\rangle(x_1)\partial_3^3\vartheta(\boldsymbol{x}, t) - \langle c\rangle(x_1)\partial_t\vartheta(\boldsymbol{x}, t) + \partial_1\left(\left\langle k^1\partial_1\gamma\right\rangle(x_1)\psi(\boldsymbol{x}, t)\right) = 0 \\ &\partial_1\left(\left\langle k^1(\gamma)^2\right\rangle(x_1)\partial_1\psi(\boldsymbol{x}, t)\right) + \left\langle k^2(\gamma)^2\right\rangle(x_1)\partial_2^2\psi(\boldsymbol{x}, t) + \left\langle k^3(\gamma)^2\right\rangle(x_1)\partial_3^2\psi(\boldsymbol{x}, t) + \\ &-\left\langle k^1(\partial_1\gamma)^2\right\rangle(x_1)\psi(\boldsymbol{x}, t) - \left\langle k^1\partial_1\gamma\right\rangle(x_1)\partial_1\vartheta(\boldsymbol{x}, t) - \left\langle c(\gamma)^2\right\rangle(x_1)\partial_t\psi(\boldsymbol{x}, t) = 0 \end{aligned} \tag{12}$$

Equations (12) with the decomposition of approximative temperature filed $\tilde{\theta}_{x1}(\cdot)$ as:

$$\tilde{\theta}(\boldsymbol{x}, t) = \vartheta(\boldsymbol{x}, t) + \gamma(x_1)\psi(\boldsymbol{x}, t) \tag{13}$$

and boundary and initial conditions are general model equations of heat conduction for orthotropic multicomponent multilayered functionally graded composites.

It has to be emphasized that the coefficients in system of equations (12) are slowly varying functions of the argument $x_1 \in (0, L)$.

If $\vartheta(\cdot, x_2, x_3, t) \in \mathrm{SV}_d^1((0, L))$ $\forall (x_2, x_3, t) \in \Xi \times (0, t_*)$ and $\psi(\cdot) \in SV_d^1((\Omega, t_*))$ equations (12) take the form:

$$
\begin{aligned}
&\partial_1\left(\left\langle k^1\right\rangle(x_1)\partial_1\vartheta(\boldsymbol{x},t)\right)+\left\langle k^2\right\rangle(x_1)\partial_2^2\vartheta(\boldsymbol{x},t)+\left\langle k^3\right\rangle(x_1)\partial_3^3\vartheta(\boldsymbol{x},t)+\\
&-\langle c\rangle(x_1)\partial_t\vartheta(\boldsymbol{x},t)+\partial_1\left(\left\langle k^1\partial_1\gamma\right\rangle(x_1)\psi(\boldsymbol{x},t)\right)=0\\
&\left\langle k^1(\partial_1\gamma)^2\right\rangle(x_1)\psi(\boldsymbol{x},t)+\left\langle k^1\partial_1\gamma\right\rangle(x_1)\partial_1\vartheta(\boldsymbol{x},t)=0
\end{aligned}
\tag{14}
$$

which with decomposition (13) and boundary and initial conditions are the local homogenization model (LHM).

Furthermore if the considerations are limited only for stationary and one-dimensional problems equations (14) take the form:

$$
\begin{aligned}
&\frac{d}{dx_1}\left(\left\langle k^1\right\rangle(x_1)\frac{d}{dx_1}\vartheta(\boldsymbol{x})\right)+\frac{d}{dx_1}\left(\left\langle k^1\frac{d}{dx_1}\gamma\right\rangle(x_1)\psi(\boldsymbol{x})\right)=0\\
&\left\langle k^1\left(\frac{d}{dx_1}\gamma\right)^2\right\rangle(x_1)\psi(\boldsymbol{x})+\left\langle k^1\frac{d}{dx_1}\gamma\right\rangle(x_1)\frac{d}{dx_1}\vartheta(\boldsymbol{x})=0
\end{aligned}
\tag{15}
$$

From equation (15_2) it follows that:

$$
\psi(\boldsymbol{x})=-\frac{\left\langle k^1\frac{d}{dx_1}\gamma\right\rangle(x_1)}{\left\langle k^1\left(\frac{d}{dx_1}\gamma\right)^2\right\rangle(x_1)}\partial_1\vartheta(\boldsymbol{x})
\tag{16}
$$

Substituting equation (16) to (15_1) equation for $\vartheta(\cdot)$ takes the form:

$$
\frac{d}{dx_1}\left(\left(K_0(x_1)\right)\frac{d}{dx_1}\vartheta(x_1)\right)=0
\tag{17}
$$

where:

$$
K_0(x_1)\equiv\left\langle k^1\right\rangle(x_1)-\frac{\left(\left\langle k^1\frac{d}{dx_1}\gamma\right\rangle(x_1)\right)^2}{\left\langle k^1\left(\frac{d}{dx_1}\gamma\right)^2\right\rangle(x_1)}\equiv\left(\frac{\varphi_1(x_1)}{k_1}+\ldots+\frac{\varphi_P(x_1)}{k_P}\right)^{-1}
\tag{18}
$$

Equation (17) with denotation (16) and (18) and decomposition of the approximate temperature (13) and boundary conditions are the base equations for one dimensional and stationary problems.

