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ORIGINAL PAPER

Height growth and site index models for main forest forming tree species in Poland

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ABSTRACT

The potential forest site productivity is a fundamental information for strategic decisions in forest management. Site productivity is commonly estimated using the site index, which is determined based on the height at a given age estimated using site index models. Site index models are also used in forest growth predictions. A proper assessment of the site index is important for management, as it is the primary criterion considered when making site- and species-specific economic decisions on silvicultural treatments, selection of species composition, setting the cutting plan and rotation age. Recently, site index models have been developed for the main forest-forming tree species of Poland. However, in developing the parameters of the models, autocorrelation was not fully taken into account and, in addition, there were inaccuracies in the parameter values of the models. In connection with the planned implementation of the models into the State Forests IT system and their application to determine the volume increment of forests in Poland, it was considered desirable to improve the existing models. The aim of this study is to develop unified site index models for the main forest-forming species of Poland using the most up-to-date approach. Using data from 2 636 single tree growth series, we developed new site index models for eight forest tree species in Poland. The use of the dynamic GADA function and taking into account the autocorrelation of the residuals allowed a very good fitting of the models to the empirical data. Furthermore, the models developed meet the main requirements for modern site index models: (a) polymorphism, which would allow for possible differences in growth patterns due to variability in site conditions; (b) variable asymptotes for different sites; (c) the equality of site index and height at a given base age; and (d) the possibility of using the same function to model height growth and site index. The determination of site productivity is the main purpose of the developed models. However, thanks to the properties of the functions used for the development of the models, they can also be used as models for the prediction of the height growth of stands. This research is a response to the demand for the development of new yield tables (growth models) for the main forest forming tree species in Poland from the State Forests in Poland, the Forest Management Office and other stakeholders. The research presented in this paper is the first stage of the construction of new empirical growth models.

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KEY WORDS

GADA, growth patterns, growth prediction, site productivity

Introduction

The potential forest site productivity forms fundamental knowledge for making strategic decisions in forest management. The site productivity is commonly estimated using the site index, which is determined based on the top height at a predetermined age estimated using site index models (Skovsgaard and Vanclay, 2008; Socha *et al.*, 2015). Site index models are also utilised to forecast forest growth, determining the economic and environmental impacts of forest management (Tompalski *et al.*, 2021). The site index measures the influence of climatic conditions on site productivity, reflecting how both climatic and edaphic factors impact it (Pau *et al.*, 2022). Therefore, the assessment of potential productivity is important for management aimed at a sustainable and balanced forest, as it is the primary criterion considered when making site- and species-specific economic decisions on silvicultural treatments, selection of species composition, setting the cutting plan and rotation age (Chen *et al.*, 1998; Kayahara *et al.*, 1998; Splechtna, 2001).

Forest ecosystems in Europe have undergone alterations due to climate change and antropopression (Lindner *et al.*, 2010; Pretzsch *et al.*, 2014; Etzold *et al.*, 2019). Climate shifts appear to have had a favourable impact on forest productivity in recent times (Etzold *et al.*, 2019; Mensah *et al.*, 2021; Socha *et al.*, 2021). The observed changes in productivity have altered the growth dynamics of European forests. Pretzsch *et al.* (2014) anticipated an increase in site productivity of 30-70% in Central Europe when compared to reports from previous centuries. At the same time, growth models, and in particular site index models, previously used in forestry practice, have become outdated (Socha *et al.*, 2020b). Yield tables for Scots pine *Pinus sylvestris* L. stands were developed in Poland in the inter-war period (Jedliński, 1932; Płoński, 1937). However, they were not used in practice: the former for methodological reasons, the latter because the borders of the country were moved westwards after the Second World War. Schwappach (1912) tables became more relevant in Poland, as about 70% of the sample plots on which he based his Scots pine tables were located within the Polish borders. For stands on the best sites they were extended by extrapolation, with a site index class of Ia (Szymkiewicz, 1949). Consequently, the application of yield tables may result in unsuitable evaluations of site productivity and estimation of volume increment. Despite it, in forestry practice in Poland, a site index class is still determined using height growth curves developed by Schwappach for Scots pine, Norway spruce *Picea abies* (L.) H.Karst, European beech *Fagus sylvatica* L., Silver fir *Abies alba* Mill., and Sessile oak *Quercus petraea* (Matt.) Liebl., and by Schober for European larch *Larix decidua* Mill. and by Tiurin for Silver birch *Betula pendula* Roth and Black alder *Alnus glutinosa* (L.) Gaertn. (Socha *et al.*, 2020b). Consequently, the application of yield tables may result in unsuitable evaluations of site productivity and estimation of volume increment.

