# Research of interaction of disc wave generator with flexible gear of heavy loaded wave gearing

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Summary: Force processes taking place in kinematic pairs of higher degree, disc – flexible gear, cause negative developments in heavy loaded wave gearing. These are sizeable axial forces possessing dynamic characteristic as well as essential energy losses in the field of wave generator. The reported results of force analysis of disc wave generator and flexible gear interaction made it possible to give estimate of force and energy factors observed in the process of large wave gearing operation.

Key words: wave gearing, axial forces, energy losses.

## INTRODUCTION

Speed reducers make the basis of heavy machines drives, the losses in which often determine total energy losses of machines and assembly units. High unit capacities of machines in heavy engineering make actual the issue of energy losses in speed reducers as it is connected not only with the increased energy consumption and financial charges but also with technical conditions: lubrication cooling, heat extraction, application of wear-resistant materials [5, 15, 16].

In theory, practice and industrialization of large wave gearings in heavy engineering, peculiarities of wave generator and flexible gear interaction are the least understood. Because of the complexity of unusual conditions of rigid parts and flexible link contact, axial forces generation mechanism is not discovered. Uncertainty of processes, proceeding in kinematic pairs of higher degree, disc – flexible gear, is conditioned by elastic deformations of flexible link and insufficiently rigid mount of wave generator discs on bearing assemblies, which causes axes deviation from the wave gearing common axis. This article will present direct conditions of axial forces generation in large wave gearings with disc wave generator as well as energy losses in kinematic pairs of higher degree, disc – flexible gear.

## MATERIALS AND METHODS

In heavy engineering gearings set in the machines, transmit torques to actuators up to $10^7$ *Nm* and more. Experiments with large industrial models cause known technical difficulties and big financial charges [1, 13, 23]. Parameters nonlinearity of flexible gear and mating members force interaction complicates conditions of experiments data transfer from small experimental models to large wave gearings [7, 8]. Size factor distorts not only the quantitative but also the qualitative characteristics of the sought parametric dependences of the physical quantities.

In the heavy loaded wave gearings with flexible gears diameters of more than 800mm disc wave generators are used. During load transmission interaction of discs with flexible gear breaks the initial symmetry of flexible gear force balance and causes additional reactions in the kinematic pairs. Force disturbances are conditioned by wave gearings design features and are intensified by scale factor that increases energy losses, wear and vibration, restricts application of large wave gearings [17]. It is known that large energy leakage takes place through the wave generator [3]. At the present moment reasons which influence this factor are not discovered. Mechanism of axial forces generation in the wave gearings is not determined either [24]. Derivation of analytic expressions connecting axial forces and energy losses with dimensions, deformations and conditions of flexible gear mating with elements of wave gearing kinematic pairs of higher degree is of practical interest for heavy engineering [22, 25, 26].

Method of construction of universal mathematical model of power contact of elements of higher degree kinematic pairs with flexible link is based on conditions of flexible gear permanent deformation by wave generator discs in the aggregate with possible spherical movements of the discs with respect to some centers situated on the axes of these discs.

## RESULTS, DISCUSSION

The purpose of the present paper is determination of mechanism of axial forces origin and reveal of nature of energy losses in the field of disc wave generator on the basis of development of universal mathematical model of higher degree kinematic pair disc – flexible gear. Simulation of force and kinematic dependences in the wave generator as well as the solution of this multifactorial problem specifies the conditions of analytic and empirical analogs extraction with sufficient approximation of mating links interaction parameters. Such statement of the problem will allow determining force and energy characteristics of mating of higher degree kinematic pairs elements of wave gearing, design of which differs fundamentally from the rigid gear gearings [4].

Wave generator discs and flexible gear, generating kinematic pair of higher degree have different speed orders. And even slight relative positional variations can have a great impact on force and energy factors in the zone of their contact. Wave generator design admits the possibility of discs axes slight deviation from parallelism with wave gearing axis. Asymmetric load distribution on discs surfaces deflects them relative to the specified planes about some angle $\gamma$. Skewness of axes of wave generator discs generates screw friction pair: wave generator – flexible gear. Drive shaft rotation induces helical motion of wave generator which is «screwed» into the flexible gear, and generates axial forces $P$, which reach intolerably large values.