EXAMPLE

The distribution of approximate temperature field $\tilde{\theta}(\cdot)$ for special multicomponent multilayered functionally graded composite is presented in this section.

Let us assume that the composite, which occupies the region $\Omega \equiv (0, L) \times R^2$ where $L = 20$ cm, is composed of $N = 20$ layers with constant thicknesses $\lambda = 1$ cm. Each layer consists of three sublayers made of three different isotropic materials. The material fractions in sublayers are depend on functions $\varphi_1(\cdot)$, $\varphi_2(\cdot)$ and $\varphi_3(\cdot)$. To this end $x \equiv x_1$. The above functions take the form: $\varphi_1 = 0.2,\ \varphi_2(x) = \frac{L - 0.8x}{L} - 0.2,\ \varphi_3(x) = 1 - \frac{L - 0.8x}{L}$.

It can be observed that the thickness of first sublayer is constant in every layer and is equal to $\lambda_1 = 0.2$ m. The thicknesses of the second and the third sublayer in several layers are not constant. It is important to notice that every two adjacent layers can be treated as indistinguishable.

The coefficients of thermal conductivity related to the corresponding sublayers are equal to $K_1 = 1.7\ \text{W}\cdot(\text{m}\cdot\text{K})^{-1}$, $K_2 = 0.042\ \text{W}\cdot(\text{m}\cdot\text{K})^{-1}$, $K_3 = 20\ \text{W}\cdot(\text{m}\cdot\text{K})^{-1}$.

The boundary conditions on the macro-temperature are: $\vartheta(0) = \vartheta_0 = 0^\circ\text{C}$, $\vartheta(20) = \vartheta_L = 20^\circ\text{C}$. The distribution of the temperature field $\vartheta(\cdot)$ determined from equation (17) is shown in Figure 3.

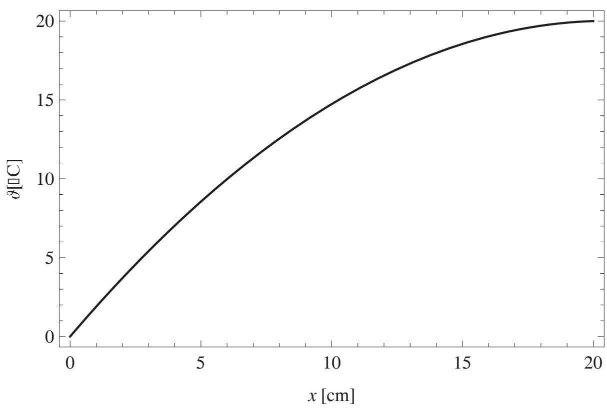

Fig. 3. Distributions of the macro-temperature fields $\vartheta(\cdot)$ for $x \in (0, 20)$

It can be observed that the distribution of macro-temperature $\vartheta(\cdot)$ is not linear function as is in periodic multicomponent multilayered composites [Wągrowska and Szlachetka 2016].

The distribution of the temperature fields $\tilde{\theta}(\cdot)$ for $x \in (0, 20)$, $x \in (2, 3)$, $x \in (5, 6)$, $x \in (15, 16)$ are shown in Figures 4, 5, 6 and 7.

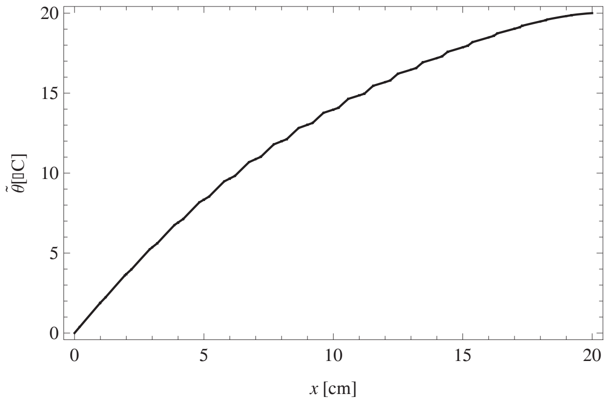

Fig. 4. Distribution of the approximated temperature field $\tilde{\theta}(\cdot)$ for $x \in (0, 20)$