An attempt to solve this problem was made by Bruchwald *et al.* (1999, 2003, 2011) developing new Polish site index models. The authors used a function describing a single growth trajectory, which is then extrapolated to different sites. However, models developed on the basis of a single growth trajectory and extrapolation to various sites may produce unrealistically high site index values, particularly for stands of younger age classes (Socha and Orzeł, 2011). Bailey and Clutter (1974) had already pointed out the problem associated with this type of modelling, which relies on a specific single base age. This is because the base age can affect the final course of the height growth trajectories obtained. They therefore proposed an improved approach based on algebraic difference, in which the course of the height function is base age invariant. However, the choice of parameter in the equation used as the site parameter resulted in site index curves that were anamorphic or polymorphic, with unique asymptotes (Cieszewski *et al*., 2000). The Generalised Algebraic Difference Approach (GADA), developed by Cieszewski and Bailey in 2000, allows the use of multiple site parameters to solve this problem. The proposed models are characterised by polymorphism and different asymptotes. GADA allows for the development of site-specific site index models that express local height growth patterns. This allows for appropriate assessments of site productivity. In recent years also nonlinear mixed-effects (NME) modelling approach (Bronisz and Mehtätalo, 2020) was used in modelling height growth (Wang *et al*., 2008; Socha *et al*., 2021). However, for Scots pine in Poland, this approach has proven to be less effective compared to the nonlinear-fixed-effects (NFE) approach, which is GADA (Socha *et al*., 2021). The GADA approach was used by Cieszewski and Zasada (2007) to develop a new dynamic site equation that fits best the Schwappach data for Scots pine.

Recently, site index models have been developed for the main forest-forming species of Poland (Socha *et al*., 2020b). However, in developing the parameters of the models, autocorrelation was not fully taken into account and, in addition, there were inaccuracies in the parameter values of the models. In connection with the planned implementation of the models into the State Forests IT system and their application to determine the volume increment of forests in Poland, it was considered desirable to improve the existing models. The aim of this study is to develop unified site index models for the main forest-forming tree species of Poland using the most up-to-date approach.

Materials and Methods

The research methodology consisted of three stages: verification of the research data, which consisted of growth series obtained by retrospective growth analysis using the stem analysis method (1), estimation of the growth function parameters using GADA (2) and evaluation of the developed models (3).

RESEARCH MATERIAL. The research material comprises data on the high-growth trajectories of 2,636 dominant and codominant trees of the eight main forest-forming tree species in Poland (Table 1), which were gathered from across the entire territory of Poland (Fig. 1). The stands sampled in a quasi-random manner are primarily pure and even-aged. However, these do not represent forest plantations, and the forest management methods used in these forests are typical for multi-functional forest management.

The study incorporates data from two sources. Part of the data was gathered from the project named ‘Actual and Potential Site Productivity in Poland For Main Forest-Forming Tree Species’, carried out during 2015-2017 (Socha *et al*., 2017a). Within this project over 300 growth trajectories for Scots pine, the main forest forming tree species in Poland covering over 60% of forested area, was collected. These trajectories represent the full range of geographic distribution and site conditions for Scots pine. The second data source consisted of a database containing the growth series of individual trees that were collected in Poland through research efforts conducted by leading forest research institutions in the country. These institutions include the Faculty of Forestry at the University of Agriculture in Krakow, the Forest Research Institute in Sękocin Stary, and the Faculty of Forestry at the Warsaw University of Life Sciences (SGGW). All growth trajectories used in this study were individually verified in order to meet the standards required for the development of site index models. The trees chosen as samples for the reconstruction of the height growth curves were dominant, without visible damage and growth anomalies.

Table 1.

