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Mathematical and simulation modeling of thermal processes in pyroelectric detectors

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Summary. This paper considers mathematical and simulation models of a typical pyroelectric detector It explains the importance of the rate of temperature change of the sensitive element in current mode.

Keywords. Pyroelectric detector, mathematical modeling, simulation modeling, temperature change, rate of temperature change.

INTRODUCTION

Pyroelectric detectors are the thermal-to-electrical energy converters [1-4]. As instruments, they are used in many areas like IR gas analyzers [1, 10, 11], precision pyrometers [1, 12], linear IR cameras [13] spectrophotometers [1], multi-element detectors [14], Fabry-Perot interferometers [15]. They are widely used in fire [1, 16] and intruder alarms [1, 17, 18] systems as non-instrument applications as well.

This paper aims to receive the mathematical models of thermal parameters for a pyroelectric detector.

MATERIALS AND METHODS

The method of modeling is known [19-21]. Every problem to be set, begins with the law of conservation of energy. We use this law for describing the heating and cooling problems. We employ Taylor series [22] to linearize the non-linear differential equations. As a result, we obtain static and dynamic equations corresponding to steady and transient responses. After mathematical manipulations, we receive transfer functions and calculate amplitude-frequency and phase-frequency responses. In order to get the transient responses, we apply the Inverse Laplace transformation method [23].

For square pulses which are hard to describe mathematically [24], we use our simulator of a pyroelectric detector.

Principle of operation. Heat flow $\Phi(t)$ [W] at the surface of the sensitive element having square A_{pyro} [m^2], emissivity α, gives its energy to the sensitive element. The sensitive element having heat capacity C_T [J/K] takes this energy in, and gives out to the mounting at the same time due to heat losses G_T [W/K], the thermal time constant $\tau_T = C_T/G_T$ [s]. The temperature change of the sensitive element $T(t)$ [K] is characterized with its rate of temperature change $\Psi(t)$ [K/s] that is of research.

RESULTS

We begin the thermal-to-electrical model of a pyroelectric detector with the law of conservation of energy:

$$\begin{aligned} & dE_{INPUT} + dE_{DIST} = \\ & = dE_{VOL} + dE_{OUTPUT} . \end{aligned} \tag{1}$$

Let us consider every stage of the thermal-to-electrical model for heating and cooling processes separately.

Heating of the sensitive element. Temperature change.

The amount of energy coming to the detector from a heat source, [J], is:

$$dE_{INPUT} = \alpha \Phi_{INPUT} dt . \tag{2}$$

The amount of energy coming to the detector from the environment (the detector's can needs to be thermally isolated) [J], is:

$$dE_{DIST} = \alpha \Phi_{DIST} dt . \tag{3}$$

The amount of energy stored in the volume of the sensitive element, [J], is:

$$dE_{VOL} = C_T dT . \tag{4}$$

The amount of energy going out of the sensitive element to the environment, [J], is:

$$dE_{OUTPUT} = C_T T dt . \tag{5}$$

Static equation is:

$$\alpha \Phi_{INPUT0} + \alpha \Phi_{DIST0} = G_T T_0 . \tag{6}$$

Dynamic equation is:

$$\begin{aligned} & \alpha \Delta \Phi_{INPUT} + \alpha \Delta \Phi_{DIST} = \\ & + C_T \frac{d\Delta T}{dt} + G_T \Delta T . \end{aligned} \tag{7}$$

Transfer function is:

$$W_{x,z \to y}(s) = \frac{K_{INPUT/DIST}}{\tau_T s + 1}, \qquad (8)$$

where:

$$K_{INPUT/DIST} = \frac{\alpha \Phi_{INPUT/DIST}}{G_T T}. \qquad (9)$$

Amplitude-frequency response is:

$$A(\omega) = \frac{1}{\sqrt{1 + \omega^2 \tau_T^2}}. \qquad (10)$$

Phase-frequency response is:

$$\varphi(\omega) = -\arctan \omega \tau_T. \qquad (11)$$

Transient response is:

$$T(t) = \frac{\alpha \Phi_{INPUT/DIST}}{G_T} \left(1 - e^{-\frac{t}{\tau_T}} \right). \qquad (12)$$

Cooling of the sensitive element. Temperature change.

