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THERMOELASTIC CONTACT PROBLEM FOR THE ELASTIC HALF-SPACE

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Summary. In this work the connected thermoelastic contact problem for an elastic half-space to which some area heat radiant is brought is considered. Applying a method of integral transformations of Fourier to system of the differential equations of thermoelasticity and a heat conduction equation, the problem is reduced to system of two two-dimensional integral equations for definition of thermoelastic normal voltages and temperature distribution.
Keywords: elastic half-space, thermoelasticity, integral transformations, thermoelastic contact problems

INTRODUCTION

Thermoelasticity is a new area of mechanics developing during the last years. It investigates interaction of a field of strains and a field of temperature and, thus, binds on the basis of thermodynamics of irreversible processes two separate earlier independent disciplines - the theory of elasticity and the thermal conduction theory. The equations and thermoelasticity problems are the further improvement of relations and problems of the classical theory of elasticity.

The basic publications on the problem are given in works [1-22] which contain the review of the basic scientific outcomes devoted to a solution of static, dynamic and thermoelastic problems for elastic and viscoelastic skew fields.

OBJECT AND PROBLEMS

The purpose of the offered work is research of a thermoelastic contact problem for an elastic half-space (fig. 1), definition of normal contact voltages and a field of temperature in the field of contact. As far as it is known to authors the similar problem was not considered earlier.

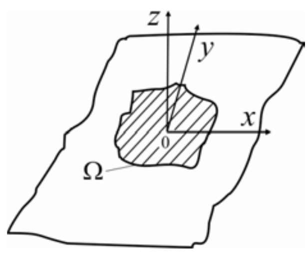

Fig.1. The rated scheme

Problem statement. The connected problem of thermoelasticity for a half-space to which some area **Ω** heat radiant is brought is considered.

Mathematically the problem is formulated in the form of three differential equations of thermoelasticity and a heat conduction equation [1,3,5]

$$(\lambda+\mu)\frac{\partial\theta}{\partial x}+\mu\Delta u=\gamma\frac{\partial T}{\partial x},$$
$$(\lambda+\mu)\frac{\partial\theta}{\partial y}+\mu\Delta v=\gamma\frac{\partial T}{\partial y},$$
$$(\lambda+\mu)\frac{\partial\theta}{\partial z}+\mu\Delta w=\gamma\frac{\partial T}{\partial z}, \qquad (1)$$
$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial T}{\partial z^2}=0.$$

In the absence of mass forces with boundary conditions $\sigma_{zz}(x,y,o)=q(x,y)$,
$w(x,y,o)=f(x,y),\ (x,y)\in\Omega$,
$T(x,y,o)=\varphi(x,y)$,
$\sigma_{xz}(x,y,o)=\sigma_{yz}(x,y,o)=\sigma_{zz}(x,y,o)=0$,

$$\psi = \frac{\partial T}{\partial Z} - \chi_o T = 0, \quad (x,y) \notin \Omega. \qquad (2)$$

Here $\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$,

$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ - a relative modification of volume and a three-dimensional differential operator of Laplace, $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$ - projections of a vector of elastic transition to axes x,y,z, accordingly, $T(x,y,z)$ - temperature, $\gamma = (3\lambda + 2\mu)\cdot\alpha_t = 3K\alpha_t$,

$K = \lambda + \frac{2}{3}\mu$, $\alpha_t = \frac{\gamma}{3K}$, μ - the module of shift of a material of an elastic half-space, $(\lambda+\mu) = \mu(1-2v)^{-1}$, v - factor of Poisson.

Applying to system of the differential equations (1) two-dimensional transformation of Fourier [13], we will receive system of two two-dimensional integral equations of the first sort concerning normal thermoelastic voltages $\sigma_{zz} = q(x,y)$ and distributions of temperature $T(x,y)$

$$\iint_\Omega k_{12}(\xi - x, \eta - y) q(\xi,\eta) d\xi d\eta + \iint_\Omega k_{22}(\xi - x, \eta - y) S(\xi,\eta) d\xi d\eta == 4\pi^2 \mu f(x,y), (x,y) \in \Omega,$$

$$\iint_\Omega k_{21}(\xi - x, \eta - y) S(\xi,\eta) d\xi d\eta = 4\pi^2 \varphi(x,y),$$

$$(x,y) \in \Omega,$$

$$k_{12}(x,y) = \int_{-\infty}^{\infty}\!\!\int K_{12}(\alpha,\beta) e^{i(\alpha x + \beta y)} d\alpha d\beta, \qquad (3)$$

$$k_{22}(x,y) = \int_{-\infty}^{\infty}\!\!\int K_{22}(\alpha,\beta) e^{i(\alpha x + \beta y)} d\alpha d\beta,$$

$$k_{21}(x,y) = \int_{-\infty}^{\infty}\!\!\int K_{21}(\alpha,\beta) e^{i(\alpha x + \beta y)} d\alpha d\beta,$$

$$K_{12}(\alpha,\beta) = (1-2v)(\lambda+\chi)\{2\lambda_1[\chi(1-2v) - \lambda_1]\}^{-1},$$

$$K_{22}(\alpha,\beta) = (1-2v)[2\lambda_1(1-v) + (\lambda_1+\chi)(2v\gamma - \beta_o)] \times \{2\lambda_1(\lambda_1 - \chi_o)[\chi(1-v) - \lambda_1]\}^{-1},$$

$$K_{21}(\alpha,\beta) = (\lambda_1 - \chi_o)^{-1}, \; \lambda_1(\alpha^2+\beta^2)^{1/2},$$

$$\chi = (3-4v).$$

The strict conclusion of system of two-dimensional integral equations of the connected thermoelastic contact problem which kernels depend on a difference of arguments is shown.

CONCLUSIONS

From the received system of two two-dimensional integral equations to which the task has been reduced normal contact voltages and a temperature field on half-space boundary have been defined.

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ТЕРМОУПРУГАЯ КОНТАКТНАЯ ЗАДАЧА ДЛЯ УПРУГОГО ПОЛУПРОСТРАНСТВА

Валерий Старченко, Вячеслав Буряк

Аннотация. В работе рассматривается связанная термоупругая контактная задача для упругого полупространства, к некоторой области которого подводится источник тепла. Применяя метод интегральных преобразований Фурье к системе дифференциальных уравнений термоупругости и уравнению теплопроводности, задача сводится к системе двух двумерных интегральных уравнений для определения термоупругих нормальных напряжений и распределения температуры.
Ключевые слова: упругое полупространство, термоупругость, интегральные преобразования, термоупругие контактные задачи.