TEKA. COMMISSION OF MOTORIZATION AND ENERGETICS IN AGRICULTURE – 2013, Vol. 13, No.4, 201-207

Study of structural stresses in the monolithic concrete of natural hardening

Vladimir Punagin

Volodymyr Dahl East-Ukrainian National University,
Molodizhny bl., 20a, Lugansk, 91034, Ukraine, e-mail: fartfakt@ukr.net

Received September 20.2013: accepted October 14.2013

Summary: This article presents the results of the study of structural stresses in the monolithic modified concrete when it is saturated with a certain amount of coarse aggregate, as well as the impact of structural stresses on concrete properties. This will solve a number of technological problems in the construction of high-rise buildings in summer temperatures.

Key words: modified concrete, structural stresses, strains.

INTRODUCTION

In the natural conditions of summer period occurs temperature-humidity gradient as a result of uneven heating (cooling) or drying (moistening) on the concrete construction section and, consequently, inherent stresses balanced in the whole construction occur.

Qualitatively other inherent stresses, structural stresses, are caused by temperature-humidity strains in anisotropic structure of concrete. Analysis of concrete structural stresses showed that their intensity is high and it often exceeds the ultimate strength of the material. Structural stresses are balanced in aggregate grains area [1, 14, 16, 18].

ANALYSIS OF PUBLICATIONS, MATERIALS, METHODS

Submicrostructural stresses in the cement matrix of concrete occur due to the development of new crystalline hydrate formations. The study of these stresses present considerable difficulties, as they are balanced in microscopic volumes of the cement matrix of concrete. Development of crystallization micro-cracks in the concrete was not observed. This can probably be explained by capillary-porous structure of the cement matrix. Moreover, there is an assertion [2-5, 27] about crystallization stresses usefulness, as they improve the cohesion of spontaneously growing formations. Sharp differences of the stress state of the hardening gel in various conditions are not to be expected because of the possibility of stress relaxation during the deformation of the pores of the material [6, 18].

In 1936 F. Thomas made a review of the works on crack formation of concrete, he found that the first cracks in the concrete become apparent in the age of a few days, while the theoretical period of crack forming was supposed to be 5 weeks. In this same survey it was established that crack formation depends both on the amount of shrinkage and physical-mechanical properties of concrete. To a greater extent crack resistance was determined by the nature of the aggregate. Aggregates of high elasticity contributed to the development of crack formation [7, 9-11]. According to this data tensile stresses in the concrete due to the cement shrinkage make 4,0…5,0 MPa. The intensity of these stresses, calculated by R. Lermit and B. Hank [8], makes 12…16 MPa, i.e. it exceed tensile strength of the material. A.V. Byelov, also considering shrinkage stresses, has established that shrinkage identical to the cooling of the concrete up to 70°C, causes stresses close to the specified [13].

Two types of inherent stress play an important role in natural conditions:

a) stresses of the first type caused by gradients of the temperature-humidity strains on the section of the elements have a certain orientation depending on the geometry of the construction. Stresses of the first type are often called mechanical, because they are determined by the methods of the elasticity theory,

b) structural stresses of the second type caused by the temperature-humidity strains in anisotropic structure of concrete. Stresses of the second type are in a certain way oriented in regard to the surface of the aggregate particles. The analysis of the material structure, apart from the theory of elasticity and plasticity, is necessary to determine them.

It is obvious, that the fields of these stresses are quite different, but their mutual overlapping and joint impact on the continuity of the constructions and relaxation of stresses in time are doubtless. Strength and deformability of concrete depend mostly on the stresses of the first and second type, therefore, the criterion of the concrete crack formation must be determined from their joint action with regard to the age of the material and construction zone [19-23]. Moreover, when studying the elasticity problem of the development of crack formation in concrete elements, the additivity of the stresses of the first and second type is considered to be fair.

As is well known, concrete is an elastoplastic material and this precondition is only partially true [28]. Anyway, plastic properties of concrete play a great role in crack formation [25, 26, 29, 30]. Due to the gradual change in time humidity strains develop droningly. Due to their continuous development, the stresses caused by them significantly relax in time [12, 13, 15, 17, 24]. Thus, the demonstration of plastic properties of concrete is the most complete here.

PURPOSE AND STATEMENT OF A PROBLEM

The purpose of the work is to study the stress state of the high-rise buildings monolithic concrete which hardens in natural conditions in summer period. The task of the research is to prevent the occurrence of technological cracks caused by temperature-shrinkage strains of concrete during its hardening.