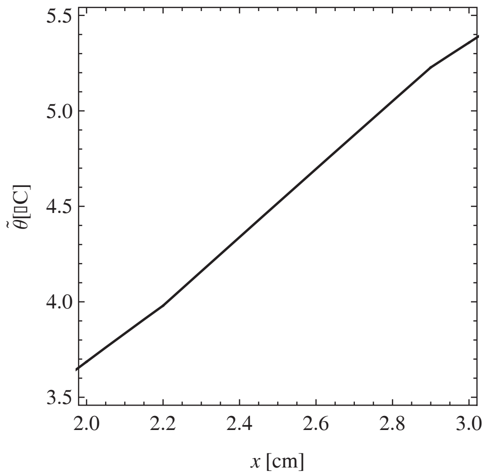

Fig. 5. Distribution of the approximated temperature field $\tilde{\theta}(\cdot)$ in the third layer

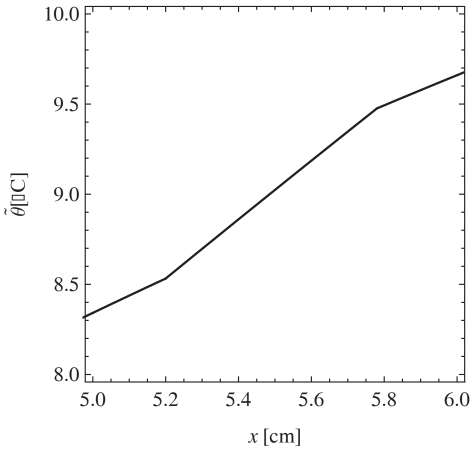

Fig. 6. Distribution of the approximated temperature field $\tilde{\theta}(\cdot)$ in the sixth layer

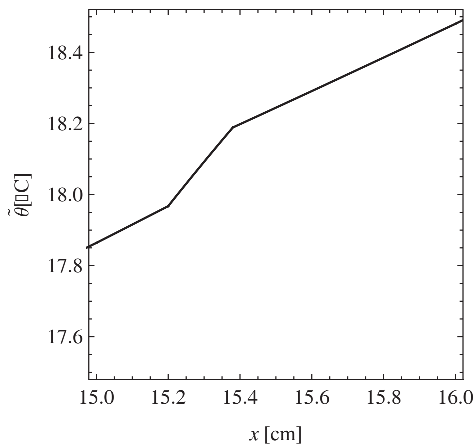

Fig. 7. Distribution of the approximated temperature field $\tilde{\theta}(\cdot)$ in the sixteenth layer

CONCLUSIONS

The process of heat conduction for multicomponent multilayered composites with functional gradation of effective material properties can be described by equations with smooth and slowly varying coefficients. The local homogenization model equations (LHM) for one-dimension stationary problems are reduced to equation for macro-temperature $\vartheta(\cdot)$ and fluctuation amplitude $\psi(\cdot)$ which have the same form like an asymptotic model equation.

REFERENCES

Bakhvalov, N.S., Panasenko, G.P. (1984). Homogenization: Averaging processes in periodic media. Nauka, Moscow [in Russian]; (1989) Kluwer, Dordrecht – Boston – London [in English].

Bensoussan, A., Lions, J.L., Papanicolaou, G. (1978). Asymptotic analysis for periodic structures. North-Holland, Amsterdam.

Briane, M. (1990). Homogeneisation de materiaux fibres et multi-couches (PHD thesis). Universite Paris 6, Pierre et Marie Curie, France.

Jędrysiak, J. (2010). Termomechanika laminatów, płyt i powłok o funkcyjnej gradacji własności. Wydawnictwo PŁ, Łódź.

Jędrysiak, J., Radzikowska, A. (2007). On the modelling of heat conduction in a nono-periodically laminated layer. J. Theor. Appl. Mech., 45 (2), 239–257.

Jędrysiak, J., Radzikowska, A. (2011). Some problems of heat conduction for transversally graded laminates with non-uniform distribution of laminas. Arch. Civ. Mech. Eng., 11 (1), 75–87.

Jędrysiak, J., Radzikowska, A. (2012). Tolerance averaging of heat conduction in transversally graded laminates. Meccanica (AIMETA), 47 (1), 95–107.

Jikov, V.V., Kozlov, C.M., Oleinik, O.A. (1994). Homogenization of differential operators and integral functionals. Springer Verlag, Berlin – Heidelberg.