The number and basic characteristics of growth series for the analysed species, obtained from stem analysis and used for the development of the site index models

Species	No trees	Age			Height [m]			Site index [m]		
		Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
Scots pine *Pinus sylvestris* L.	875	76	12	175	23.2	7.1	36.8	27.9	10.9	41.8
European beech *Fagus sylvatica* L.	187	82	60	160	24.2	15.8	33.6	28.5	20.3	36.6
Black alder *Alnus glutinosa* (L.) Gaertn.	365	47	7	100	19.6	5.2	31.4	28.5	21.3	34.1
European larch *Larix decidua* Mill.	227	60	20	170	26.7	16.4	38.6	34.9	27.5	41.1
Silver birch *Betula pendula* Roth	99	49	20	105	23.9	10.5	33.5	32.5	22.8	40.7
Oak *Quercus* sp.	323	68	18	157	22.0	9.5	32.9	27.9	19.5	35.9
Norway spruce *Picea abies* (L.) H.Karst	416	76	30	150	25.3	5.1	41.5	30.7	15.7	40.5
Silver fir *Abies alba* Mill.	144	70	30	140	15.8	2.3	36.0	23.6	7.6	34.9

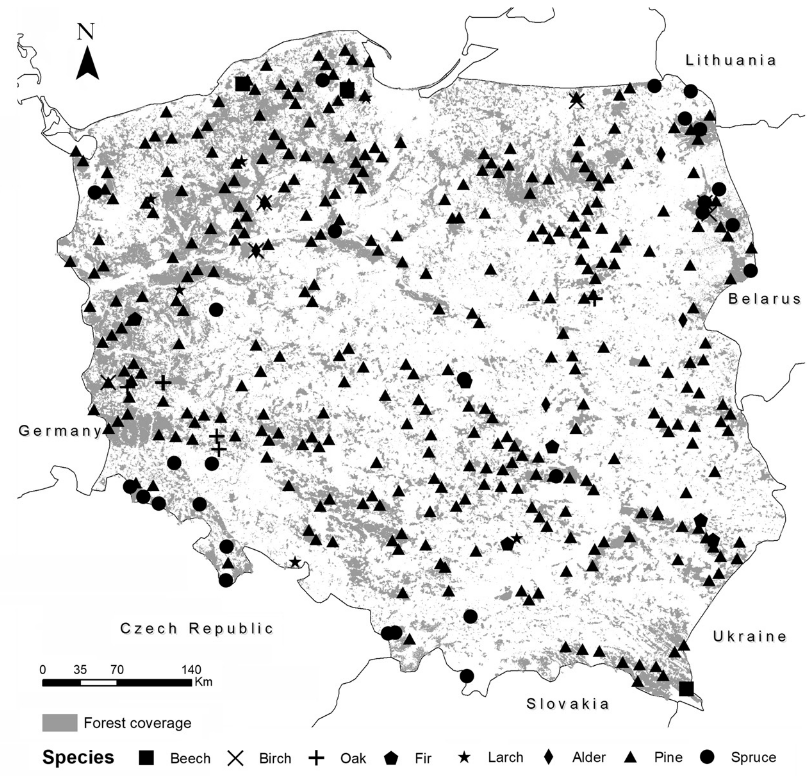

Fig. 1.

The location of sample plots for the stem analyses of the forest-forming tree species in Poland

STEM ANALYSIS. Stem analysis was utilised on the material obtained from individual trees in order to reconstruct their past height growth. The trees for stem analysis were selected from the immediate vicinity of the permanent sample plots as established within the research projects. For each plot, one to three trees with a diameter and height close to the top height defined by the average height of the 100 thickest trees per 1 ha were chosen and felled for stem analysis. To construct the height growth trajectories of trees, only trees with normally developed crowns that exhibit no conspicuous signs of damage or growth irregularities were selected. After cutting, the entire length of the tree was measured and stem analysis samples were collected from rings at the base of the felled tree at a height of 0.5 m, at the diameter of breast height (DBH), and then for trees higher than 15 m, additional sections were taken at intervals of 2 m, 4 m, and 2 m thereafter up to the tree top. For trees below 15 m in height, we gathered additional sections at heights of 1.5, 2.5, 3.5, and every 1 m up the tree. To depict the tree growth, we employed the heights of the sections and the annual rings amount. The growth trajectories of individual trees were analysed visually to detect patterns of suppression, release, and growth anomalies. As cross-section lengths does not coincide with the periodic growth of height, the height-age data obtained from the stem analysis were adjusted with Carmeans technique (1972).