THE MAIN SECTION

The development of inherent stresses has different influence on concrete properties. When temperature-shrinkage strains increase in time and, consequently, when structural stresses in the material of the cement sheath around the aggregate increase, there may develop two contrary processes: the area of plastic flows or the area of cracks. The possibility of any process depends on strength and elastic properties of the material. Using the strength theory [14], the condition of the formation of a plastic zone around the grain will be written as follows:

$$\tau_{max} = \frac{V \cdot \Delta E(t)}{\frac{1+V}{K_1} + \frac{1-V}{K_2}} \le \frac{k_{nn} R_t}{2}. \qquad (1)$$

The condition of the beginning of crack formation will be expressed using Fere formula for the concrete tensile strength:

$$\sigma_T^{max} = \frac{(1+2V)\cdot 0,5\cdot \Delta E(t)}{\frac{1+V}{K_1}+\frac{1-V}{K_2}} = \frac{1}{2}\sqrt[3]{R_t^2}. \qquad (2)$$

Hence, we shall express the quantity of ΔE

a) from the condition of the plastic zone formation:

$$\Delta E_{пл} = \frac{k_{пп}\cdot R_t}{2V}\cdot\left(\frac{1+V}{K_1}+\frac{1-V}{K_2}\right), \qquad (3)$$

b) from the condition of the crack zone formation:

$$\Delta E_{mp} = \sqrt[3]{R_t^2}\left(\frac{\varphi}{K_1}+\frac{\xi}{K_2}\right). \qquad (4)$$

Apparently, for the summer temperatures the limiting crack formation strains are always far less than plastic flow strains. There is a possibility of plastic flow at low values of R/K in unreal concrete compositions, when V>0,8, therefore, only the condition of crack formation as a sequent of a bulk concrete strain is to be considered hereinafter.

Equation of equilibrium for a spherical element separated from the shell around the aggregate looks like [16]:

$$2\cdot(\sigma_p-\sigma_T)+r\frac{d\sigma_p}{dr}=0. \qquad (5)$$

For the crack zone at $\sigma_T = 0$ after the separation of variables it will become:

$$\frac{d\sigma_p}{\sigma_p}=-2\frac{dr}{r}. \qquad (6)$$

Integrating the equation between the limits $a \le r \le r_T$, where r_T is the radius of the crack zone, we shall find $\ln\sigma_p = -2\ln r + \ln H$. Therefore, $\sigma_p = \frac{H}{r^2}$. Using the boundary conditions we shall define the arbitrary constant of integration at $r = a, \quad \sigma_p = \sigma_{pa}$. Therefore, considering $a = \frac{d}{2}$; $H = \frac{\sigma_{pa}\cdot d^2}{4}$, definitely $\sigma_r = \sigma_{pa}\frac{d^2}{4r^2}$. The equation is true only within the zone of cracks.

Radial stresses on the boundary of the elastic zone and the crack zone are equal, and unit stresses at $r = r_T$ in the limit equal the tensile strength of material at a time $t : \sigma_T(t) = R_p(t)$. From this condition the radius of the crack zone can be defined:

$$\sigma_t(\tau) = \frac{V\cdot\left(1+2\frac{r_T^3}{\eta^3 d^3}\right)\Delta E(t)}{2\frac{r_T^3}{\eta^3 d^3}\left(\frac{1+V}{K_1}+\frac{1-V}{K_2}\right)\cdot(1+0,5\varphi_t)} = \qquad (7)$$
$$= R_p.$$

Here we adopt the convention that $b = \eta\cdot d$. In addition coefficient η can be defined by the value with regard to the content of aggregate in a unit of the concrete volume $V : \eta = \frac{1}{2\cdot\sqrt[3]{V}}$.

The crack zone radius around the aggregate equals to:

$$r_T = \frac{d}{2}\sqrt{\frac{\Delta E_t}{2\left\{\left[\frac{R_p}{K_1}\cdot(1+V)+\frac{R_p}{K_2}\cdot(1-V)\right]-\Delta E_t\cdot V\right\}}}. \qquad (8a)$$

It is obvious from the obtained formula that the size of the crack formation zone increases with the extension of the size of aggregate particles (d) and strains $\Delta\mathrm{E}(t)$. The formula (8a) can be simplified by adopting, for example K_1=K and $\frac{R_p}{K} = \mathrm{E}_R$:

$$r_T = \frac{d}{2}\sqrt{\frac{\Delta\mathrm{E}(t)}{4\,\mathrm{E}_R(1+0,5\,\varphi_t)-2\,\Delta\mathrm{E}(t)\cdot V}}. \qquad (8b)$$

Let’s analyze radial and unit stresses in an elastic zone of the aggregate shell.