Lewiński, T., Telega, J. (2000). Plates, laminates and shells. World Scientific Publishing Company, Singapore.

Łaciński, Ł., Woźniak, Cz. (2006). Boundary-layer phenomena in the laminated rigid heat conduction. J. Thermal Stresses, 29, 665–682.

Matysiak, S.J. (1991). On certain problems of heat conduction in periodic composites. J. Appl. Math. Mech. (ZAMM) 71, 524–528. doi: 10.1002/zamm.19910711218).

Michalak, B. (2011). Teromechanika ciał z pewną niejednorodną mikrostrukturą. Technika tolerancyjnej aproksymacji. Wydawnictwo PŁ, Łódź.

Michalak, B., Ostrowski, P. (2007). Modelling and analysis of the heat conduction in a certain functionally graded material. In: Mechanics of Solids and Structures. Ed. V. Pauk. Kielce Technical University Press, Kielce, 23–35.

Michalak, B., Woźniak, M. (2006). On the heat conduction in certain functionally graded composites. In: Selected Topics in the Mechanics of Inhomogeneous Media. Ed. Cz. Woźniak, R. Świtka, M. Kuczma. University of Zielona Góra Press, Zielona Góra, 229–238.

Michalak, B., Woźniak, Cz., Woźniak, M. (2007). Modelling and analysis of certain functionally graded heat conductor. Arch. Appl. Mech., 77, 823–834.

Nagórko, W., Piwowarski, M. (2006). On the heat conduction in periodically nonhomogeneous solids. In: Selected Topics in the Mechanics of Inhomogeneous Media. Ed. Cz. Woźniak, R. Świtka, M. Kuczma. University of Zielona Góra Press, Zielona Góra, 241–254.

Nagórko, W., Woźniak, Cz. (2011). Mathematical modelling of heat conduction in certain functionally graded composites. PAMM 11, 253–254.

Nagórko, W., Zieliński, J. (1999). On heat conduction modelling in plates formed by periodically nonhomogeneous layers. Visnyk Livi Univ. Ser. Mech.-Math., 55, 100–105.

Ostrowski, P. (2009a). On the heat transfer in a two-phase hollow cylinder with piecewise continuous boundary conditions. PAMM, 9, 259–260.

Ostrowski, P. (2009b). Zagadnienie przewodnictwa ciepła w laminatach z funkcyjną gradacją własności. PhD Thesis. Wydz. BAiIS PŁ, Łódź.

Ostrowski, P., Michalak, B. (2011). Non-stationary heat transfer in a hollow cylinder with functionally graded material properties. J. Ther. Appl. Mech., 49 (2), 385–397.

Ostrowski, P., Michalak, B. (2015). The combined asymptotic–tolerance model of heat conduction in a skeletal micro-heterogeneous hollow cylinder. Composite Structures, 134, 343–352.

Ostrowski, P., Michalak, B. (2016). A contribution to the modelling of heat conduction for cylindrical composite conductors with non-uniform distribution of constituents. International Journal of Heat and Mass Transfer, 92, 435–448.

Sanchez-Palencia, E. (1980). Non-homogeneous media and vibration theory. Lecture Notes in Physics, 127, Springer-Verlag, Berlin.

Suresh, S., Mortensen, A. (1998). Fundamentals of functionally graded materials. The University Press, Cambridge.

Szlachetka, O. (2012). Tolerance and asymptotic modelling for functionally graded heat conductors. In: Badania doświadczalne i teoretyczne w budownictwie. Ed. J. Bzówka. Publ. House of Silesian Univ. Technol., 205–220.

Szlachetka, O., Wągrowska, M. (2010). Efekt warstwy brzegowej w warstwowej przegrodzie o poprzecznej gradacji własności. Acta Sci. Pol. Architectura, 9 (4), 15–23.

Szlachetka, O., Wągrowska, M. (2011). Efekt brzegowy w warstwowej przegrodzie o podłużnej gradacji własności. Acta Sci. Pol. Architectura, 10 (3), 27–34.

Szlachetka, O., Wągrowska, M. (2014a). Przewodnictwo ciepła w wieloskładnikowych kompozytach warstwowych. Konferencja „Aktualne problemy budownictwa", 26–27 czerwca 2014, Warszawa.

Szlachetka, O., Wągrowska, M. (2014b). Wpływ struktury periodycznych wieloskładnikowych kompozytów warstwowych na rozkład temperatury. VII Sympozjum „Kompozyty, konstrukcje warstwowe", 15–18 października 2014, Wrocław.

