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Varying of reliability indexes in passively reserved difficult technical systems

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Summary. In the article the passively reserved system working in the mode of gradual damages accumulation is considered. The rules of change of basic reliability indexes, such as the probability of faultless work and mean time of work till renunciation are set.
Key words. Readiness to work, mean work on a renunciation, probability of faultless work, model of fail safety.

INTRODUCTION

Reservation is one of the most effective methods in increase of reliability of the technical systems. Its wider use in engineering is restrained by the necessity of introduction of additional elements (reserve parts), methods and availability of periodic regulations, and also the renewals of details' working capability in the shape of regrinds, use of additional working verges etc [8]. Another reason of insufficient application of the passive reservation is the absence of scientifically-reasonable recommendations on its effective use. This article is directed on the removal of the indicated defects.

RESULTS AND DISCUSSION

Pre-condition for the establishment of main criteria of reliability of the passively reserved system are probabilities of its being in this or that expediently chosen state [5]. The sizes of such probabilities settle accounts in accordance with the rule of Cramer [6]:

$$\varphi_з(S) = \frac{\Delta_i}{\Delta}, \qquad (1)$$

where:

$\varphi_i(S) \leftrightarrow P_i(t)$ – the possibility of i-state in the transformations of Laplas;

Δ_i – determinant of the equation system (7) [1] for the observed unknown authenticity;

Δ – basic determinant of the same equation system.

The choice of the state of the system and corresponding to its determinant are set in obedience to the established task of researches and oriented to one or another studied reliability index. However, in any case it is necessary to find the decision of the determinant which stands in the denominator of formula (1). Its size comes from the general extended matrix of the system of equalizations [1] and makes a basic matrix which is written down as follows:

$$\Delta = \begin{vmatrix} S+\lambda_{00} & 0 & -\mu_{10} & 0 & 0 \\ -\lambda_{00} & S+\lambda_{0'0} & 0 & 0 & 0 \\ S & S & S & S & S \\ 0 & 0 & -\lambda_{10} & S+\lambda_{00'} & 0 \\ 0 & 0 & 0 & -\lambda_{10'} & S+\mu_{11} \end{vmatrix}. \qquad (2)$$

The presented matrix has a fifth grade and needs for a further decision in lowering [7].

Executing the corresponding mathematical operations of grade lowering, by means of algebraic transformations and bringing expressions over, we will get:

$$\Delta = aS^5 + bS^4 + cS^3 + dS^2 + eS,$$

where:

$a = 1;$

$b = (\mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{0'0} + \lambda_{00} + \mu_{10});$

$c = (\lambda_{10'}\mu_{11} + \lambda_{10}\lambda_{10'} + \mu_{11}\lambda_{10} + \lambda_{0'0}\mu_{11} + \lambda_{0'0}\lambda_{10'} + \lambda_{0'0}\lambda_{10} + \lambda_{00}\mu_{11} +$
$+ \lambda_{10}\lambda_{10'} + \lambda_{00}\lambda_{10} + \lambda_{0'0}\lambda_{0'0} - \mu_{10'}\mu_{11} + \mu_{10}\lambda_{10'} + \mu_{10}\lambda_{0'0});$

$d = (\lambda_{0'0}\lambda_{10'}\mu_{11} + \lambda_{0'0}\lambda_{10}\lambda_{10'} + \lambda_{0'0}\mu_{11}\lambda_{10} + \lambda_{0'0}\lambda_{10'}\mu_{11} + \lambda_{00}\lambda_{10}\lambda_{10'} +$
$+ \lambda_{00}\mu_{11}\lambda_{10} + \lambda_{00}\lambda_{0'0}\mu_{11} + \lambda_{00}\lambda_{0'0}\lambda_{10'} + \lambda_{00}\lambda_{0'0}\lambda_{10} - \mu_{10}\lambda_{00}\mu_{11} - \mu_{10}\lambda_{00}\lambda_{10'} +$
$+ \mu_{10}\mu_{11}\lambda_{00} + \mu_{10}\lambda_{10'}\lambda_{00} + \mu_{10}\lambda_{10'}\mu_{11} + \mu_{10}\mu_{11}\lambda_{0'0} + \mu_{10}\lambda_{10'}\lambda_{0'0});$