MODELS. The researchers have used various growth functions to develop height growth models. Kiviste *et al.* (2002) documented 74 different models for this purpose. New growth functions have been utilised to construct site index models for several countries, including Sweden (Elfving and Kiviste, 1997), Finland (Karlsson, 2000), Spain (Palahí *et al.*, 2004), the USA (Weiskittel *et al.*, 2016), Turkey (Ercanli *et al.*, 2014), and Poland (Cieszewski and Zasada, 2002; Socha and Orzeł, 2011; Socha *et al.*, 2017b, 2020b).

Adequate site index models should have the following attributes (Goelz and Burk, 1992; Cieszewski and Bailey, 2000; Cieszewski, 2003, 2004; Dieguez-Aranda *et al.*, 2006): (a) polymorphism, which would account for possible differences in growth patterns due to variability in site conditions; (b) variable asymptotes for different sites; (c) the equality of site index and height at a given base age, and (d) the possibility of using the same function as a model for height growth and site index. The criteria are met with the generalized algebraic difference approach (GADA), which utilises one or two site-specific variables that indicate the quality of the site (Cieszewski and Bailey, 2000). For the development of the model, we chose the dynamic function derived by Cieszewski (2001; Eq. 1). The chosen function is well established in site index modelling for the main forest forming tree species in Poland (Cieszewski and Zasada, 2002; Cieszewski *et al.*, 2007; Socha and Orzeł, 2011, 2013; Socha *et al.*, 2015; 2020b; Tymińska-Czabańska *et al.* 2021, 2022). The utilized dynamic GADA function is formulated as follows:

$$H_2 = H_1 \frac{T_2^{b_1}(T_1^{b_1} \cdot R + b_2)}{T_1^{b_1}(T_2^{b_1} \cdot R + b_2)} \tag{1}$$

where:

$$R = Z_0 + \left(Z_0^2 + \frac{2b_2 \cdot H_1}{T_1^{b_1}} \right)^{0.5} \tag{2}$$

$$Z_0 = H_1 - b_3 \tag{3}$$

where:

b_1, b_2, b_3 are parameters of the models;

H_1 and H_2 are heights [m] at ages T_1 and T_2 [years], respectively.

Parameter estimation. The chosen equation determines that the base age selection does not affect the model parameter estimates (Cieszewski and Bailey, 2000; Cieszewski, 2001). Site-specific and global parameters of site index models were simultaneously fitted using the dummy variable approach (DVA), also known as the varying parameter method (Cieszewski *et al.*, 2000). In the DVA, global and site-specific parameters are estimated simultaneously, resulting in the same parameter estimates regardless of the selected base age; therefore, it is a base-age invariant method and provides unbiased estimates for base-age invariant equations (Nunes *et al.*, 2011). The DVA uses a starting value of height as the site specific parameter for each growth series. This assumes that all particular growth series measurements have the same starting value. As a result of parameter estimation, a unique site parameter can be calculated for each tree using an expression containing the simulated variables and a site-specific parameter.

Given that the data represent a time series, the models used first-order autocorrected errors. This was done to deal with the possibility of autocorrelated residuals from the models. This is a common problem with models fitted to data representing successive (and thus often correlated) observations in time, especially when all possible time intervals are used. This autocorrelation has the potential to adversely affect estimates of model parameters and their standard errors (Parresol and Vissage, 1998).

For elimination of data autocorrelation and better curve fit, it was explored to add an autoregressive and a moving average process (ARMA) to the estimated growth curves. It was assumed that each species and growth series represents some kind of time series. This can be generalised or simplified after data decomposition, identifying ARMA process, assessment of parameter and re-submission of entire processes.

After estimating the parameters for the GADA function (Eq. 1), the trend obtained was removed from the data. If the residuals for each species were stationary, the partial autocorrelation and autocorrelation functions were analysed to identify the autoregressive and moving average processes. There was no need to differentiate as all data were stationary. Once the ARMA process was identified, it was added to the general trend obtained. In this way, the second data growth series (which by design should be free of autocorrelation and random disturbances) was obtained and used to estimate the dynamic GADA site index function. The parameters of the models were estimated using a procedure written in R (R Core Team, 2021).

