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Mathematic modelling of onion bulb shape (*Allium cepa* L.)

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Abstract: *Mathematic modelling of onion bulb shape (*Allium cepa L.*).* The author of the study proposes a method of mathematic modelling of the bulb shape of onion of the Dutch variety Alonso (*Allium cepa* L.). Selected for modelling was an onion bulb of dimensions close to the average dimensions for this variety, that is: the length of 65.8 mm, width of 56 mm and thickness of 55.4 mm. The mathematical model was developed to describe the latitudinal contours using coordinates *x, y, z* for 56 points distributed along each of these. The angle between planes intersecting the latitudinal contours, for which the intersection edge is within the center line of the onion bulb was 36°. For the onion bulb, a mathematical description of longitudinal contours was also developed. The saved coordinates of the onion bulb model surface grid nodes may serve as a basis for designing of working units of equipment used for onion processing.

Key words: onion bulb, shape, 3D model, longitudinal contours, latitudinal contours

INTRODUCTION

Onion (*Allium cepa* L.) is a popular vegetable used in everyday human diet as an addition to raw, cooked, fried and steamed vegetables. It is cultivated in more than 175 countries, in which the harvest reaches approx. 742.51 million t. According to FAO [2012], the first place among onion producers in the world is occupied by China, and the second – by India [Baloch et al. 2014]. Further positions are occupied by the United States and Russia. Onion has healing properties, it is used as a raw material for production of some pharmaceutical preparations. According to Świetlikowska et al. [2008], onion contains essential oils, sulfuric oils, flavonols and flavonoid glycosides, as well as many other compounds. The onion bulb shape determines to a great extent its quality and commercial value [Akademicki and Nawrocka 2005, Rana and Sinija 2014].

Methods used by production and trade companies for stock management [Buliński et al. 2012, 2013] and production automation [Shvorov et al. 2012], as well as application of product classification and diagnostic methods due to significantly different features [Janaszek and Trajer 2010] allow for systematization of data, defining of work parameters and design decisions, which influence the product quality [Mieszkalski 2014].

Świetlikowska et al. [2008] and Ghaffari et al. [2013], refer to the onion shape as: elliptic, obovate, wide ellipsoidal, spherical, wide oval, wide oval-obovate, rhomboid, spherical flattened, flat. The onion bulb shape is taken into account when designing onion processing installations. Bohdziewicz and Czachor [2010] have noted that onion bulbs of larger dimensions assume the shape of an elongated or flat ellipsoid. Ghabel et al. [2010] have designed a mathematical model for the purpose of prediction of onion mass on the basis of its shape and size.

The shape and geometric properties are among the basic parameters, taken into account when designing of sorters, conveyors, packaging, as well as containers for single fruit and vegetables [Mieszkalski 2011, Katalog Betterware 2012]. Onion bulb shape is taken into account in design of sorters equipped with sensors using near infrared energy. Onions can be sorted according to color, damages and the presence of foreign bodies [katalog Odenberg 2012].

A problem to be solved is indication of methods describing onion bulb shapes, which meet the requirements of design of the equipment, listed above.

The aim of the study is to develop a model of shape of an average size onion bulb of Alonso variety on the basis of measurements of coordinates of the points within its contours, specified after rotating it along the longitudinal center line by angle of 0°, 36°, 72°, 108°, 144°.

MATERIAL AND METHODS

The research material consisted of onion bulbs of Alonso variety. For modelling purposes, 108 onions of dimensions close to the average, varying in shape, were selected, and one onion bulb was picked at random. Using a slide caliper, its length, width and thickness was measured with accuracy of 0.5 mm. The dimensions of the selected onion bulb were as follows: length 65.8 mm, width 56 mm, thickness 55.4 mm. The onion bulb was placed on the test bench (Fig. 1) along the vertical center line between two sharp-tipped grips, allowing for its rotation by the angle of 0°, 36°, 72°, 108°, 144°. The onion, placed in the position corresponding with its rotation by the angles listed above, was photographed using a Panasonic LUMIX DMC-TZ3 camera placed on the test bench. The distance between the lens and the photographed object was 500 mm. Photos of resolution of 2,560 × 1,712 pixels were saved in JPEG format.

