

## MATHEMATICAL MODELLING OF THE UNSTATIONARY FRICTION INTERACTION OF THE WORKING ELEMENTS OF THE LOCOMOTIVE DISK BRAKE

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**Summary.** The article considers the mathematical model which gives the possibility to predict the work of the frictional elements of the locomotive disk brake the functioning base of which is ensured by joint use of both materials which are situated independently on the brake pad and it exerts an integral influence on the friction characteristics. A developed mathematical model permits to predict the work of the above frictional unit due to the friction coefficient criteria.

**Key words:** friction coefficient, temperature, locomotive disk brake.

### INTRODUCTION

Speed growth on the railway requires new demands to the frictional disk brake connected with improvement of its technical and economical indexes concerning stability of brake characteristics, reliability and value decrease. In the nearest future a disk brake is to ensure high operational qualities under the conditions when its power capacity attains 100 MJ (at present 40 MJ), working temperature – 1000<sup>0</sup> C (at present 600<sup>0</sup> C).

The solution of this task is realized due to complex approach, main components of which are the improvement of the disk brake, development of effective algorithms of its work, development of the working materials with high working qualities and creation of favourable conditions for full realization of their frictional possibilities in the regime of braking. In this case the last two tends are considered of some priority and great attention is paid to them [1, 2].

At present, the development of new frictional materials does not bring some positive results in a unique manner and they (the materials) require a sharp cost increase of production. Such a position proved to be correct only with respect to the brake disk, the working period of which is to be equal to the working period of the wheel set.

In this connection the perspective tend of the research is the ensurance of stable braking characteristics of the railway rolling stock by means of creation of new integral

frictional properties of the disk brake due to simultaneous use of both frictional materials which produce some integral influence on the friction characteristics.

### CALCULATION MODEL

The solution of the given task is attained by the solution of the heat problem concerning frictional brake of the rolling stock taking into account the temperature influence as a dominating factor which determines the friction unit behaviour on the physical and mechanical properties of the materials which are applied in the calculations.

The differential equality of heat conductivity (Fourier - Kirchhoff) in the cylindrical system of coordinates without internal heat sources (calculated diagram is given in the fig. 1) serves as the model basis of heat interaction of the working elements of the frictional brake.

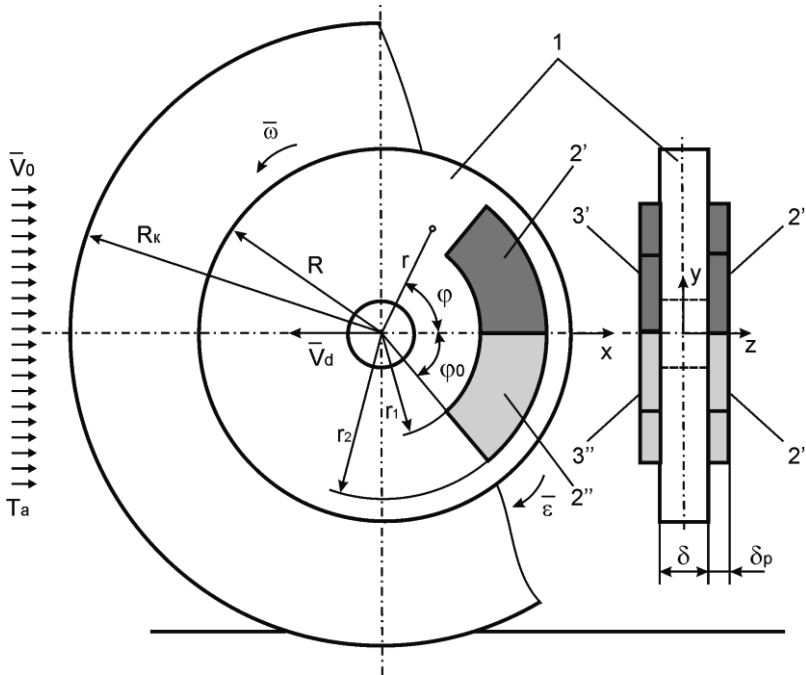


Fig. 1. Design model of the disk brake of rolling stock:  
1 - brake disk 2, 3 - brake pads, which contain two frictional material with of the different properties (marked "''" and "'''" respectively)

This is achieved by the solution of the heat problem concerning the frictional brake of the rolling stock taking into account the temperature influence as a dominating

factor which determines the friction unit behaviour on the physical and mechanical properties of the materials applied in the calculations.