Model evaluation. In the first stage of the evaluation, the goodness of fit of the model was characterised by the mean absolute error (MAE; Eq. 4) and the adjusted coefficient of determination (R^2_{adj}; Eq. 5):

$$MAE = \sum_{i=1}^{n} \frac{|h_i - \hat{h}_i|}{n} \tag{4}$$

$$R^2_{adj} = 1 - \frac{\sum_{i=1}^{n} (h_i - \hat{h}_i)^2}{\sum_{i=1}^{n} (h_i - \overline{h}_i)^2} \cdot \left(\frac{n-1}{n-p} \right) \tag{5}$$

where:

$h_i, \hat{h}_i, \overline{h}_i$ are the measured, mean measured, and predicted values of the heights, respectively,

n is the total number of observations used to fit the model,

p is the number of parameters in the model.

In the next step, we focused on the remaining bias in the residuals. We presented the mean of the residuals (called bias) and their confidence intervals for each species. The *t*-Student test was used to test the null hypothesis that the mean of the residuals is zero. The stationarity of the residuals was tested using the Dickey-Fuller test to show that the mean and variance of the residuals were independent of height. The Ljung-Box Q-test was used to check if the autocorrelations of the data are different from zero (null hypothesis: the data are independently distributed).

Results

Parameters of eight site index models were estimated as a result of model fitting using 2,636 growth chronologies (Table 2). The analysis of the residuals after the model decomposition showed that the partial autocorrelation function very rarely exceeded one and the autocorrelation less often exceeded ten. Most of the values indicated a low or weak autocorrelation (less than 0.2 in absolute value). Therefore, regardless of the species and to simplify the calculation, similar to Adame *et al.* (2006), the ARMA (1,10) process was applied in the estimated GADA model. The estimated model parameters (Table 2) were statistically significant ($p<0.001$). With the exception of the second parameter (b_2) for the silver fir model, which was statistically significant at a slightly higher level of significance. The models accounting for autocorrelation explained over 99.25% of the variance. The mean absolute error (MAE) varies between 0.22 and 0.54 metres. The model growth trajectories for individual species representing a wide range of site conditions are shown in Figure 2.

The results of the *t*-Students tests (Table 3) show that the means of the residuals did not significantly differ from zero, except for Scots pine, for which the growth curve was slightly biased.

Table 2.

Parameter estimates and goodness-of-fit statistics for the individual site functions and species

Species	Coeff.	Estimate	Std. Error	*t*-value	R^2_{adj}	*AIC*	*MAE*
Scots pine	b1	1.3601	0.0027	503.6330*	0.9966	19979.51	0.3290
	b3	31.6767	0.3120	101.5311*			
	b2	5320.602	149.7675	35.5257*			
European beech	b1	1.611	0.0071	227.3728*	0.9968	4431.018	0.3342
	b3	20.2754	2.4338	8.3306*			
	b2	75431.678	6166.6581	12.2322*			
Black alder	b1	1.5354	0.0029	532.4671*	0.9975	10173.5	0.2136
	b3	30.6081	0.1658	184.5713*			
	b2	1186.0699	61.6645	19.2342*			
European larch	b1	1.3248	0.0070	188.6894*	0.9938	6270.949	0.4827
	b3	43.4248	0.5117	84.8642*			
	b2	653.0129	135.9801	4.8023*			
Silver birch	b1	1.6186	0.0175	92.3626*	0.9925	2303.426	0.5360
	b3	11.5414	3.2306	3.5726*			
	b2	14326.78	1853.918	7.7278*			
Oak	b1	1.6413	0.0061	268.2070*	0.9955	7599.869	0.3965
	b3	30.814	0.4301	71.6503*			
	b2	7012.034	533.8533	13.1348*			
Norway spruce	b1	1.8131	0.0059	308.9833*	0.9957	12290.15	0.4185
	b3	41.5612	0.3126	132.9439*			
	b2	4978.229	610.8700	8.1494*			
Silver fir	b1	2.2177	0.0120	185.2099*	0.9970	3321.225	0.3757
	b3	35.9102	0.3811	94.2403*			
	b2	11362.6423	3897.2398	2.9156**			