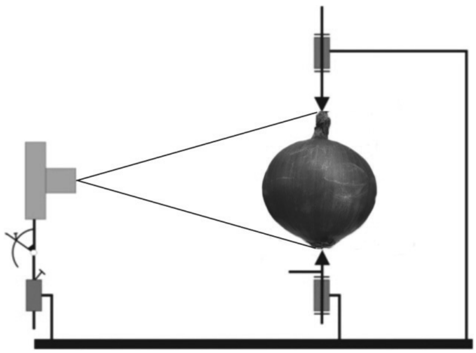

FIGURE. 1 Onion bulb photographing test bench

The scaled photographs of the onion bulb were placed in the *XYZ* coordinate system. The origin of the coordinate system is found in the point of intersection of the center line of the onion bulb with the stem contour. On each scaled photograph, along the longitudinal contours, 56 points were distributed, measuring their coordinates $xk\alpha$, $zk\alpha$. For this purpose, in a graphic system, such as CorelDRAW or Solid Edge, it is necessary to load the prepared template of horizontal lines with a 5-mm scale and the onion image. After scaling of the horizontal line template, the scaled onion image is inserted so that the stem is tangential to *AB*, while the natural center line of the onion bulb should overlap with *CD* (Fig. 2).

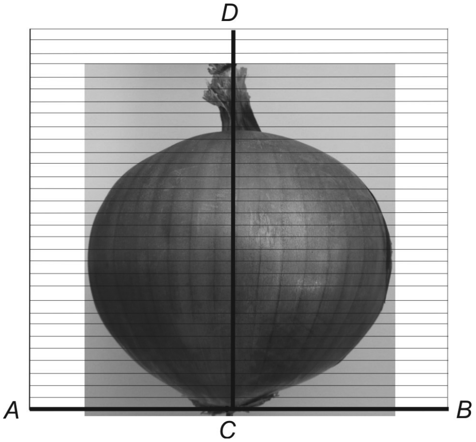

FIGURE 2. Onion bulb selected for modelling with the center line indicated, placed against the horizontal line template

Coordinates of points at the intersection between horizontal lines and the contours are saved. Afterwards, vectors of *Xkα* and *Zkα* coordinates of individual meridians are developed for rotation angles of the onion bulb by 0°, 36°, 72°, 108°, 144°. In order to develop a 3D model of the onion bulb, vectors of coordinates corresponding with a given angle of positioning of the longitudinal contour were determined in the *XYZ* coordinate system according to the following equations:

$$X\alpha = Xk\alpha \cdot \cos\left(\alpha \frac{\pi}{180}\right) \tag{1}$$

$$Y\alpha = Xk\alpha \cdot \sin\left(\alpha \frac{\pi}{180}\right) \tag{2}$$

$$Z\alpha = Zk\alpha \tag{3}$$

where: α = (0°, 36°, 72°, 108°, 144°).

On the basis of results obtained from equations (1), (2) and (3), matrices *Xpo*, *Ypo*, *Zpo* were established, which are used for development of 3D charts of the onion longitudinal contours.

$$Xpo = \begin{bmatrix} X0_{1.1} & X36_{1.2} & X72_{1.3} & X108_{1.4} & X144_{1.5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ X0_{56.1} & X36_{56.2} & X72_{56,3} & X108_{56.4} & X144_{56.5} \end{bmatrix} \tag{4}$$

$$Ypo = \begin{bmatrix} Y0_{1.1} & Y36_{1.2} & Y72_{1.3} & Y108_{1.4} & Y144_{1.5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y0_{56.1} & Y36_{56.2} & Y72_{56.3} & Y108_{56.4} & Y144_{56.5} \end{bmatrix} \tag{5}$$

$$Zpo = \begin{bmatrix} Z0_{1.1} & Z36_{1.2} & Z72_{1.3} & Z108_{1.4} & Z144_{1.5} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z0_{56.1} & Z36_{56.2} & Z72_{56.3} & Z108_{56.4} & Z144_{56.5} \end{bmatrix} \tag{6}$$

In designations of the *Xpo*, *Ypo*, *Zpo* matrix components, the first index corresponds with the number point on the contour, and the second – with the number of angle α of positioning of the longitudinal contour. Matrix of *Xpo*, *Ypo*, *Zpo* is used to develop matrix **A**, used to develop charts of the onion bulb parallel arcs.

$$\mathbf{A} = \left[\begin{matrix} \mathrm{X0}_{n,1} & \mathrm{X36}_{n,2} & \mathrm{X72}_{n,3} & \mathrm{X108}_{n,4} & \mathrm{X144}_{n,5} & \mathrm{X0}_{56-n,6} \\ \mathrm{Y0}_{n,1} & \mathrm{Y36}_{n,2} & \mathrm{Y72}_{n,3} & \mathrm{Y108}_{n,4} & \mathrm{Y144}_{n,5} & \mathrm{Y0}_{56-n,6} \\ \mathrm{Z0}_{n,1} & \mathrm{Z36}_{n,2} & \mathrm{Z72}_{n,3} & \mathrm{Z108}_{n,4} & \mathrm{Z144}_{n,5} & \mathrm{Z0}_{56-n,6} \end{matrix}\right.$$
$$\left.\begin{matrix} \mathrm{X36}_{56-n,7} & \mathrm{X72}_{56-n,8} & \mathrm{X108}_{56-n,9} & \mathrm{X144}_{56-n,10} & \mathrm{X0}_{n,11} \\ \mathrm{Y36}_{56-n,7} & \mathrm{Y72}_{56-n,8} & \mathrm{Y108}_{56-n,9} & \mathrm{Y144}_{56-n,10} & \mathrm{Y0}_{n,11} \\ \mathrm{Z36}_{56-n,7} & \mathrm{Z72}_{56-n,8} & \mathrm{Z108}_{56-n,9} & \mathrm{Z144}_{56-n,10} & \mathrm{Z0}_{n,11} \end{matrix}\right] \tag{7}$$