The amount of energy coming to the detector from a heat source, [J], is:

$$dE_{INPUT} = \alpha d\Phi_{INPUT} dt. \qquad (13)$$

The amount of energy coming to the detector from the environment (the detector's can needs to be thermally isolated) [J], is:

$$dE_{DIST} = \alpha d\Phi_{DIST} dt. \qquad (14)$$

The amount of energy stored in the volume of the sensitive element, [J], is:

$$dE_{VOL} = G_T dT dt. \qquad (15)$$

The amount of energy going out of the sensitive element to the environment, [J], is:

$$dE_{OUTPUT} = \frac{G_T^2}{C_T} T dt^2. \qquad (16)$$

Static equation is:

$$\frac{G_T^2}{C_T} T_0 = 0. \qquad (17)$$

Dynamic equation is:

$$\frac{\alpha d\Delta\Phi_{INPUT}}{dt} + \frac{\alpha d\Delta\Phi_{DIST}}{dt} = = G_T \frac{d\Delta T}{dt} + \frac{G_T^2}{C_T} \Delta T. \qquad (18)$$

Transfer function is:

$$W_{x,z \to y}(s) = \frac{K_{INPUT/DIST} s}{\tau_T s + 1}, \qquad (19)$$

where:

$$K_{INPUT/DIST} = \frac{C_T}{G_T^2} \frac{\alpha \Phi_{INPUT/DIST}}{T}. \qquad (20)$$

Amplitude-frequency response is:

$$A(\omega) = \frac{\omega \tau_T}{\sqrt{1 + \omega^2 \tau_T^2}}. \qquad (21)$$

Phase-frequency response:

$$\varphi(\omega) = \arctan \frac{1}{\omega \tau_T}. \qquad (22)$$

Transient response is:

$$T(t) = \frac{\alpha \Phi_{INPUT/DIST}}{G_T} e^{-\frac{t}{\tau_T}}. \qquad (23)$$

Heating of the sensitive element. Rate of temperature change.

The amount of energy coming to the detector from a heat source, [J], is:

$$dE_{INPUT} = \alpha d\Phi_{INPUT} dt. \qquad (24)$$

The amount of energy coming to the detector from the environment (the detector's can needs to be thermally isolated) [J], is:

$$dE_{DIST} = \alpha d\Phi_{DIST} dt. \qquad (25)$$

The amount of energy stored in the volume of the sensitive element, [J], is:

$$dE_{VOL} = G_T d\Psi dt. \qquad (26)$$

The amount of energy going out of the sensitive element to the environment, [J], is:

$$dE_{OUTPUT} = G_T \Psi dt^2. \qquad (27)$$

Static equation is:

$$G_T \Psi_0 = 0. \qquad (28)$$

Dynamic equation is:

$$\frac{\alpha d\Delta\Phi_{INPUT}}{dt} + \frac{\alpha d\Delta\Phi_{DIST}}{dt} = = C_T \frac{d\Delta\Psi}{dt} + G_T \Delta\Psi. \qquad (29)$$

Transfer function is:

$$W_{x,z\to y}(s)=\frac{K_{INPUT/DIST}s}{\tau_T s+1}. \qquad (30)$$

where:

$$K_{INPUT/DIST}=\frac{\alpha\Phi_{INPUT/DIST}}{G_T\Psi}. \qquad (31)$$

Amplitude-frequency response is:

$$A(\omega)=\frac{\omega\tau_T}{\sqrt{1+\omega^2\tau_T^2}}. \qquad (31)$$

Phase-frequency response is:

$$\varphi(\omega)=\arctan\frac{1}{\omega\tau_T}. \qquad (32)$$

Transient response is:

$$\Psi(t)=\frac{\alpha\Phi_{INPUT/DIST}}{C_T}e^{-\frac{t}{\tau_T}}. \qquad (33)$$

Cooling the sensitive element. Rate of temperature change.

The amount of energy coming to the detector from a heat source, [J], is:

$$dE_{INPUT}=\alpha d^2\Phi_{INPUT}dt. \qquad (34)$$

The amount of energy coming to the detector from the environment (the detector's can needs to be thermally isolated) [J], is:

$$dE_{DIST}=\alpha d^2\Phi_{DIST}dt. \qquad (35)$$

The amount of energy stored in the volume of the sensitive element, [J], is:

$$dE_{VOL}=G_T d\Psi dt^2. \qquad (36)$$

The amount of energy going out of the sensitive element to the environment, [J], is:

$$dE_{OUTPUT}=G_T\Psi dt^2. \qquad (37)$$

Static equation is:

$$\frac{G_T^2}{C_T}\Psi_0=0. \qquad (38)$$

Dynamic equation is:

$$\frac{\alpha d^2\Delta\Phi_{INPUT}}{dt^2}+\frac{\alpha d^2\Delta\Phi_{DIST}}{dt^2}= \qquad (39)$$

$$=G_T\frac{d\Delta\Psi}{dt}+\frac{G_T^2}{C_T}\Delta\Psi.$$

Transfer function is:

$$W_{x,z\to y}(s)=\frac{K_{INPUT/DIST}s^2}{\tau_T s+1}, \qquad (40)$$

where:

$$K_{INPUT/DIST}=\frac{C_T}{G_T^2}\frac{\alpha\Phi_{INPUT/DIST}}{\Psi}. \qquad (41)$$