Generator discs deform flexible gear, located on the free end of the cylindrical shell, opposite end of which is mounted on the driven shaft and fastened by means of spline joint. On the arc of $CD$ disc contact with flexible gear radial forces $q_r$ (Fig. 1) effect, the influence of which is approximated by parabolic relation:

$$q_r = q_{r\max} \cdot \left(1 - \frac{\theta^2}{\theta^{*2}}\right), \qquad (1)$$

where: $\theta$ - current angular coordinate, $\theta^*$ - angular coordinate which determines dimension of radial load with respect to its maximum value $q_{r\,max}$.

Forces $q_r$ form flexible gear and balance radial forces in the gearing, carried by wave generator discs. It is due to the radial float of flexible gear, forces for generation of which in the absence of external load $(M_2 = 0)$ are weak as compared to the radial forces in the gearing. Maximum value of radial forces $q_{r\,max}$ is connected with loading torque $M_2$:

$$M_2 = \frac{d^2}{tg\,\alpha} \cdot \int_0^{\theta*} q_r\, d\theta, \qquad (2)$$

by the following relation:

$$q_{r\,\max} = \frac{3M_2 tg\,\alpha}{2d^2\theta*}, \qquad (3)$$

where: $\alpha$ – gearing angle, $d$ – diameter of the circle passing through the middle of the rigid gear teeth depth.

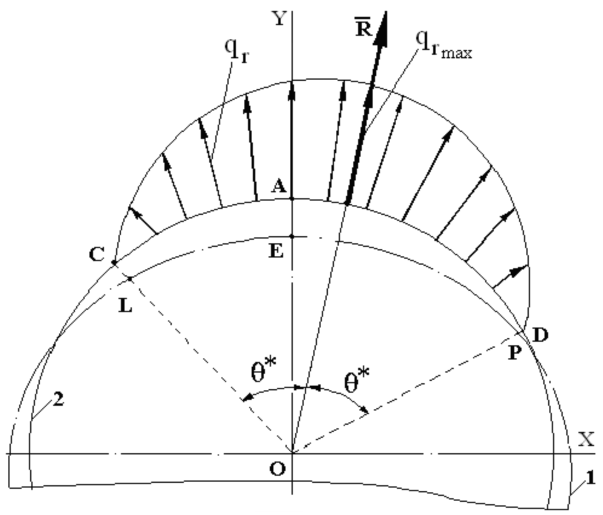



**Fig. 1.** Radial forces distribution on the flexible gear: 1 – before deformation, 2 – after deformation

Maximum radial load $q_{r\,\max}$ is deviated with respect to the major axis of the wave generator $OA$ about the angle of $\chi$ (Fig. 2). During the flexible gear formation by wave generator discs, generators BB$_1$, situated in front of the major axis of wave generator $OA$, deviate from the initial position $KB_1$, before the deformation, about the angle $\psi$, which continuously increases and takes the maximum value $\psi_{\max}$ in the vicinity of wave generator major axis. Equivalent friction forces $F_1$, $F_2$, are determined by formula integration (Eg. 3) within relevant limits:

$$F_1 = f\, q_{r\,max}\alpha \cdot \left\{ \frac{2}{3}\theta* + \chi\left(1 - \frac{\chi^2}{3\theta^2}\right)\right\}, \qquad (4)$$

$$F_2 = f\, q_{r\,max}\alpha \cdot \left\{ \frac{2}{3}\theta* - \chi\left(1 - \frac{\chi^2}{3\theta^2}\right)\right\}, \qquad (5)$$

where: $a$ – distance from drive center line to surface of disk contact with flexible gear, $f$ – friction ratio.

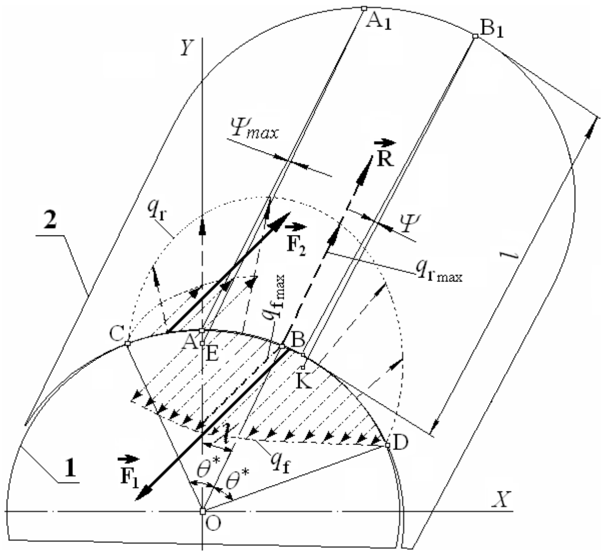



**Fig. 2.** Model of the disks interaction with the flexible gear: 1 – generator disk, 2 – flexible gear

Hereby, we compose the formula for friction torques along axis $X$ and $Y$:

$$M_{1x} = fq_{r\,max}\, a^2 \int_{-\chi}^{\theta*} \left(1 - \frac{\theta^2}{\theta^{*2}}\right) cos(\theta - \chi)\, d\theta, \qquad (6)$$

$$M_{1v} = fq_{r\,max}\, a^2 \int_{-\chi}^{\theta*} \left(1 - \frac{\theta^2}{\theta^{*2}}\right) sin(\theta + \chi)\, d\theta, \qquad (7)$$

$$M_{2x} = fq_{r\,max}\, a^2 \int_{-0*}^{-\chi} \left(1 - \frac{\theta^2}{\theta^{*2}}\right) cos(\theta + \chi)\, d\theta, \qquad (8)$$

$$M_{2v} = fq_{r\,max}\, a^2 \int_{-0*}^{-\chi} \left(1 - \frac{\theta^2}{\theta^{*2}}\right) sin(\theta + \chi)\, d\theta. \qquad (9)$$

After formulas integration for the torques $M_{1x}$, $M_{1v}$, $M_{2x}$, $M_{2v}$, we determine point grid reference for friction torque application $F_1$, $F_2$

$$x_{c_1} = \frac{3a\{\zeta - 2[cos(\theta^* + \chi) + \theta^* sin(\theta^* + \chi)]\}}{2\theta^{*3} + \chi(3\theta^{*2} - \chi^2)}, \quad (10)$$

$$y_{c_1} = \frac{6a[sin(\theta^* + \chi) - \chi - \theta^* cos(\theta^* + \chi)]}{2\theta^3 + \chi(3\theta^{*2} - \chi^2)}, \quad (11)$$

$$x_{c_2} = \frac{3a\{\zeta - 2[cos(\theta^* - \chi) + \theta^* sin(\theta^* - \chi)]\}}{2\theta^{*3} - \chi(3\theta^{*2} - \chi^2)}, \quad (12)$$

$$y_{c_2} = \frac{6a[sin(\theta^* - \chi) - \chi - \theta^* cos(\theta^* - \chi)]}{2\theta^3 - \chi(3\theta^{*2} - \chi^2)}, \quad (13)$$

where: $\zeta = (2 + \theta^{*2} - \chi^2)$.

Friction torques $q_f$ along wave generator large axis:

$$M = F_1 x_{c1} + F_2 x_{c2}. \quad (14)$$

Torque $M$ gives rise to disks skewing with slewing around $OA$ axis for an angle $\gamma$, which depends upon the type of the bearings and mounting fits. The disks can be supported in roller radial double-raw spherical bearings, admitting the twists, whereby, the axial clearances between the disks are taken up and their and their end planes are matched. The disks together with flexible gear establish friction couple with an agle $\gamma$ equivalent to lead angle. The angle is linked to summarized axial clearance of the disks $\Delta$ [2]:

$$\Delta = (c_1 + c_2)(cos\gamma - cos^2\gamma) + \varepsilon \cdot tg\gamma, \quad (15)$$

where: $c_1, c_2$ – distance from spherical bearings center lines up to disks end planes, separated by clearance $\Delta$, $\varepsilon$ – eccentricity of the disks mounting position on the wave generator shaft.

