Limited contact dynamic solution tasks of horizontal vibrations of the stamp on the elastic half-space

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S u m m a r y . The results of theoretical research of dynamic contact problems for isotropic elastic semi-space swing joint and totally hard punch under the action of horizontal forces, changing in time for the harmonic law have been given. A limited solution tangent contact stresses in the characteristic points on the ends of the section of contact interaction, has been received.
K e y w o r d s : elastic half-space, integral transforms, tangents, contact voltage oscillation frequency.

INTRODUCTION

Contact Mechanics of deformable solid body interaction is currently the most active and growing field of continuum mechanics. It is constantly the focus of researchers. This is because all the mechanisms and structures are composed of interacting components and the distribution of effort between the contact details is not known in advance and can only be found as a result of the contact problems. Static contact problems have been pretty well researched at the same time the problem is solving dynamic contact tasks which have a scientific and practical value.

Major publications on the subject are in the works [1-18] developed methods for solving problems of statics are insufficient in solving integral equations of dynamic contact problems, which are oscillating engine, resulting in the development of new problem solving techniques of dynamic contact problems.

OBJECT AND PROBLEMS

The purpose of work is theoretical study of dynamic contact problems of horizontal vibrations of the stamp, tightly bound with elastic poluprostranstvom (flat task) under the influence of the horizontal loads varies in harmonic $T = T_o \exp(i\omega t)$ law taking into account the terms of the limited tangential contact stresses, in the characteristic points on the ends of the plot surface area (fig. 1).

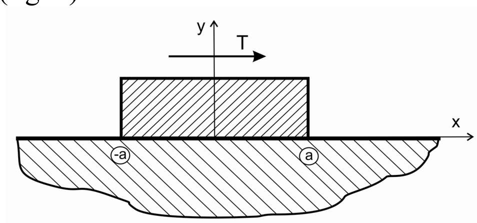

Fig. 1. Design scheme

Problem statement. Using the principle of maximum absorption and integral Fourier transform leads to the following integral equation in dimensionless variables relative to the amplitude value of the unknown strain of tangent $\tau_\varepsilon(x)$ [12]:

$$\int_{-1}^{1}\tau_\varepsilon(\xi)k_\varepsilon\left[\kappa(\xi-x)\right]d\xi = 2\pi\beta_\varepsilon(x),\ |x|\le 1,\ \Delta=\mu a^{-1}, \quad (1)$$

$$k_\varepsilon(t)=\int_{-\infty}^{\infty}K_\varepsilon(u)\exp(-i|t|u)du,\quad \kappa=\omega a(\rho/\mu)^{1/2}, \quad (2)$$

$$K_\varepsilon(u)=\frac{\sqrt{u^2-(1-i\varepsilon)}}{4u^2\sqrt{u^2-(1-i\varepsilon)b^2}\cdot\sqrt{u^2-(1-i\varepsilon)}-\left[2u^2-(1-i\varepsilon)\right]^2}, \quad (3)$$

where: $\tau_\varepsilon(x,t)=\tau_\varepsilon(x)\exp(i\omega t)$ – unknown distribution function tangential contact stresses; $\beta_\varepsilon \exp(i\omega t)$ – the horizontal movement of a stamp called the applied force; ρ,μ – material density and modulus elastic half-space; ϖ – the oscillation frequency; t – time; ν – Poisson's ratio, when $\nu=0.3$, $b=\left[(1-2\nu)/2(1-\nu)\right]^{1/2}=0.5345$, $\tau_\varepsilon(x)=\tau_{1\varepsilon}(x)+i\tau_{2\varepsilon}(x)$, $\beta_\varepsilon(x)=\beta_{1\varepsilon}(x)+i\beta_{2\varepsilon}(x)$.

To find a uniform limit of functions $\beta_\varepsilon(x)$ when $\varepsilon\to 0$ the path of integration in the view (2) under [12] deformiruem so that when the $\varepsilon\to 0$ on the real axis of Poles and branching point functions $K_\varepsilon(u)$ do not cross it. From equations, (1) and (3) will take the following form:

$$\int_{-1}^{1}\tau(\xi)k\left[\kappa(\xi-x)\right]d\xi=2\pi\beta(x),(|x|\le 1,\Delta=\mu a^{-1}), \quad (4)$$

$$k(t)=\int_{\Gamma}K(u)\exp(-i|t|u)\cdot du, \quad (5)$$

$$K(u)=\frac{\sqrt{u^2-1}}{4u^2\sqrt{u^2-b^2}\sqrt{u^2-1}-(2u^2-1)^2}. \quad (6)$$

The path, in the ratio (5) is the same as the real axis deviating from it, bypassing all the positive and negative features from top to bottom.