The model gives the possibility to determine the temperature pattern of the frictional and interacting brake surfaces (both local and mean integral) and also heat flows which pass through marked surfaces in the process of braking. There considered the case of application of the friction brake property – a disk brake the pad of which hold two frictional materials with different properties, each of which is loaded independently from each other. However the given model can be also used for a block brake [3 – 5]:

$$\frac{\partial T}{\partial \tau} = \vartheta \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (1)$$

where:  $\vartheta = \lambda / (c_p \cdot \rho)$  – coefficient of thermal diffusivity material, here  $\lambda$  – heat conduction coefficient,  $c_p$  – isobar specific heat,  $\rho$  – density;  $\tau$  – current value of the time when braking;  $T$  – absolute temperature ( $T = f(r, \varphi, z, \tau)$ ),  $\varphi$  – angle between the radius vector  $r$  and axis  $x$ .

The diagram (Fig. 1.) adopted the following labels:  $R, R_k$  – radius of the brake disc and wheel locomotive respectively;  $\delta, \delta_p$  – thickness of brake pad and disc respectively;  $V_0$  – the speed of oncoming air flow;  $T_a$  – the average temperature on the distance from brake disc;  $V_d$  – current engine speed;  $\omega$  – angular velocity of brake disc;  $\varepsilon$  – angular acceleration (deceleration) brake disc during braking.

Point of origin located at the geometric center of the brake disk.

Equation (1) supplemented by the following boundary conditions. For the side and end surfaces of brake disc (excluding areas of its frictional contact with braking plates) used the boundary conditions of 3rd sort (without internal heat sources):

$$\pm \lambda \left( \frac{\partial T}{\partial z} \right) = \alpha_z (T - T_a), \quad z = \pm \delta / 2, \quad (2)$$

where:  $\alpha_z$  – coefficient of heat transfer between the end surfaces of the brake disk and the ambient air;

On the side (cylindrical) surface of the brake disk:

$$\lambda \left( \frac{\partial T}{\partial r} \right) = \alpha_r (T - T_a), \quad r = R, \quad (3)$$

where:  $\alpha_r$  – coefficient of heat transfer between the lateral surface of the brake disk and the ambient air;

For the surface area of sliding contact with the disc brake pads are used boundary conditions of the 4th sort of surface heat source (for the case of non-ideal thermal contact). This is because in the set the next time the basic principles that are based on the research area of thermal tribocontact, the source of heat is considered the finest continuous surface layers directly adjacent to the actual platform contact each of the friction-interacting bodies and reproduce like a single system [6 - 8].

$$\left. \begin{aligned} -\lambda_1 \left( \frac{\partial T_1}{\partial z} \right) &= \alpha_{er} q - \frac{1}{R_c} (T_1 - T_2), \\ \lambda_2 \left( \frac{\partial T_2}{\partial z} \right) &= (1 - \alpha_{er}) q + \frac{1}{R_c} (T_1 - T_2), \end{aligned} \right\} z = \pm \delta / 2, \quad (4)$$

where:  $q$  – specific heat flow generated during sliding contact with the brake disk plate;  
 $R_c$  – thermal resistance contact. During thermal contact resistance is understood value:  
 $R_c = \Delta T / q$ , where  $\Delta T$  – average temperatures drop contacting surfaces;  
 $\alpha_{er}$  – energy distribution coefficient of friction ( $0 < \alpha_{er} < 1$ ) shows how much heat energy is formed on the surface friction of the first body as a result of destruction of the adhesive bonds in the actual contact area and deformation microirregularities roughness of the interacting surfaces. On the surface of the second body is formed another portion of heat energy equal to  $(1 - \alpha_{er})$  [9, 10]:

$$\alpha_{er} = \frac{\sqrt{\lambda_1 \cdot c_{p1} \cdot \rho_1}}{\sqrt{\lambda_2 \cdot c_{p2} \cdot \rho_2} + \sqrt{\lambda_1 \cdot c_{p1} \cdot \rho_1}}.$$