Disks sliding speed $V_f$ and power of energy losses $N_f$ in areas of the disks contact with the flexible gear is proportional to the angle $\gamma$:

$$V_f = a \cdot \omega_1 \cdot tg\gamma, \quad (16)$$

$$N_f = 2(F_1 + F_2)V_f. \quad (17)$$

Axial force $F$, resulted from wave generator disks skewing and from friction forces:

$$F = F_1 + F_2. \quad (18)$$

Coordinate data as for axial force application $F$ during the contact of wave generator disk with flexible gear:

$$X = \frac{F_1\, x_{C_1} - F_2\, x_{C_2}}{F_1 - F_2};\ y = \frac{F_1\, y_{C_1} - F_2\, y_{C_2}}{F_1 - F_2}. \quad (19)$$

Friction forces $F_1$ и $F_2$ create friction torque $M_f$ relative to wave generator large axis:

$$M_f = F_1 x_{C_1} - F_2 x_{C_2}. \quad (20)$$

Friction torque $M_f$ gives rise to disks skewing, rotating them about $OA$ major axis of wave generator at some angle $\gamma$, which depends on type of bearings and mounting clearance. In heavily loaded wave gearing, the disks are supported in roller radial double-raw spherical bearings. Such bearing assemblies assume disk skewing, where axial clearances between the disks are taken up and their end planes are matched. This allows to obtain the dependence of total axial clearance $\Delta$ between the disks from design parameters of wave generator and angle $\gamma$, which determines disk axis alignment error with general wave gearing center line:

$$\Delta = (c_1 + c_2)(1 - cos\,\gamma)\,cos\,\gamma + \varepsilon \cdot tg\,\gamma, \quad (21)$$

where: $c_1$, $c_2$ – corresponding distances from spherical bearings centers up to disks end planes, separated by clearance $\Delta$, $\varepsilon$ – mounting eccentricity of wave generator disks at high speed shaft of wave gearing.

Sliding speed $V_f$ of wave generator disks relating to flexible gear is proportional to disks skewing angle $\gamma$:

$$V_f = a\,\omega_1\,\gamma, \quad (22)$$

where: $\omega_1$ – angular rotational velocity of high speed shaft.

Sliding speed $V_f$ of wave generator disks determines loss of power during friction at higher pair of kinematic elements disk – flexible gear:

$$N_f = 2\, a\, \omega_1\, \gamma\left(F_1 + F_2\right). \qquad (23)$$

Higher energy losses in deformed gear [9, 10] are mainly determined by two factors. Firstly, it is large extent of contact patch and remoteness from pitch point that increase sliding speed of the teeth. Secondly, it is the second class meshing interference resulting from flexible gear complex deformation due to wave generator and loading torque twist [14]. Small module $m = 1{,}5...4\,mm$, with flexible gear diameter $1{,}1...3{,}0\,m$, and small gear spacing of internal toothing $Z_1 - Z_2 = 2$, support meshing interference. Under higher loading torque $M_2 > M_{2nom}$, the meshing interference remarkably increases energy losses in gearing. Under higher loading torque significantly exceeding nominal values $M_2 > 1{,}2M_{2nom}$, the meshing interference can lead to binding and skipping of the teeth in large wave gearing.

To reduce energy losses, the width of toothing should be limited and gearing backlash should be increased. Contact patch range expansion, for example, during gear ratio increase, moves away extreme points of teeth entry/exit from the meshing against pitch point. Thus, the influence of teeth wedging effect is increased, along with teeth sliding speed, and energy loss does grow in toothing of wave gearing.

In a view of wave gearing multithreading, we assume average sliding speed of the teeth as equal to half of the maximum. On the basis of oscillography results as for teeth loading, the power loss in gearing can be determined $P_f$:

$$P_f = \frac{I \cdot f \cdot \omega_1 \cdot M_1}{cos\, \alpha} sin\, \frac{\beta_0}{4}, \qquad (24)$$

where: $I = 1{,}1 + 0{,}0005(u - 80)$ – is ratio, taking into account negative effect of meshing interference under higher loads, it is valid for large wave gearing with disk-type wave generator, upon condition of $b \le 40m$, where $b$ – is width of flexible gear toothing, $\alpha$ – rack tooth profile angle, $f$ – gearing friction ratio, $\omega_1$ – wave generator rotation frequency, $M_1$ – rotation torque at wave generator shaft, $\beta_0$ – gearing range extent.