The elasticity theory considers the problem of the stresses state in axially symmetrical bodies which states that:

$$\mathrm{E}_p = \frac{dU}{dr} = -\frac{2A}{r^3} + B, \tag{9a}$$

$$\mathrm{E}_p = \frac{U}{r} = \frac{A}{r^3} + B, \tag{9b}$$

where: U is a radial displacement of the given point of the element.

Using the generalized Hook’s law, the stresses in the mortar shell equal to:

$$\sigma_p = \frac{\mathrm{E}_1}{1-\mu_1-2\mu_1^2}\left[2\mu_1\mathrm{E}_T + (1-\mu_1)\mathrm{E}_{p1}\right], \tag{10a}$$

$$\sigma_T = \frac{\mathrm{E}_1}{1-\mu_1-2\mu_1^2}\left[\mathrm{E}_T + \mu_1\,\mathrm{E}_p\right]. \tag{10b}$$

Substituting into equation (10) the values of the relative strains from the (9), we obtain:

$$\sigma_p = \frac{\mathrm{E}_1}{1-\mu_1-2\mu_1^2}\left[-\frac{2A}{r^3}\cdot(1-2\mu_1) + B(1+\mu_1)\right], \tag{11a}$$

$$\sigma_T = \frac{\mathrm{E}_1}{1-\mu_1-2\mu_1^2}\left[\frac{A}{r^3}\cdot(1-2\mu_1) + B(1+\mu_1)\right]. \tag{11b}$$

It is necessary to obtain the constants of integration real for every layer (A, B, A_1: B_1, A_2, B_2, A_3) to describe the properties of the material equivalent to the adopted structural model. For this purpose, the conditions of equality of radial displacements (9b) and stresses (11a) at the junction of each layer of the element are used like:

$$U = Ar + Br^{-2},\ \sigma_p = 3KA - 6\lambda\cdot K\cdot Br^{-3}, \tag{12}$$

where: K is a modulus of volume elasticity of the material of a given layer, λ is a transverse elasticity parameter.

The reduced equations are true only in the free of cracks layers of the element. For the crack zone, where the convention $\sigma_T = 0$ is made, we shall formulate new integrated forms of equations (12) out of the following conditions.

If $\sigma_T = 0$, it follows from the general Hook’s law that:

$$\mathrm{E}_p = \frac{\sigma_p}{\mathrm{E}} \quad \text{or} \quad \mathrm{E}_T = -\mu\frac{\sigma_p}{\mathrm{E}}. \tag{13}$$

Expressing radial strain through the interchange $\mathrm{E}_r = \frac{dU}{dr}$, we find:

$$\sigma_p = \frac{dU}{dr}\cdot\mathrm{E}. \tag{14}$$

The obtained dependence of σ_p on U, as well as the equation of equilibrium (5), allow to write down the equation of strain compatibility on the area between the cracks:

$$\frac{d^2U}{dr^2} + \frac{2}{r}\cdot\frac{dU}{dr} = 0. \tag{15}$$

The integration of the obtained differential equation of second order results in the dependence:

$$U = A + B\,r^{-1}. \tag{16}$$

For the searching of A and B it is necessary to substitute relatively the first and the second columns in this determinant by the right side of the equations.

$K_{\max}$ is defined by formula obtained from the condition that $r_T = a$:

$$K_{\max} = \frac{K_1K_2}{K_2-(K_1-K_2)\cdot V}. \tag{17}$$

At $V=0;\ K_{\max}=K_1$, and at $V=1{,}0;\ \ K_{\max}=K_2$.

The constants of integration of *A* and *B* allow solving a number of tasks. Expressions to determine constants when $r_T=0$, after elementary simplifications look like:

$$\left.\begin{aligned} A&=\frac{4}{3}\sigma_y\cdot 3b^3-(1-V)/K_1-V/\cdot K_2\\ B&=-3\sigma_y b^3\cdot\frac{K}{K_1}\end{aligned}\right\}. \qquad (18)$$

Using these values in the equation (12) together with the criterion of crack resistance $\sigma_T\le R_p$, as well as the dependence (18), it is possible to calculate the condition of the material continuity conservation, which depends on the shrinkage strain limits of concrete.

Let us write the unit stresses equation for the case of the full development of the crack in the mortar shell $r_T=b$, and ΔE_y we shall express through the ultimate shrinkage of the mortar and coarse aggregate saturation of concrete. Then the real unit stresses, considering their relaxation equal to:

$$\sigma_T=\frac{3V[\Delta E]}{2\left(\frac{1+V}{K_1}+\frac{1-V}{K_2}\right)S}=R_p, \qquad (19)$$

where: S is a function of the inherent stresses relaxation which equals one month period of hardening:

$$S=1+0{,}5\,\varphi_t. \qquad (20)$$

Here φ_t is a creep characteristic, it is accepted for one month period of concrete hardening, $\varphi=2{,}0$.