$e = (\lambda_{00}\lambda_{0'0}\lambda_{10'}\mu_{11} + \lambda_{00}\lambda_{10}\lambda_{0'0}\lambda_{10'} + \lambda_{00}\lambda_{0'0}\mu_{11}\lambda_{10} + \mu_{10}\lambda_{10'}\mu_{11}\lambda_{0'0}).$

If to ignore the values of intensities of refuses at triple and more multiplying, as the values of high order of trifle, we will get considerable simplification of mathematical expressions for determination of matrix decision. Then we get:

$$\Delta = S^3\left(aS^2 + bS + c\right). \quad (3)$$

The numerator of ratio (1) for determination of probability of fully capable working state $\varphi_{00}(S)$ comes from the basic matrix by the substitution of the column of free members of the extended matrix [10] in the column of the found probability of the ”00” state. Then we have:

$$\Delta_{00} = \begin{vmatrix} 1 & 0 & -\mu_{10} & 0 & 0 \\ 0 & S+\lambda_{0'0} & 0 & 0 & 0 \\ 1 & S & S & S & S \\ 0 & 0 & -\lambda_{10} & S+\lambda_{10'} & 0 \\ 0 & 0 & 0 & -\lambda_{10'} & S+\mu_{11} \end{vmatrix}.$$

Thus, the components for finding out the probabilities of the first capable working state are set and when we put them according to formula (1) for the state “00” we get:

$$\varphi_{00}(S) = \frac{\begin{pmatrix} S^4 + S^3(\mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{0'0} - \mu_{10}) + S^2(2\lambda_{10}\mu_{11} + \lambda_{10}\lambda_{10'} + \\ \lambda_{0'0}\mu_{11} + \lambda_{0'0}\lambda_{10'} + \lambda_{0'0}\lambda_{10} - \mu_{10}\lambda_{10'} - \mu_{10}\lambda_{0'0} - \mu_{10}\mu_{11}) + \\ + S(2\lambda_{0'0}\lambda_{10}\mu_{11} - \mu_{10}\lambda_{0'0}\lambda_{10'} - \mu_{10}\mu_{11}\lambda_{0'0}) - \mu_{10}\mu_{11}\lambda_{0'0}\lambda_{10'} \end{pmatrix}}{aS^5 + bS^4 + cS^3 + dS^2 + eS}. \quad (4)$$

However, this probability is obtained as a reflection in transformations of Laplace and for passing to the original it needs proper mathematical operations. Such transition becomes possible if to present the function of probability as a sum of vulgar fractions [2]:

$$\varphi_{00}(S) = \frac{A_{00}}{S-S_1} + \frac{B_{00}}{S-S_2} + \frac{C_{00}}{S-S_3} + \frac{D_{00}}{S-S_4} + \frac{E_{00}}{S-S_5}, \quad (5)$$

where:

$A_{00}, B_{00}, C_{00}, D_{00}, E_{00}$ – inserted unknown permanent values, necessary to be set for back transformation of Laplas;

S_1, S_2, S_3, S_4, S_5 – roots of the right part of the equation (3).

The roots come from the equation (3) in the following way:

$$S^3\left(aS^2 + bS + c\right) = 0\cdot$$

It is obvious, that $S_1 = S_2 = S_3 = 0$, but S_3 and S_4 – roots of the right part of the equation, situated in brackets according to the formula [3]:

$$S_{4,5} = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right) - c}\cdot \quad (6)$$

For determination of the inserted additional permanent values we will make a comparison in the equivalence of numerators of expressions (4) and (5). When denominators are equal, the equivalence of polynomials of numerators is possible as a result of equality of coefficients at the same degrees of the unknown. After inserting of the substitutes $A_{00} + B_{00} + C_{00} = Z_{00}$ it is possible to write down the additional system from three equations:

$$\begin{cases} Z_{00} + E_{00} = 1; \\ (Z_{00} + D_{00})S_5 - (Ж_{00} + E_{00})S_4 = \mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{00'} - \mu_{10}; \\ -ZS_4S_5 = L, \end{cases}$$

where:

$$L = 2\lambda_{10}\mu_{11} + \lambda_{10}\lambda_{10'} + \lambda_{0'0}\mu_{11} + \lambda_{0'0}\lambda_{10'} + \lambda_{00}\lambda_{10} - \\ - \mu_{10}\lambda_{10'} - \mu_{10}\lambda_{0'0} - \mu_{10}\mu_{11}.$$