Main power leakage in wave gearing comes from flexible gear deformation [18]. Contact patch is expanded and, therefore, sliding speed is increased at its peripheral areas and meshing interference areas are appeared. Rigid tooth-wheel gearings do not produce meshing interference [11, 12]. In wave gearings, the meshing interference can be compensated by mechanical compliance of flexible gear, increasing power loss in gearing .Under higher load and low structure rigidity , meshing interference causes their slippage. If mechanical compliance of flexible gear [6] does not compensate meshing interference, then binding of teeth will occur. The attempts to eliminate binding of teeth with module $1{,}5\,mm$ and $2\,mm$ in large wave reducers at «NKMZ» PJSC, by means of treatment with special pastes, did not bring favorable results. However, after elimination of meshing interference areas, the wave gearings achieved specified technical characteristics [21].

The power of energy losses in gearing $N_\Sigma$ comprises the losses in gearing $P_f$ and wave generator area $N_f$:

$$N_\Sigma = P_f + N_f. \qquad (25)$$

Other energy losses in wave gearings are insignificant compared to those in gear engagement and wave generator area, therefore, they are not taken in consideration [19, 20].

Theoretical and experimental investigations have been performed at industrial models of wave reducers: tilt drive of mobile mixer MP – 600AC with capacity – $600\,t$ (molten metal) and drive of ore-crushing mill MGR 5500x7500 with capacity – $160\,m^3$, and charged ore weight – $220\,t$.

The power of energy losses in wave gearing engagement of the mill $P_{Lf}$ and the mixer $P_{kf}$ is determined according to formula (Eg. 24), where: gearing friction ratio $f_1 = 0{,}08$, $f_2 = 0{,}1$, wave generator rotation frequency $\omega_1 = 78{,}5\,s^{-1}$ and rotation torque at mill wave generator shaft $M_{L1} = 2600\,Nm$ and mixer $M_{k1} = 1880\,Nm$: $P_{kf1} = 6946\,W$; $P_{kf2} = 8683\,W$, $P_{Lf1} = 10029\,W$, $P_{Lf2} = 12536\,W$. If: $\omega_1 = 13{,}8\,s^{-1}$, $f_1 = 0{,}08$ and $f_2 = 0{,}1$, $M_{k1} = 1880\,Nm$ – power loss in wave gearing engagement of the mixer $P_{kf_1} = 1221\,W$, $P_{kf_2} = 1526\,W$.

Wave gearing engagement efficiency of the mixer $\eta_k$ and the mill $\eta_L$, with different friction ratios $f_1 = 0{,}08$ and $f_2 = 0{,}1$: $\eta_{k1} = \eta_{L1} = 0{,}95$, $\eta_{k2} = \eta_{L2} = 0{,}94$. Upon the absence of meshing interference $(I = 1)$, wave gearing engagement efficiency grows: $\eta'_{k1} = \eta'_{L1} = 0{,}96$, $\eta'_{k2} = \eta'_{L2} = 0{,}95$. It is possible for low-loaded wave gearing with narrow toothing, correctly calculated and precisely executed geometrical parameters of the gearing and harmonic drive in general.

The power of energy losses in wave generator area $N_f$ is proportional to contacting surfaces friction ratio $f$ and disks skewing angle $\gamma$ towards their motion plane. The dependence of power loss volume in contact areas of the disks with flexible gear from the angle $\gamma$, for mixer and mill reducers are determined according to formulas (Eg. 4), (Eg. 5) and (Eg. 23). The power of energy losses with reducer load torque $M_2 = 5 \cdot 10^5\,Nm$, friction ratios $f_1 = 0{,}049$ and $f_2 = 0{,}08$, mixer reducer wave generator rotation frequency: $\omega_{k1} = 13{,}8\,s^{-1}$ are shown at Fig. 3 – straight line 1 and 2, $\omega_{k2} = 78{,}5\,s^{-1}$ – straight line 5 and 6, for the mill $\omega_{L1} = 78{,}5\,s^{-1}$ – straight line 3 and 4.

Upon reduction of the friction ratio $f$ from $0{,}08$ up to $0{,}049$, the energy losses in wave reducers under investigation are decreased. Reduction of the friction ratio and energy losses is achieved by means of bronze ring installation between the disks of wave generator and flexible gear.