Let's define the intensity of unit stresses as a function of coarse aggregate saturation of concrete, expressing ΔE from the dependence (4):

$$\sigma_T=\frac{3\cdot E_p\cdot K\cdot V\cdot(1-V)}{2\cdot S\cdot[(1-V)+nV]\cdot[(1+V)\cdot n+(1-V)]}, \qquad (21)$$

where: E_p is a shrinkage strain of the mortar shell of concrete,

n is a ratio of moduluses of volume elasticity of aggregate and mortar $\left(n=\frac{K_2}{K_1}\right)$.

For this purpose we should define the derivative $d\sigma/dV$ and equate it to zero. However, as in general form the equation is rather lengthy, this difficulty can be overcome, giving the amounts of ***n*** certain values and differentiating specific equation at a given value ***n***. For example, if *n=1* than unit stresses equal to $\sigma_T=\frac{3\cdot E_p\cdot K_2}{4\cdot S}\cdot V\cdot(1-V)$.

Consequently, $\frac{d\sigma}{dV}=\frac{3\cdot E_p\cdot K_2}{4S}\cdot(1-2V)=0,$

whence at $n=1;V_{onm}=0{,}50$, analogously $n=2;V_{onm}=0{,}38$.

CONCLUSIONS

1. When temperature-shrinkage strains increase in time and, consequently, when structural stresses in the material of the cement sheath around the aggregate increase, there may develop two contrary processes: the area of plastic flows or the area of cracks. The possibility of any process depends on strength and elastic properties of the material.
2. Using values in the equation (12) together with the criterion of crack resistance $\sigma_T\le R_p$, as well as the dependence (18), it is possible to calculate the condition of the material continuity conservation, which depends on the shrinkage strain limits of concrete.
3. The obtained equations allow solving the problem of low level of structural stresses in monolithic concrete when saturating it with a certain amount of coarse aggregate, and also to evaluate the impact of structural stresses on concrete properties.