Here and below the index "1" refers to the corresponding value of brake disk, the index "2" - to the brake linings. Moreover, for each of the friction materials that are in interaction with the brake disk must use the corresponding system of equations (4).

Other (not marked in Figure 1.) Surfaces are insulated.

Specific heat flow generated during braking one type of friction material [4, 11]:

$$q(\varphi, r, \tau) = \frac{1}{n_k \iint_S r(\varphi) ds} \frac{m R_k^2 \cdot \varepsilon}{n_n} r(\varphi) (\omega^* - \varepsilon \cdot \tau),$$

where:  $m$  – mass brakes;

$n_k$  – number of types of friction material used in pads;

$n_n$  – number of brake pads with a material that is involved in braking (at the design scheme (Fig. 1). using two types of friction material);

$r$  – radius vector describing the contact area of the disk pad  $S$ ;

$\omega^*$  – angular velocity of the disc that precedes braking ( $\varepsilon > 0$ );

$\tau \in [0; \tau_k]$ ,  $\tau_k = (V^* - V_k) / a^*$ , – braking time (time interval from the start of braking to achieve the ultimate engine speed).

Where  $V_k$  – the final speed of the locomotive,  $a^*$  – linear deceleration when braking ( $a^* > 0$ ),  $V^*$  – speed of the locomotive that precedes braking.

To determine the heat transfer coefficients, which are included in expressions (2) and (3) are used, respectively, the following criterial equation [6, 12]:

$$\left. \begin{aligned} Nu_z &= 0,135 \cdot [(0,5 \cdot Re_\omega^2 + Re_a^2 + Gr) \cdot Pr]^{0,33}, \\ Nu_r &= 0,037 \cdot (Re_a^{0,8} + Re_\omega^{0,4}) \cdot Pr^{0,33}, \end{aligned} \right\} \quad (5)$$

where:  $Re_\omega$ ,  $Re_a$  – Reynolds number, which are caused by rotation of the brake disk and the blowing wind blowing, respectively:

$$\left. \begin{aligned} \text{Re}_\omega &= \frac{4 \cdot \omega \cdot R^2}{\nu} = 4 \cdot \left( \frac{V_d}{R_k} \right) \cdot \frac{R^2}{\nu}, \\ \text{Re}_a &= \frac{2 \cdot V \cdot R}{\nu}, \end{aligned} \right\}$$

where:  $\nu$  – kinematic viscosity of air;

$V = V_d + V_0 = (V^* - a^* \cdot t) + V_0$ , – current speed of the airflow, which run on a disc brake ( $a^* > 0$ ).

Hrashof, Prandtl and Nusselt numbers for air:

$$\text{Gr} = 8 \cdot \beta \cdot g \cdot R^3 \cdot (T - T_a) / \nu^2;$$

$$\text{Pr} = c_a \cdot \mu / \lambda; \text{Nu} = 2 \cdot \alpha \cdot R / \lambda,$$

where  $\beta$  – coefficient of volumetric expansion of air;

$\lambda$  – thermal conductivity of air;

$c_a$  – specific isobar heat capacity of air;

$\mu$  – viscosity of air;

$\alpha$  – heat transfer coefficient of the surface of the brake disk, which is determined by the system (5).

Value of thermal resistance contact, which is part of the system (4) serves for finding the ratio of heat flow transmitted by contact with heat transfer surfaces interacting combination of brake disk - overlay. Since it is the contact thermal resistance caused by imperfect mechanical connection friction surfaces significantly affect the relationship between transferred through a heat flow and temperature difference of the interacting surfaces.