Amplitude-frequency response:

$$A(\omega)=\frac{\omega^2\tau_T^2}{\sqrt{1+\omega^2\tau_T^2}}. \qquad (42)$$

Phase-frequency response:

$$\varphi(\omega)=-\arctan\omega\tau_T. \qquad (43)$$

Transient response is:

$$\Psi(t)=-\frac{\alpha\Phi_{INPUT/DIST}}{C_T}e^{-\frac{t}{\tau_T}}. \qquad (44)$$

We apply these results to our software - simulator of a pyroelectric detector. Fig. 1 shows heat flow at the surface of the sensitive element. Fig. 2 includes its temperature change. Fig. 3 explains the rate of temperature change.

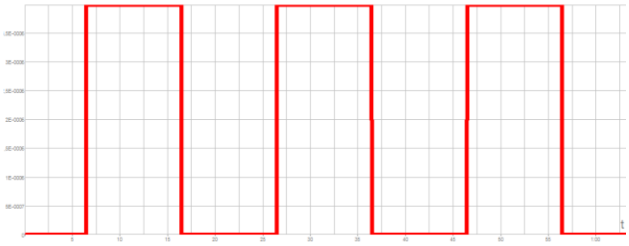

Fig. 1. Heat flow at the surface of the sensitive element

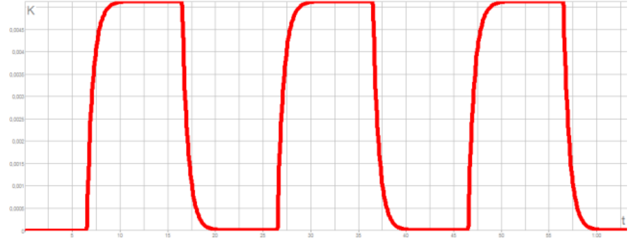

Fig. 2. Temperature change of the sensitive element

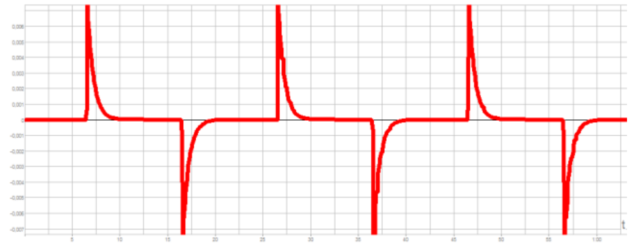

Fig. 3. Rate of temperature change of the sensitive element

CONCLUSIONS

1. We achieved amplitude-frequency responses for every stage of the thermal model of a pyroelectric detector. These plots show amplitude behavior of the detector at different frequencies of modulation.

2. We obtained phase-frequency responses for every stage of the thermal model of a pyroelectric detector. These diagrams indicate phase behavior of the detector at different frequencies of modulation.
3. We could see in the above mentioned responses that phase angle between the stage of temperature change and that of rate of temperature change equaled 90 deg that corresponded to be true.
4. We received transient responses of a pyroelectric detector. The rate of temperature change is of particular importance in current mode. Being exceeded, the sensitive element becomes short-circuited. This parameter is not well considered in the literature [1-4]. The simulator can be used to replace expensive experiments in manufacturing with cheap simulation.

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МАТЕМАТИЧЕСКОЕ И ИМИТАЦИОННОЕ МОДЕЛИРОВАНИЕ ТЕПЛОВЫХ ПРОЦЕССОВ В ПИРОЭЛЕКТРИЧЕСКИХ ДЕТЕКТОРАХ

Александр Бондаренко, Петр Елисеев, Марина Лория, Алексей Целищев

Аннотация. В статье рассматриваются математические и имитационные модели тепловых характеристик типового пироэлектрического детектора. Уделяется внимание скорости изменения температуры чувствительного элемента ,как главному тепловому параметру пироэлектрических детекторов в режиме работы по току.

Ключевые слова. Пироэлектрический детектор, математическое моделирование, имитационное моделирование, изменение температуры, скорость изменения температуры.