In the **A** matrix components, the first index corresponds with the number of the latitudinal contour of the 3D solid, for n = 1, 2, 3 …, 55, and the second – with the number of the point located along the latitudinal contour.

In order to verify the shape of the onion bulb model, the length of the bulb perimeter and its model were compared along three cross sections at the following heights: 3.6 mm (latitudinal contour of the onion bulb stem); 30.5 mm (latitudinal contour of the highest perimeter length of the onion bulb); 62.7 mm (latitudinal contour of the dry and cut chives of the onion bulb). The dimensions of perimeter of the latitudinal contour perimeter of the 4D model of the onion bulb on the *XY* plane were compared with the corresponding length of measured perimeter of the real onion, determining the relative error.

Section lengths along the latitudinal contours were calculated on the basis of the following equation:

$$L_j = \sqrt{(\mathbf{A}_{1,j+1} - \mathbf{A}_{1,j})^2 + (\mathbf{A}_{2,j+1} - \mathbf{A}_{2,j})^2} \tag{8}$$

The perimeter of the selected latitudinal contour is calculated on the basis of the following equation:

$$O = \sum_{j=1}^{10} L_j \quad \text{for } j = 10 \tag{9}$$

RESULTS

Figure 3 presents distribution of the points along the contours of the onion bulb, rotated along its vertical center line by angle of: 0°, 36°, 72°, 108°, 144°.

The basis for the proposed method is measurement of contour geometry from image data. After plotting of the points on digital photographs of subsequent contours, reading, using the graphic computer program (e.g. CorelDRAW) and saving their *xkα, zkα* u coordinates, a matrix was developed, in which the components were coordinates of the *Xpo* contour points, and another matrix with *Ypo* coordinates of the points and the third matrix with *Zpo* coordinates. Saving of coordinates in separate matrices allows for 3D visualization of the solid (Fig. 4), which is a model of the onion

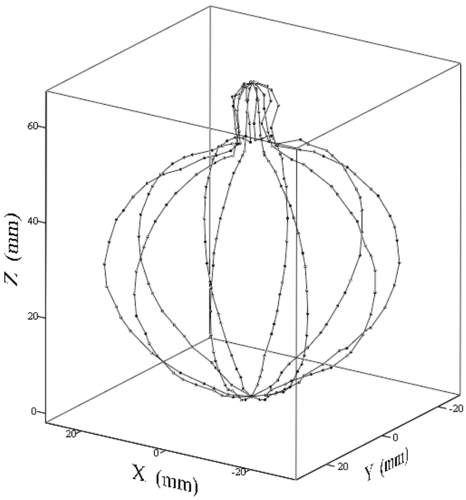

FIGURE 4. The onion bulb 3D model of – longitudinal contours

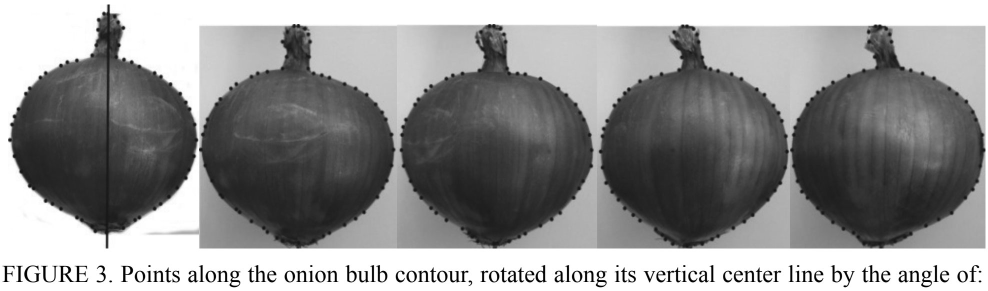

FIGURE 3. Points along the onion bulb contour, rotated along its vertical center line by the angle of: 0°, 36°, 72°, 108°, 144°

bulb. The longitudinal contour of the onion bulb is a polygon, consisting of 56 sides (Fig. 4). The latitudinal contour of the onion bulb is a polygon, consisting of 10 sides. Matrices of coordinates of points located within the onion bulb contours serve as a basis for visualization of 56 latitudinal contours and 10 longitudinal contours, serving as nodes of the grid distributed along the onion bulb surface.