Its definition is as follows [8, 13]:

$$\left. \begin{aligned} \frac{1}{R_c} &= 1,15 \cdot 10^{-4} \frac{\sigma_1 + \sigma_2}{\sigma_1 / \lambda_1 + \sigma_2 / \lambda_2} \left( \frac{P_n^2}{\Omega} \cdot \frac{E_1 + E_2}{2 \cdot E_1 \cdot E_2} \cdot \frac{T_c}{T_m} \cdot K^2 \right)^{0,302}, \\ K &= 1 \text{ при } (\sigma_1 + \sigma_2) > 3 \cdot 10^{-5}, \\ K^2 &= \left( \frac{30 \cdot 10^{-6}}{\sigma_1 + \sigma_2} \right)^{2/3} \text{ при } 1 \cdot 10^{-5} < (\sigma_1 + \sigma_2) \leq 3 \cdot 10^{-5}, \\ K &= \frac{15 \cdot 10^{-6}}{\sigma_1 + \sigma_2} \text{ при } (\sigma_1 + \sigma_2) \leq 1 \cdot 10^{-5}, \end{aligned} \right\}$$

where:  $\sigma_{1,2}$  – average height microirregularities performances in the area of contact mating surfaces;

$\lambda_{1,2}$  – thermal conductivity of materials interacting surfaces;

$P_n$  – nominal contact pressure;

$\Omega$  – tensile strength of more pliable material;

$E_{1,2}$  – modulus of elasticity of contacting materials;;

$T_c$  – average contact temperature;

$T_m$  – melting point of a fusible material;

$K$  – factor that determines the change of geometrical characteristics of contacting surfaces.

Using the equation given above, the obtained mathematical model of temperature field of brake disc surfaces:

$$T^*(r^*, z^*, \Theta) = q \cdot \sqrt{\frac{\delta}{2R}} \cdot \frac{\sqrt{A+B}}{\pi \sqrt{A+B} \cdot (\pi - 2\varphi_0)} \cdot \left[ \frac{2\varphi_0}{\sqrt{A+B}} \cdot \exp\left(-\sqrt{\frac{2A \cdot R}{\delta}} \times \right. \right. \\ \left. \left. \times (1-r^*) \right) + \frac{1}{P} \cdot \sqrt{\frac{\delta}{R \cdot r^*}} \cdot \sum_{K=-\infty}^{\infty} \frac{\sin(2\alpha_{er} \cdot \varphi_0)}{\alpha_{er}} \cdot \exp\left(2P \cdot R \cdot \frac{\sqrt{\alpha_{er}}}{\delta} \cdot (r^* - 1)\right) \times \right. \\ \left. \times \cos\left(2\alpha_{er} \cdot \Theta + \frac{\pi}{4} - P \cdot \frac{2R \cdot \sqrt{\alpha_{er}}}{\delta} \cdot (r^* - 1)\right) \right], \quad (6)$$

where:  $T^* = (T - T_a) / T_a$ , – dimensionless temperature;

$r^* = r / R$ ,  $z^* = 2z / \delta$ , – dimensionless coordinates;

$\tau^* = \omega \cdot \tau$ , – dimensionless time;  $\Theta = \varphi - \tau^*$ ;

$P = \omega \cdot c_p \cdot \rho \cdot \delta^2 / (4\lambda)$ , where  $c_p$  i  $\rho$  – isobar specific heat and density of materials of brake disk;  $A = \alpha_z \cdot \delta / (2\lambda)$ ;  $B = \alpha_r \cdot R / \lambda$ ;