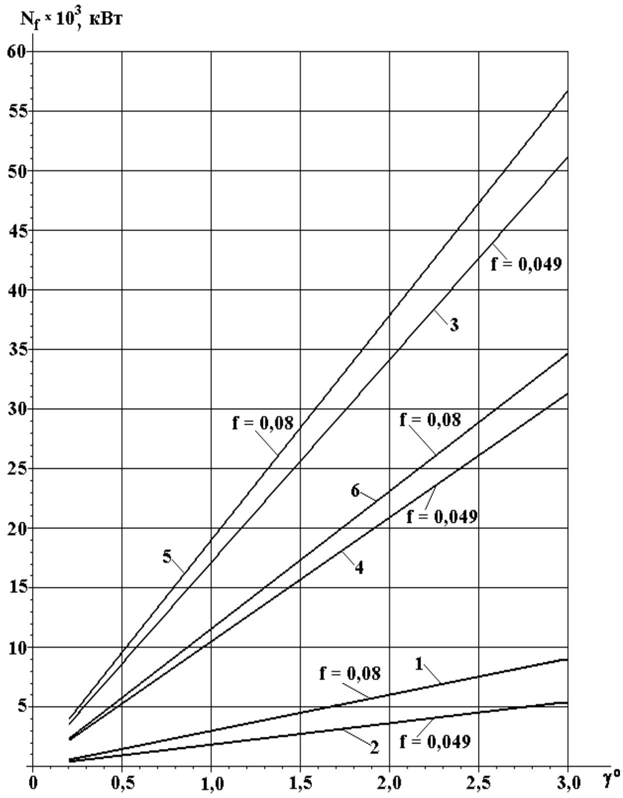


**Fig. 3.** Characteristic lines of energy losses power $N_f$, depending on disks skewing angle in flexible gear contact areas: 1, 2, 3, 4 – for mixer reducer, 5, 6 – for mill reducer

Upon growth of mixer tilt drive wave generator rotation frequency from $\omega_{k1} = 13{,}8\,s^{-1}$ (Fig. 3) (straight lines 1, 2) up to $\omega_{k2} = 78{,}5\,s^{-1}$ (straight lines 3, 4), the power of energy loss increases in $5{,}7$ times, i.e. in proportion to growth of the power transmitted by this reducer.

Upon wave generator rotation frequency $\omega_{L1} = \omega_{k2} = 78{,}5\,s^{-1}$, energy losses in mixer slewing reducer wave generator area are for $9\%$ *less*, than in the mill reducer . It is stipulated by different values of transmitted powers and gear ratios of the reducers, as well as different correlation of geometrical parameters, effecting the load distribution in kinematic pairs. Thus, upon the same load, the mixer slewing reducer $q_{r\,max}$ is for $12\%$ less, than in the mill reducer and, correspondently, the axial force *F* of the mixer reducer is less for $8\%$.

At Fig. 4 and Fig. 5, the efficiency characteristic lines of the mixer and mill drives wave reducers, depending on disks skewing angle $\gamma$, with different friction ratios $f$ in flexible gear contact areas are shown. Dependence characteristic lines 1 and 2 are given without consideration of the losses in gearing. Dependence characteristic lines 3 and 4 are given with consideration of the losses in gearing and wave generator.

without consideration of the losses in gearing, changes as follows: at $f = 0{,}08$, $\eta = 0{,}94 \div 0{,}72$, at $f = 0{,}049$, $\eta = 0{,}96 \div 0{,}81$ (Fig. 4) curves 1 and 2. With consideration of the losses in gearing: at $f = 0{,}08$, $\eta = 0{,}89 \div 0{,}68$, at $f = 0{,}049$, $\eta = 0{,}91 \div 0{,}77$ (curves 3 and 4). With consideration of the losses in gearing: at $f = 0{,}08$, $\eta = 0{,}89 \div 0{,}68$, at $f = 0{,}049$, $\eta = 0{,}91 \div 0{,}77$ (curves 3 and 4).