The received system is solved by the method of successive substitution. From the third equalization we will write down:

$$Z_{00} = -\frac{L}{S_4S_5}\cdot$$

From the second equation we will get:

$$\left(Z_{00} + D_{00}\right)S_5 - S_4 = \mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{00'} - \mu_{10}\cdot$$

And

$$D_{00} = \frac{\mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{00'} - \mu_{10} + S_4 - \frac{L}{S_4}}{S_5}.$$

Thus, the permanent values of Z_{00}, D_{00}, E_{00} are obtained and it is possible to conduct reverse transformation of Laplace from the images to the originals. Then we have:

$$P_{00}(t) = Z_{00} + D_{00}\exp(-S_4t) + E_{00}\exp(-S_5t).$$

Inserting the meanings of permanent values we can write down:

$$P_{00}(t) = -\frac{L}{S_4S_5} + \frac{1}{S_5}\left(\mu_{11} + \lambda_{10'} + \lambda_{10} + \lambda_{00'} - \mu_{10} + S_4 - \frac{L}{S_4}\right)\times \\ \times\exp(-S_4t) + \left(1 - \frac{L}{S_4S_5}\right)\exp(-S_5t). \quad (7)$$

By analyzing the received result it is necessary to notice that the value of probability of $P_{00}(t)$ is determined by three elements. Their signs depend on those components which are included in them, but the general value of probability must not exceed the unit that is the rationed condition. The analysis of the result is complicated by unknown λ, μ – characteristics, included both directly into the formula (7) and into the substitution L, and also roots of S_4 and S_5. That’s why in this research it is possible to conduct only the preliminary quality analysis of the change of probability of the working state “00” depending on the time of exploitation of the technical system. It is obvious that independently of its sign the first component is some permanent value which depends on the value of the attended roots of S_4 and S_5 displacing general dependence of $P_{00}(t)$ upwards or downward on a y-axis.

The analysis of behavior of probability function for the state of the system “00” at the time of exploitation of $t \to \infty$ shows that the increase of time diminishes the probability on an exponential law. The final probability in this case is:

$$P_{00}(t \to \infty) = Z_{00}\ .$$

Or, it is possible on the basis of the value of permanent Z_{00} which has been got before to write down:

$$P_{00}(t \to \infty) = \frac{1}{S_4S_5}(2\lambda_{10}\mu_{11} + \lambda_{10}\lambda_{10'} + \lambda_{0'0}\mu_{11} + \lambda_{0'0}\lambda_{10'} + \\ + \lambda_{00}\lambda_{10} - \mu_{10}\lambda_{10'} - \mu_{10}\lambda_{0'0} - \mu_{10}\mu_{11}).$$

Thus, the conducted analysis set, that general character of changing in probability of capable working state “00” of the passively dubbed system is described by a double exponential law. Its chart is presented in Fig. 1. The concrete form of the curve is greatly dependent on the correlations λ and μ which are descriptions included in the equalization. The double exponential dependence assists at deceleration of loss of capacity by the system while entering the dubbed element.

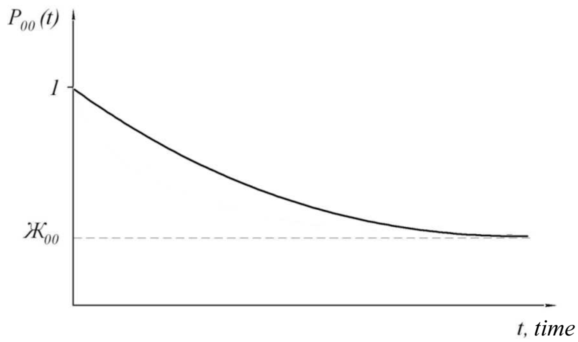

Fig. 1. Changing in the probability of the passively dubbed system in fully capable working state from time of exploitation