Szlachetka, O., Wągrowska, M. (2015). Przewodnictwo ciepła w wieloskładnikowych kompozytach wielowarstwowych. Materiały Kompozytowe, 2, 27–29.

Szlachetka, O., Wągrowska, M., Woźniak, Cz. (2012). A two-step asymptotic modelling of the heat conduction in a functionally graded stratified layer. Acta Sci. Pol. Architectura, 11 (3), 3–9.

Wągrowska, M. (1988). On the homogenization of elastic-plastic periodic composites by the microlocal parameter approach. Acta Mech., 73, 45–65.

Wągrowska, M., Szlachetka, O. (2016). Distribution of temperature in multicomponent multilayered composites. In: Continuous media with microstructure, 2. Ed. B. Albers, M. Kuczma. Springer Intern. Publishing, Switzerland, 199–214.

Wągrowska, M., Woźniak, Cz. (2014). A new 2D-model of the heat conduction in multilayered medium-thickness plates. Acta Sci. Pol. Architectura, 13 (1), 3–44.

Wierzbicki, E., Siedlecka, U. (2004). Isotropic models for a heat transfer in periodic composites. PAMM, 4 (1), 502–503.

Woźniak, Cz. (1987a). Homogenized thermoelasticity with microlocal parameters. Bull. Polish. Acad. Tech., 35, 133–141.

Woźniak, Cz. (1987b). On the linearized problems of thermoelasticity with microlocal paramiters. Bull. Pol. Ac. Tech., 35, 143–151.

Woźniak, Cz. (2010). Asymptotic modelling and boundary-layer effect for functionally graded microlayered composites. Acta Sci. Pol. Architectura, 9 (2), 3–9.

Woźniak, Cz. (2012). Macroscopic models of the heat conduction in periodically stratified multicomponent composites (1D- models). 6th Sympozjon „Kompozyty. Konstrukcje warstwowe", November 2012, Wrocław – Srebrna Góra, 8–10.

Woźniak, Cz. (2013). Uniaxial elastic strain and heat conduction problems in multicomponent periodic composites. Conference "Mechanika ośrodków niejednorodnych", 6–9 June 2013, Zielona Góra – Łagów.

Woźniak, Cz., Nagórko, W. (2007). Modelowanie matematyczne materiałów z funkcjonalną gradacją własności efektywnych – wyniki badań i perspektywy rozwoju w Polsce, Acta Sci. Pol. Architectura, 6 (4), 23–32.

Woźniak, Cz., Wierzbicki, E. (2000). Averaging techniques in thermomechanics of composite solids. Tolerance averaging versus homogenization. Wydawnictwo PCz., Częstochowa.

Woźniak, Cz., Wągrowska, M., Szlachetka, O. (2012). Asymptotic modelling and design of some microlayered functionally graded heat conductors. Math. Mech., 92 (10), 841–848.
Woźniak, Cz., Wagrowska, M., Szlachetka, O. (2015). On the tolerance modeling of heat conduction in functionally graded laminated media. J. Appl. Mech. Tech. Phys., 56 (2), 274–281.
Woźniak, Cz., Michalak, B., Jędrysiak, J., ed. (2008). Thermomechanics of heterogeneous solids and structures, tolerance averaging approach. Wydawnictwo PŁ, Łódź.
Woźniak, Cz. i inni, ed. (2010). Mathematical modelling and analysis in continuum mechanics of microstructured media. Wydawnictwo Politechniki Śląskej, Gliwice.

ROZKŁAD TEMPERATURY W WIELOSKŁADNIKOWYM WIELOWARSTWOWYM KOMPOZYCIE O FUNKCYJNEJ GRADACJI WŁASNOŚCI EFEKTYWNYCH

Streszczenie. Przedmiotem rozważań jest modelowanie tolerancyjne przewodnictwa ciepła w wieloskładnikowych wielowarstwowych kompozytach o funkcyjnej gradacji własności efektywnych. Zaproponowane równania modelu dla analizowanych kompozytów są równaniami różniczkowymi cząstkowymi z wolnozmiennymi współczynnikami. Wyznaczono rozkład przybliżonej temperatury dla jednowymiarowego stacjonarnego zagadnienia przewodnictwa ciepła w kierunku prostopadłym do uwarstwienia.

Słowa kluczowe: przewodnictwo ciepła, modelowanie tolerancyjne, kompozyt o funkcyjnej gradacji własności efektywnych

Accepted for print: 21.03.2016

For citation: Wągrowska, M., Szlachetka, O. (2016). Distribution of temperature in multicomponent functionally graded multilayered composites. Acta Sci. Pol. Architectura, 15 (1), 27–39.