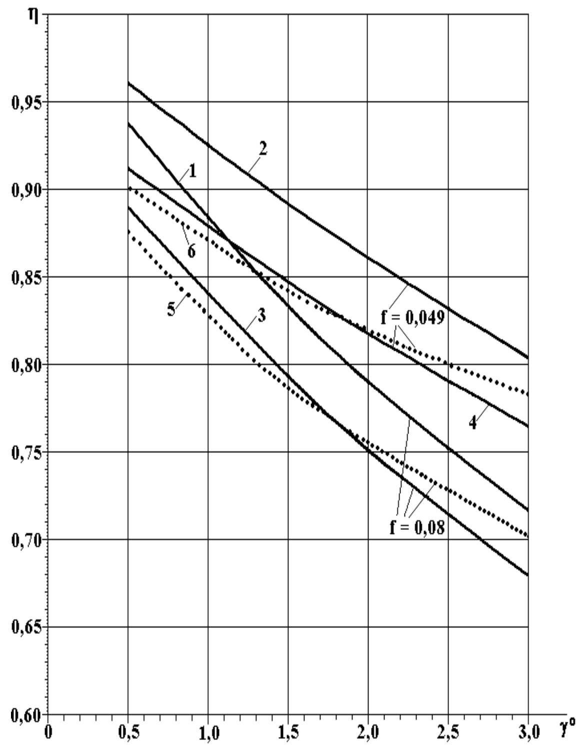


**Fig. 4.** Mill wave reducer efficiency depending on the angle : 1, 2 – losses in wave generator included, 3, 4 – losses in gearing and wave generator included, 5, 6 – all losses in speed reducer included, results of experiment

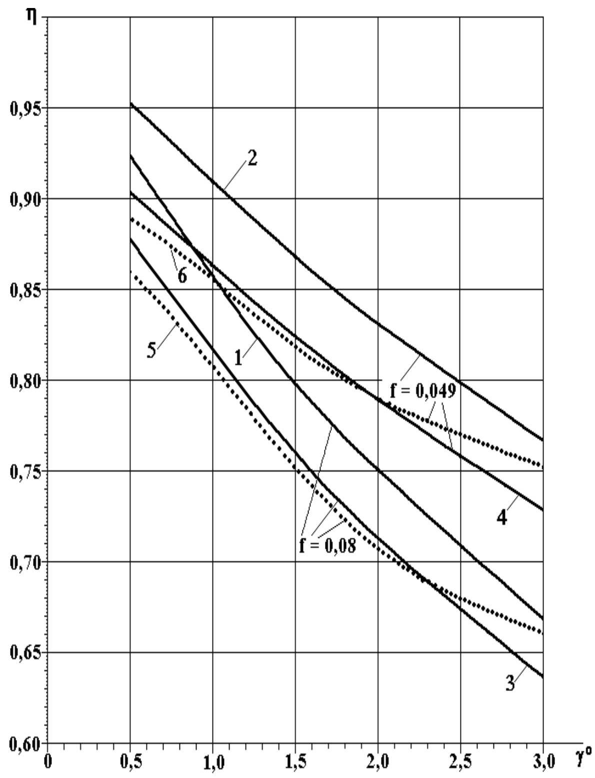


**Fig. 5.** Mixer wave reducer efficiency depending on the angle: 1, 2 – losses in wave generator included, 3, 4 – losses in gearing and wave generator included, 5, 6 – all losses in speed reducer included, results of experiment

Dependence characteristic curves 5 and 6 were obtained experimentally and conform the general efficiency of the reducers under investigation. During experiment, the bronze or steel spacing ring was used, placed between the disks and flexible gear. Dependence characteristic lines 5 at Fig. 4 and Fig. 5 were obtained for steel spacing ring, and dependence characteristic lines 6 for bronze ring.

Upon angle $\gamma$ variation from $0{,}5^\circ$ up to $3^\circ$, the efficiency of the mill wave reducer,

The results of experimental studies of the mill speed reducer efficiency are as follows: with steel spacer ring $\eta = 0{,}88 \ldots 0{,}70;$ with bronze ring $\eta = 0{,}90 \ldots 0{,}78$ (curves 5 and 6).

The mixer wave reducer efficiency excluding engagement losses is as follows: at $f = 0{,}08$, $\eta = 0{,}92 \ldots 0{,}67;$ at $f = 0{,}049$, $\eta = 0{,}95 \ldots 0{,}77$ (Fig. 5) (curves 1 and 2) and the same including engagement losses is as follows: at $f = 0{,}08$, $\eta = 0{,}88 \ldots 0{,}64;$ at $f = 0{,}049$, $\eta = 0{,}90 \ldots 0{,}73$ (curves 3 and 4). The following results of experimental studies of the mixer speed reducer efficiency are obtained: with steel spacer ring $\eta = 0{,}86 \ldots 0{,}66$; with bronze ring $\eta = 0{,}89 \ldots 0{,}75$ (curves 5 and 6).