Coming from the known determination of mean work on the refuse through the probability of faultless work [4] we can write down:

$$\overline{T} = \int_0^\infty P(t)dt\,.$$

In detail, to the solving task on determination of mean work in the capable working state "00" it is possible to write down:

$$\bar{t} = \int_{t_1}^{t_2} P_{00}(t)dt\,.$$

For establishment of mean value of work on the refuse it is expedient to choose the time of domain $t_2 - t_1$, after the period of extra work of the system and its entrance in the mode of exploitation, when aging processes, related to the subsequent loss of capacity, begin gradually show the selves [9].

Putting the value of probability from the expression (7) we have:

$$\bar{t}_{00} = \int_{t_1}^{t_2}\left[-\frac{L}{S_4S_5}+\frac{1}{S_5}(\mu_{11}+\lambda_{10'}+\lambda_{10}+\lambda_{0'0}-\mu_{10}+S_4)\exp(-S_4t)+\left(1-\frac{L}{S_4S_5}\right)\exp(-S_5t)\right]dt.$$

From where:

$$\bar{t}_{00} = -\frac{L}{S_4S_5}t\Big|_{t_1}^{t_2} + \frac{1}{S_5}\left(\mu_{11}+\lambda_{10'}+\lambda_{10}+\lambda_{0'0}-\mu_{10}+S_4-\frac{L}{S_4}\right)\left(-\frac{1}{S_4}\right)e^{-S_4t}\Big|_{t_1}^{t_2} + \left(1-\frac{L}{S_4S_5}\right)\left(-\frac{1}{S_4}\right)e^{-S_5t}\Big|_{t_1}^{t_2}. \qquad (8)$$

For the quality analysis of result the possible dependences of influence of each components of equalization were built (8) on a general result, and the total curve of change of mean time of system staying in the fully capable working state of *t00*. The graphics is represented in Fig. 2.

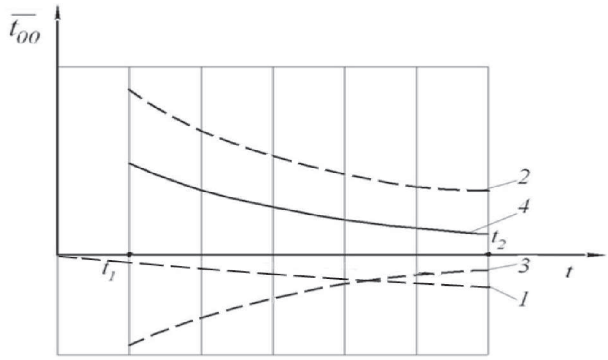

Fig. 2. Dependence of changing of time of the system being in the complete capable working state "00" on the time of its exploitation:

1 – is the first component of equalization (9); 2 – is the second component; 3 – is the third component; 4 – is total resulting dependence

As we can see from the summarizing chart (curve 4), mean time of the system being in the fully capable working state "00", when both basic and reserve elements are in good condition, is gradually going down. It corresponds to the physical essence of the problem of the system's research that it gets older losing the capacity while exploitation.

CONCLUSIONS

1. Passively dubbed technical system losing the capacity as a result of aging has the reliability indexes, depending on the time exploitation according to the double exponential law.
2. The maximally able value of probability of faultless work of the system, when both basic and dubbed elements are in good condition, serves the final (asymptotic) value of this probability.
3. Mean work on the refuse of the system consists of three components, one of those depends on the time of exploitation, and two others change on exponent.

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ВАРЬИРОВАНИЕ ПОКАЗАТЕЛЕЙ НАДЕЖНОСТИ ПАССИВНО РЕЗЕРВИРОВАНЫХ СЛОЖНЫХ ТЕХНИЧЕСКИХ СИСТЕМ

Анотація. У статті розглянута пасивно резервована система працююча в режимі поступового накопичення пошкоджень. Встановлені закономірності зміни основних показників надійності таких як вірогідність безвідмовної роботи і середній час роботи повністю.

Ключові слова: готовність до роботи, середнє напрацювання на відмову, вірогідність безвідмовної роботи, модель надійності системи.