Figure 5 presents the charts of selected contours of the onion model (dried and cut chives, maximum length of the latitudinal contour perimeter, latitudinal contour of the onion bulb stem).

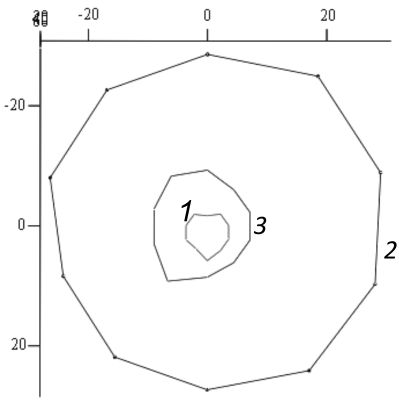

FIGURE 5. A chart of the selected latitudinal contours of the onion bulb: 1 – dried and cut chives, 2 – maximum length of onion bulb perimeter, 3 – onion bulb stem

Analysis of images provided in Figure 6 and results of comparisons contained in the Table 1 indicates that accuracy of cross sections of models in the place of maximum length of the latitudinal contour of the onion bulb, the contour of the dried and cut chives and the latitudinal contour of the onion bulb stem is sufficient for practical purposes, as the relative error in the cross-sections marked is within 1–2.9%.

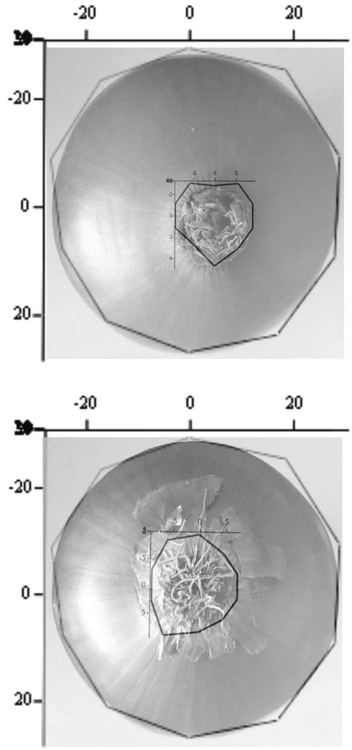

FIGURE 6. Projections of *YZ* plane of the real onion bulb solid with dried chives and stem visible and the appropriate 3D model contours

SUMMARY

The proposed method of mathematical modelling of the onion bulb can be used for design purposes, which require accuracy not exceeding 5%. The method developed allows for observation of shape and other features, such as perimeter, for a selected cross section out of 28 possible cross sections. Knowledge of coordinates of 280 points lying within the onion bulb surface allows for numerical computations using a discrete 3D model, associated with process visualization and simulation (building of deposit, analysis of deposit porosity etc.).

TABLE. Relative error of comparison of length of selected latitudinal perimeters of an onion bulb and its 3D model (dried and cut chives, maximum length of the latitudinal contour of the onion bulb, latitudinal contour of the onion bulb stem)

Contour line number (n)	Contour line name	Contour line parameter based on eq. (9) (O) [mm]	Contour line parameter measured (Oz) [mm]	Relative error [%]
3	dried and cut chives	23.9	23.4	2.1
16	maximum onion bulb diameter	176.6	174.8	1
25	onion bulb stem	55.9	54.3	2.9

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Streszczenie: *Matematyczne modelowanie konturów główki cebuli (*Allium cepa *L.).* W pracy zaproponowano metodę matematycznego modelowania kształtu główki cebuli cukrowej holenderskiej odmiany Alonso (*Allium cepa* L.). Do modelowania wybrano cebulę o wymiarach zbliżoną do średnich wymiarów dla tej odmiany, tj.: długość wynosiła 65,8 mm, szerokość 56 mm, a grubość 55,4 mm. Model matematyczny dotyczył opisu konturów południkowych za pomocą współrzędnych *x, y, z* 56 punktów rozmieszczonych na każdym z nich. Kąt między przecinającymi się płaszczyznami z konturami południkowymi, których krawędź przecięcia leży w osi symetrii główki cebuli, wynosił 36°. Dla główki cebuli opracowano także matematyczny opisu konturów równoleżnikowych. Zapisane współrzędne węzłów siatki powierzchni modelu główki cebuli mogą być podstawą projektowania zespołów roboczych urządzeń stosowanych w przetwórstwie cebuli.

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