Function $K(u)$-even with the real axis two poles and two pairs of points, branching symmetrically arranged about the origin. Kernel of the integral equation is of the form [12]:

$$k(t)=A+Bt^2+Ct^4+(D+Et^2+Ft^4)-$$
$$-(L+Mt^2+Nt^4)\cdot i, \quad (7)$$

A=0,2819, C=0,01416, E=-0,1325, L=1,09056,
B=-0,1490, Д=0,6999, F=0,009403, M=-0,2081, (8)
N=0,01477.

Then the solution of integral equation (4) takes the form:

$$\tau_1(x)=\frac{1}{\pi\sqrt{1-x^2}}\cdot\left[T_1-\int_{-1}^{1}\frac{\psi_1'(\tau)\sqrt{1-\tau^2}\,d\tau}{\tau-x}\right], \quad (9)$$

$$\tau_2(x)=\frac{1}{\pi\sqrt{1-x^2}}\cdot\left[T_2-\int_{-1}^{1}\frac{\psi_2'(\tau)\sqrt{1-\tau^2}\,d\tau}{\tau-x}\right]. \quad (10)$$

According to the decision by equations (9) and (10) voltage $\tau(x)=\tau_1(x)+i\tau_2(x)$ at the ends of the section of contact interaction (with the x= ± 1) is unrestricted. The decision to keep physical meaning in the vicinity of characteristic points of contact interaction, stress condition is limited at the points indicated (x = ± 1) [11]:

$$\int_{-1}^{1}\frac{\psi_1'(\tau)\,d\tau}{\sqrt{1-\tau^2}}=0, \qquad T_1=-\frac{1}{\pi}\cdot\int_{-1}^{1}\frac{\psi_1'(\tau)\tau\,d\tau}{\sqrt{1-\tau^2}}, \quad (11)$$

$$\int_{-1}^{1}\frac{\psi_2'(\tau)\,d\tau}{\sqrt{1-\tau^2}}=0, \qquad T_2=-\frac{1}{\pi}\cdot\int_{-1}^{1}\frac{\psi_2'(\tau)\tau\,d\tau}{\sqrt{1-\tau^2}}. \quad (12)$$

Then limited solution for determining tangential contact stresses takes the form:

$$\tau_1(x)=-\frac{1}{\pi}\sqrt{1-x^2}\cdot\int_{-1}^{1}\frac{\psi_1'(\tau)\,d\tau}{\sqrt{1-\tau^2}\,(\tau-x)}, \quad (13)$$

$$\tau_2(x)=-\frac{1}{\pi}\sqrt{1-x^2}\cdot\int_{-1}^{1}\frac{\psi_2'(\tau)\,d\tau}{\sqrt{1-\tau^2}\,(\tau-x)}, \quad (14)$$

$$\psi_1'(\tau)=-\frac{1}{\pi D}\int_{-1}^{1}\Big\langle\tau_1(\xi)\Big\{\kappa^2\Big[2(\xi-\tau)\times$$
$$\times\left(B+\frac{1}{2}E+E\ln\kappa+E\ln|\xi-\tau|\right)\Big]+$$
$$+\kappa^4\left[4(\xi-\tau)^3\left(C+\frac{1}{4}F+F\ln\kappa+F\ln|\xi-\tau|\right)\right]\Big\}+$$
$$+\tau_2(\xi)[2M(\xi-\tau)\kappa^2+4N(\xi-\tau)^3\kappa^4]\Big\rangle d\xi, \quad (15)$$

$$\psi_2'(\tau)=-\frac{1}{\pi D}\cdot\int_{-1}^{1}\Big\langle\tau_2(\xi)\Big\{\kappa^2\Big[(\xi-\tau)\times$$
$$\times\left(B+\frac{1}{2}E+E\ln\kappa+E\ln|\xi-\tau|\right)\Big]+$$
$$+\kappa^4[4(\xi-\tau)^3\left(C+\frac{1}{4}F+F\ln\kappa+F\ln|\xi-\tau|\right)]\Big\}-$$
$$-\tau_1(\xi)\left[2M(\xi-\tau)\kappa^2+4N(\xi-\tau)^3\kappa^4\right]\Big\rangle d\xi. \quad (16)$$

Substituting (15) and (16) respectively (13) and (14) and then searching for solutions to these equations in the form:

$$\tau_j(x)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\tau_{jmn}(x)\kappa^{2m}\ln^n\kappa, \quad (17)$$

We obtain an infinite system of ratios to determine $\tau_{jmn}(x)$ in the form of:

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\tau_{jmn}(x)\kappa^{2m}\ln^n\kappa=\frac{1}{\pi}\sqrt{1-x^2}+\frac{1}{\pi D}\times$$
$$\times\int_{-1}^{1}\frac{d\tau}{\sqrt{1-\tau^2}(\tau-x)}\cdot\int_{-1}^{1}\Big\langle\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\tau_{jmn}(\xi)\kappa^{2m}\ln^n\kappa\Big\{\kappa^2\Big[2\times$$
$$\times(\xi-\tau)\cdot(B+\frac{1}{2}E+E\ln\kappa+E\ln|\xi-\tau|)\Big]+\kappa^4\Big[4(\xi-\tau)^3\times$$
$$\times(C+\frac{1}{4}F+F\ln 2+F\ln|\xi-\tau|)\Big]\Big\}\pm\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\tau_{smn}(\xi)\kappa^{2m}\times$$
$$\times\ln^n\kappa\cdot[2M(\xi-\tau)\kappa^2+4N(\xi-\tau)^3\kappa^4]\Big\rangle d\xi\Big\rangle, \quad (18)$$
$$(j=1,2;\ s=1,2).$$

Index $j=1$ fits indexed $s=2$ and a plus sign, and index $j=2$ fits indexed $s=1$ and minus sign. Defining $\tau_{jmn}(x)$ from equations (18) and substituting into equation (17) we get up to $\kappa^4 \ln^2 \kappa$ the following asymptotic formulas:

$$\tau_1(x)=2\Delta D^{-2}\cdot\beta_1\sqrt{1-x^2}[A_1+B_1x^2+C_1x^4+O(\kappa^6\ln^3\kappa)]+$$
$$+2\Delta D^{-2}\cdot\beta_2\sqrt{1-x^2}[A_2+B_2x^2+O(\kappa^6\ln^3\kappa)],\quad(19)$$
$$\tau_2(x)=2\Delta D^{-2}\cdot\beta_2\sqrt{1-x^2}[A_1+B_1x^2+C_1x^4+O(\kappa^6\ln^3$$
$$\kappa)]-2\Delta D^{-2}\cdot\beta_1\sqrt{1-x^2}[A_2+B_2x^2+O(\kappa^6\ln^3\kappa)].\ (20)$$

Here $\tau(x)=\tau_1(x)+i\tau_2(x)$-tangential contact stresses amplitude value, $\beta(x)=\beta_1+i\beta_2$, $\beta=const$-the amount of amplitude values move stamp κ-relative frequency of oscillation of the stamp, ρ and μ-density and modulus elastic half-space.

Options A_1, A_2, B_1, B_2, C_1-are determined from expressions of

$$A_1=D-(B+\frac{7}{6}E-E\ln 2/\kappa)\kappa^2+\left\{\frac{1}{D}\left[B+\frac{29}{12}E-\right.\right.$$
$$\left.-2E\ln 2/\kappa+E^2\times\left(\ln^2 2/\kappa-\frac{29}{12}\ln 2/\kappa+\frac{179}{120}\right)-M^2\right]-$$
$$\left.-\frac{5}{2}\left(C+\frac{349}{300}F-F\ln 2/\kappa\right)\right\}\kappa^4,$$
$$B_1=\frac{1}{3}E\kappa^2+\left[-2(C+\frac{49}{30}F-F\ln 2/\kappa)+\frac{1}{3}\frac{E}{D}\times\right.$$
$$\left.\times\left(B+\frac{21}{20}E-E\ln 2/\kappa\right)\right]\kappa^4,$$
$$C_1=-\frac{1}{5}\left(F-\frac{1}{6}\frac{E^2}{D}\right)\kappa^4,\qquad B_2=-2\left(N-\frac{1}{6}\frac{EM}{D}\right)\kappa^4,$$
$$A_2=-M\kappa^2+\left[\frac{2M}{D}\left(B+\frac{29}{24}E-E\ln 2/\kappa\right)-\frac{5}{2}N\right]\kappa^4.$$

CONCLUSIONS

Solution of integral equation of the dynamic contact interaction of tasks for a hard punch with elastic isotropic poluprostranstvom when imposing conditions limited stresses in the characteristic points inx= ± 1 to receive a valid evaluation values of amplitude values of the tangential contact stresses and their distribution along the length of the surface area of the asymptotic formulas can be used not only for determining tangential contact stresses depending on lateral moves hard punch when interacting with elastic poluprostranstvom, but also on dynamic contact problems of other units and mechanisms of transport and General machinery.

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ОГРАНИЧЕННОЕ РЕШЕНИЕ ДИНАМИЧЕСКОЙ КОНТАКТНОЙ ЗАДАЧИ ГОРИЗОНТАЛЬНИХ КОЛЕБАНИЙ ШТАМПА НА УПРУГОМ ПОЛУПРОСТРАНСТВЕ

Валерий Старченко, В'ячеслав Буряк

А н н о т а ц и я . В работе приведены результаты теоретического исследования динамической контактной задачи о совместном колебании упругого изотропного полупространства и абсолютно жесткого штампа под действием горизонтальной силы, изменяющейся во времени по гармоническому закону. Получено ограниченное решение касательных контактных напряжений в характерных точках на концах участка контактного взаимодействия.

К л ю ч е в ы е с л о в а : упругое полупространство, интегральные преобразования, касательные контактные напряжения, частота колебаний.