To calculate the friction coefficient in case of simultaneous use of two different friction materials (according to that shown in Figure 1.) Considering temperature field work surfaces interacting elements of disk brakes, received the following dependence [14 - 18]:

$$f = \frac{1}{3N \cdot R_m} (\varphi_2^2 - \varphi_1^2) \left[ (r_2^3 - r_1^3) \left( f_m \cdot P_n + \frac{0,84v \cdot (v-1) \cdot h^{0,5} \cdot k \cdot N^{\frac{2 \cdot v+1}{2 \cdot v}} \cdot 2^{\frac{2 \cdot v+1}{2 \cdot v}}}{\pi \cdot R_h^{0,5} \cdot HB^{0,5/v} \cdot b^{0,5/v}} \right) \right], \quad (7)$$

where:  $f_m$  – molecular component of friction coefficient, which depends on the tangential stresses at the interface of interacting surfaces of the brake disk pad;

$N$  – normal strain clamping pads to brake disk;

$HB$  – the smallest of the two values of hardness of the material of the contacting surfaces;

$R_h$  – radius of curvature microirregularities a solid surface (average);

$R_m$  – average radius of the friction brake pad;

$k$  – coefficient that depends on the geometrical and mechanical properties of surfaces that are in friction interaction;

$h$  – convergence of the contacting surfaces of the brake disc and pads;

$b, v$  – parameters of the curve bearing surface;

To obtain total coefficient of friction disc brakes should be determined by the formula (7) the coefficients for each material separately, and summarize the values obtained. Physical and mechanical properties of materials that come in the above equation is considered as a function of temperature.

## INPUT DATA AND RESULTS OF CALCULATION

Sample calculation depends disc brake friction coefficient on the relative velocity of sliding friction of its elements, filled with using the above mathematical models, shown in fig. 2.

Considered by frictional contact of metal surfaces in the following combinations: steel brake disc - steel pad, steel brake disc - cast iron pad, steel brake disc - a metal and cast iron pads.

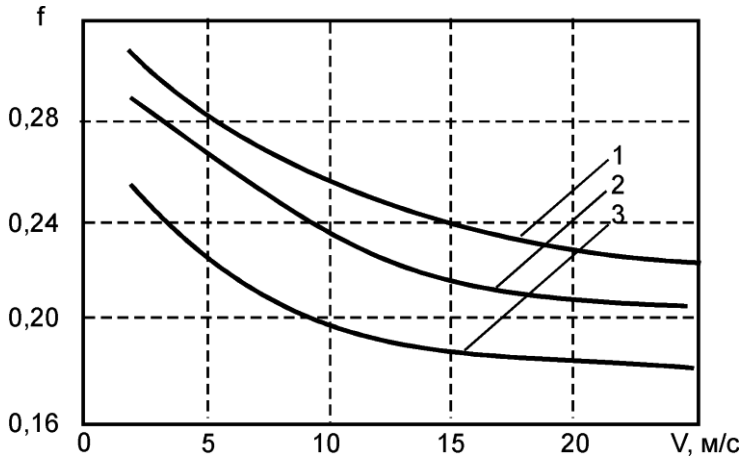


Fig. 2. Dependence of coefficient of friction disc brakes on the relative velocity of sliding friction of its elements obtained for these combinations of contacting materials:  
 1 - steel brake disk - cast iron pad;  
 2 - steel brake disk - a metal and cast iron pad;  
 3 - steel brake disk - steel pad

Calculations are based typical performance curve of the core surface (take from the literature) as well as physical, mechanical and thermal properties of materials considered in the functional dependence on temperature [19, 20]. Thus the average values:  $v = 2$ ,  $b = 4$ ,  $h = 12,5 \mu\text{m}$  (maximum value),  $k = 0,8$ . Geometric dimensions of brake linings and disc match these used traction rolling stock of railways. Maximum value  $N = 15 \text{ kN}$ . The molecular component of friction coefficient was considered constant.

Fig. 3 shows the results of the calculation of the integral value of the coefficient of friction disc brakes depending on surface temperature in the contact zone for combinations of "iron-steel," "carbon-steel" and "iron and carbon-steel."