Energy losses in the field of the wave generator are caused by friction torques in the zones of contact between the disks and the flexible gear that turn the disks orthogonal to the motion plane. Disks turning and conelike strain of the flexible gear stimulate the formation of some similarity of friction helical pair with the helix angle equivalent to the angle of disks turn $\gamma$.

Axial fixing of the disks and the flexible gear prevents their relative helical motion, the axial component of which is transformed into axial sliding of the disks relative to the flexible gear. This causes considerable energy losses in wave gearing.

Friction torques in the zones of contact between the disks and the flexible gear turn the disks in the direction of «screwing in» the flexible gear while stretching it in the axial direction by means of friction forces, the resultant value of which is equal to double the amount of forces $F_1$ and $F_2$.

For the ore-pulverizing mill relining drive speed reducer the axial stretching force $F_{\sum}$ acting on the flexible gear on the wave generator side, where $f = 0{,}08$, loading torque $M_{2max} = 5 \cdot 10^5\ Nm$, makes $F_{\sum L} = 12603\ N$, and for the mixer swing drive speed reducer at the same load $F_{\sum k} = 11619\ N$. In the presence of end plays, the action of axial friction forces becomes hazardous not only for the support between the wave generator and the flexible gear. They cause strong impacts, vibrations, noise as well as rapid wear.

## CONCLUSIONS

1. The studies undertaken made it possible to reveal the mechanism of axial forces and increased energy losses generation in large wave gearings. Axial forces and basic energy losses in wave gearing are formed in the zones of contact between the disks and the flexible gear, they are proportional to the angle of deviation of the disks $\gamma$ and friction factor $f$ in the contact between the disks and the flexible gear.

2. Developed has been a new method of force analysis of kinematic pairs of higher degree including a flexible link, as applied to the wave gearing. In so doing, the influence of the flexible gear strains, design parameters of the wave generator, the amount and nature of contact forces distribution as well as the amount and directions of friction forces on the generation of axial forces and energy losses in the zones of contact between the wave generator disks and the flexible gearing has been taken into account.

3. By realizing the developed analytical method of force processes analysis in kinematic pairs of higher degree, the numerical results of the values of axial forces and energy losses in the field of the wave generator as well as the energy losses in the gearing, mechanical drives wave-type gear reducers: of tilting MP – 600AC mobile mixer with the capacity of $600\,t$ molten metal and of relining MGR 5500x7500 ore-pulverizing mill with the capacity of $160\,m^3$ and ore loading weight of $220\,t$ have been obtained.

4. On the basis of the results obtained in the course of studies a possibility has appeared to minimize the axial forces and energy losses in the large wave gear reducer kinematic pairs of higher degree at the stage of their design and to enforce high scientific and technical level of intellectual and commercial products of heavy engineering industry.

5. To restrict negative influence of flexible gear «windup» intensifying interference along the tooth length, on the increase of energy losses and in order to eliminate overshoot of teeth in meshing, it is recommended to limit the width of gear girths $b$ depending on the teeth module $m$, by the following inequation $b \le 40m$.

6. To reduce the axial forces and energy losses in the field of wave generator the following means are applied: manufacture of spacer ring of bronze, minimization of the

angle of disks skewness by using more rigid bearing supports that prevent rotation of the disks relative to the predetermined positions of the process planes perpendicular to the common axis of the wave gearing as well as by using lubricant with high antifriction properties.

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# ИССЛЕДОВАНИЕ ВЗАИМОДЕЙСТВИЯ ДИСКОВОГО ГЕНЕРАТОРА ВОЛН С ГИБКИМ КОЛЕСОМ ТЯЖЕЛО НАГРУЖЕННОЙ ВОЛНОВОЙ ПЕРЕДАЧИ

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Аннотация: Силовые процессы в высших кинематических парах диск – гибкое колесо вызывают негативные явления в тяжело нагруженной волновой зубчатой передаче. Это значительные по величине осевые силы, обладающие динамической характеристикой, а также существенные энергетические потери в области генератора волн. Приведенные результаты силового анализа взаимодействия дискового генератора волн с гибким колесом, позволили дать оценку силовым и энергетическим факторам, наблюдающимся в процессе работы крупной волновой зубчатой передачи.
Ключевые слова: волновая передача, осевые силы, энергетические потери.