REFERENCES

1. **Aitcin P.C., 2003.:** The Art and Science of Durable High-Performance Concrete / Pierre-Claude Aitcin // Nelu Spiratos Symp. Committee for the Organization of CANMET/ACI Conferences: Proc. - 69-88.
2. **Akroid T.M.B., 1980.:** Ultimate loabs in rock and concrete // Engineering. – Vo1. 189. – № 4908. – 146-159.
3. **Collepardi M., 2003.:** Innovative Concretes for Civil Engineering Structures: SCC, HPC and RPC / M. Collepardi // Workshop on New Technologies and Materials in Civil Engineering: Proc. – Milan. – 1-8.
4. **Collepardi M., 2006.:** The New Concrete / Collepardi M. - Published by Grafishe Tintoretto. - 421.
5. **Daimon M., 1972.:** Assembly of a vapor adsorption apparatus and its application to the determination of the pore size distribution of hydrated cement / M. Daimon, R. Kondo // Nippon Kagaku, Tokyo. – 238-243.
6. **Derucher K.M., 1989.:** Composite materials: Testing and Design / Derucher K.M. – New Orleans - Philadelphia. - 697.
7. **Edwards R.H., 1981.:** Stress Concentrations Around Spheroidal Inclusions and Cavities / R.H. Edwards // J. Appl. Mech. – V. 73. – N 3. - 19-30.
8. **Henk B., 1976.:** Betrachtung uber Gefuhnespannunengen im Beton / B. Henк // Zement - Kalk - Gips. - № 3. - 76-79.
9. **Holland T.C., 1998.:** High-Performance Concrete: As High as It Gets/T.C. Holland // The Concrete Producer. - Vol. 16, No 7, July. - 501-505.
10. **Hooks John M., 1999.:** HPS bridges for the 21-st century / John M. Hooks // Bridge Views. - Issue N 6. - 1-3.
11. **Hughes B.P., Gregory R., 1982.:** Concrete Subjected to High Rates of Loading in Compression//Mag. Concr. Res. – V. 24. – 25-36.
12. **Mechanical Properties of Modified Reactive Powder Concrete, 1997.:** S. Collepardi, L. Coppola, R. Troli, M. Collepardi//The Fifth Conf. on Superplasticizers and Other Chemical Admixtures in Concrete: ACI Publication SP-173, 1997: Proc. – Roma. - 1-21.
13. **Meguid S.A., 1989.:** Engineering fracture mechanics / Meguid S.A. – London and New York: Elsevier applied science. – 397.
14. **Mikhaylov K.V., 1987.:** Concrete and reinforced concrete in construction / K.V. Mikhaylov, Y.S. Volkov – M.: Stroyizdat. – 103. (in Russian).
15. **Microcracking of Plain Concrete and the Shape of the Stress Strain Curve, 1983.:** T.T.C. Hsu, F.O. Slate, G. Sturman, G. Winter // J. Amer. Conc. Inst. – N2. – Proc. 60. – 209-224.
16. **Murin A.N., 1985.:** Chemistry of imperfect ionic crystals / A.N. Murin - L.: Publ. LGU. - 272. (in Russian).
17. **Popovics S., 1989.:** Fracture Mechanism in Concrete: How Much Do We Know? / S. Popovics // J. Eng. Mech. Div. Proc. Amer. Soc. Eng. – V. 95. – N3. – 531-544.
18. **Punagin V.V., 2010.:** Nonrigid properties of modified fine grain concrete for the groiting of damaged structures//Herald of Volodymyr Dahl East Ukrainian National University. - № 3 (145). – 181-186. (in Russian).
19. **Punagin V.V., 2010.:** Peculiarities of the monolithic concrete curing at the construction of high-rise buildings at elevated temperatures//Herald of Volodymyr Dahl East Ukrainian National University. - № 12 (154). – Part 2. – 159-166. (in Russian).
20. **Punagin V., 2010.:** Properties and Technology of Concrete for high altitude monolithic construction//TEKA Kom. Mot. I Energ. Roln. – Lublin: OL PAN. – Vol. XB. – P. 114-119.
21. **Punagin V., 2012.:** Physicochemical Characteristics of Bond and Friction between the Modified Concrete and Sliding Formwork for the Construction of High-rise Buildings//TEKA Kom. Mot. I Energ. Roln. – Lublin: OL PAN. – Vol. 12. – No. 4. - 229-234.
22. **Reading T.J., 1987.:** Shotcrete as a construction material / T. J. Reading // Shotcreting. – Publication SP-14. – ACI. – 98-118.
23. **Rudenko N., 2010.:** The Development of Conception of New Generation Concretes//TEKA Kom. Mot. I Energ. Roln. – Lublin: OL PAN. – Vol. XB. – 128-133.
24. **Schiller K.K., 1991.:** Mechanical Properties of Non-Metallic Brittle Materials / K. K. Schiller // Cement and Concrete Research. - Vol. 2. - 35-42.
25. **Shayan A., 1992.:** Microscopic Features of Cracked and Uncracked Concrete Railway Sleepers / A. Shayan, G. W. Quick // ACI Materials Journal. - Vol. 89. - No. 4. - 348-360.
26. **Sontige C.D., 1993.:** Fracture Mechanism of Concrete Under Compressive Loads / C.D. Sontige, H. Hilsdorf // Cem. and Concr. Res. – V.3. – N 4. – 363-388.
27. **Vyrodov I.P., 1979.:** Perspectives of development of the physicochemical bases of hydration processes and formation of physical and mechanical properties of mineral binding systems / I.P. Vyrodov//Physicochemical processes of hydration and hydration hardening

of binders. - Krasnodar: Publ. KPI. - Iss. 96. - 5-27. (in Russian).

28. **Willis J.R., 1982.:** Elasticity Theory of Composites / J.R. Willis // Mechanics of Solids / Ed. H.G. Hopkins and M. J. Sewell: Pergamon Press. – 653-686.
29. **Yong I.F., 1991.:** Makrodefect-free Cement / I. F.Yong // A Review. Mat. Soc. - Vol. 179. – 101-122.
30. **Zollo R.F., 1997.:** Fiber-reinforced Concrete: an Overview after 30 year of Development / R.F. Zollo // Cem. Concr. Com. - Vol. 19. – 107-122.

ИССЛЕДОВАНИЕ СТРУКТУРНЫХ НАПРЯЖЕНИЙ В МОНОЛИТНОМ БЕТОНЕ ЕСТЕСТВЕННОГО ТВЕРДЕНИЯ

Владимир Пунагин

Аннотация. В статье представлены результаты исследований структурных напряжений в монолитном модифицированном бетоне при определенном насыщении его крупным заполнителем, а также влияния структурных напряжений на свойства бетона. Это позволит решить ряд технологических задач при возведении высотных зданий в условиях летних температур.

Ключевые слова: модифицированный бетон, структурные напряжения, деформации.