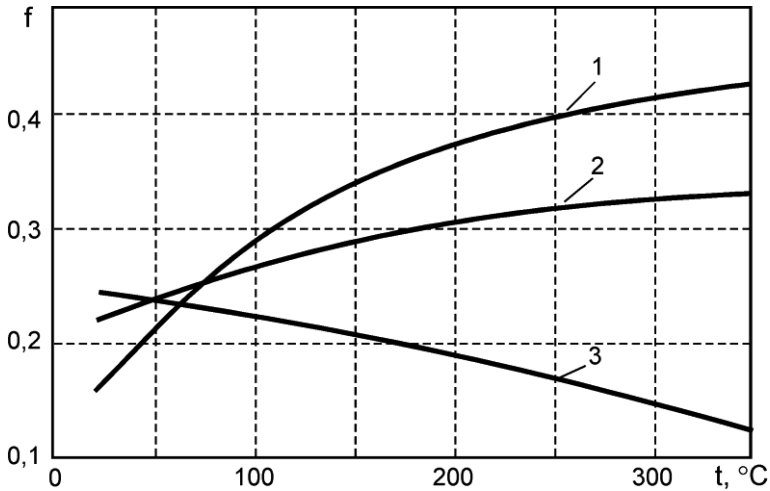


Fig. 3. Estimated value of coefficient of friction disc brake as a function surface temperature of the contact elements of his work, obtained by using the mathematical model for the following combinations of friction: 1 - steel disc and pads of carbon; 2 - steel disc and pad with two working materials: iron and carbon; 3 - steel disc and block with cast iron

## CONCLUSIONS

Mathematical models, considered above, give the possibility to determine the friction coefficient value of the frictional brake (especially a disk brake) of the rolling stock which was attained as an integral value from the joint action of several frictional materials with different properties and loading conditions which are placed simultaneously on the disk brake pad (the given models differ from the existing ones in this aspect). This is carried out due to the temperature influence, which is generated while braking, on the major physical and mechanical properties of the materials which are in the position of the frictional interaction. In this case the temperature value of the brake system elements, motion speed of the locomotive and contrary air flow at the moment before braking are used.

There obtained mathematical models which account all the main design ( $R, \delta, r_1, r_2, \varphi_0$ ), technological ( $v, b, k, R_h, \sigma_{1,2}$ ), material ( $HB, E_{1,2}, \Omega, T_m$ ) and operational ( $f_m, h, P_n$ ) parameters of the rolling stock frictional brakes (especially disk brakes).

The analysis of the obtained results allow to make the following conclusion: the application of simultaneous combination of two (as minimum) frictional materials having different frictional properties, in the brake pad design gives the possibility to obtain new integral properties of the disk brake as a whole. The combination of the materials cast iron – coal has higher frictional properties than in the case of two pads of cast-iron. This combination of the materials ensures higher friction coefficient values in the beginning of the braking moment in the difference from the friction couple coal –



coal (though further the given combination of the materials attains higher friction coefficient values).

As a result it is possible to state, that the way of effectiveness increase of the frictional brakes (especially disk brakes) by means of application of the materials which are in operation becomes possible due to new integral characteristics which are stipulated by their joint simultaneous application in the given brake pads.

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**МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕСТАЦИОНАРНОГО  
ФРИКЦИОННОГО ВЗАИМОДЕЙСТВИЯ РАБОЧИХ ЭЛЕМЕНТОВ  
ДИСКОВОГО ТОРМОЗА ЛОКОМОТИВА**

**Юрий Ю. Осенин, Игорь И. Соснов**

**Аннотация.** В статье рассмотрена математическая модель, которая позволяет прогнозировать работу фрикционных элементов дискового тормоза локомотива, основу функционирования которого обеспечивает совместное использование двух материалов, независимо расположенных на тормозной накладке, которые оказывают интегральное влияние на характеристики трения. Разработанная математическая модель позволяет прогнозировать работу указанного выше фрикционного узла по критерию коэффициента трения.

**Ключевые слова:** коэффициент трения, температура, дисковый тормоз локомотива.