R^2 – coefficient of determination, *AIC* – Akaike information criterion, *MAE* – mean absolute error
*statistical significance (p-value<0.001), **statistical significance (p-value=0.00359)

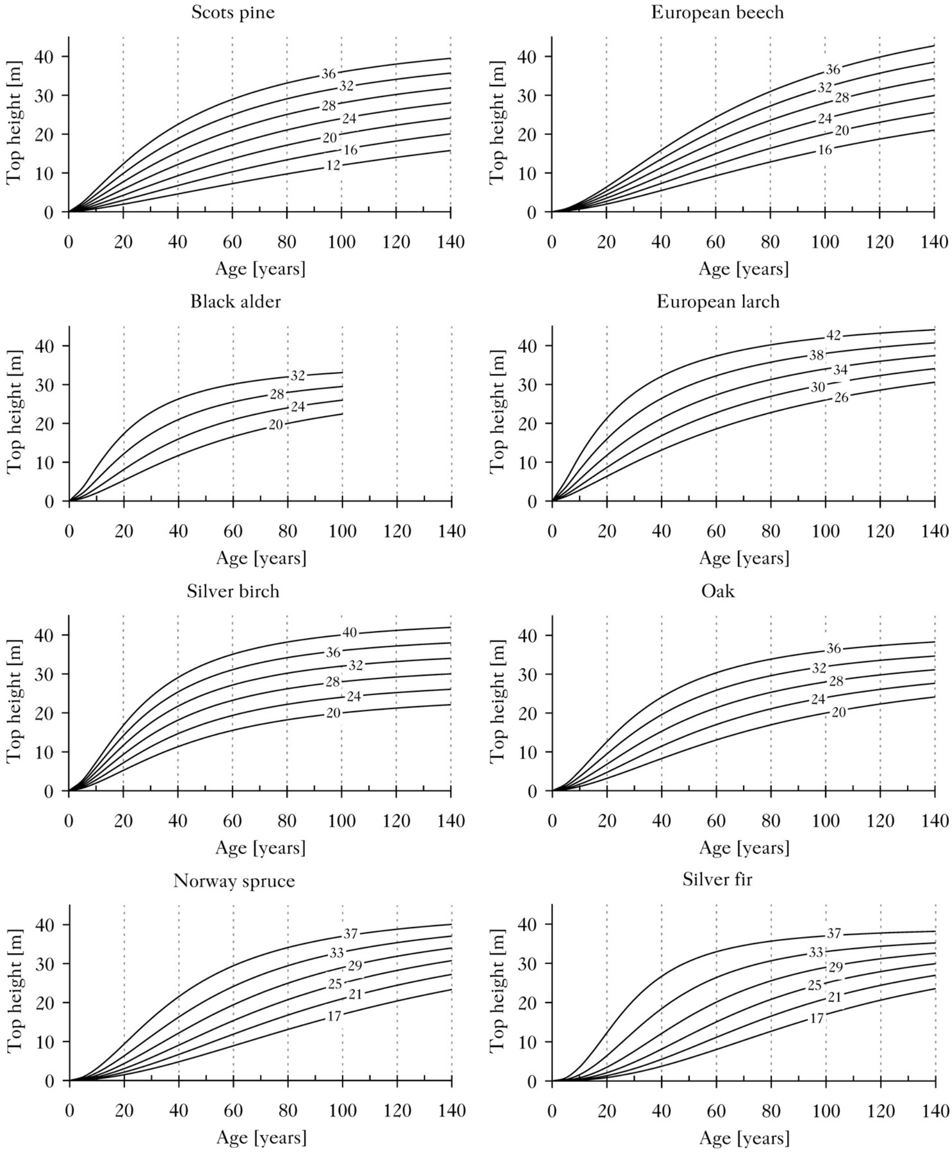

Fig. 2.

Site-specific height growth curves representing the developed site index models; the numbers in rectangle show the corresponding values of the site index

The Dickey-Fuller test (testing for a unit root, Table 3) allowed to reject the hypothesis that the residuals were autoregressive. This means that both the mean and the variance of the residuals were constant and unchanged over the predicted tree age. Figure 3 shows the mean of the residuals as the bias of the model and its variance. The derived 95% confidence interval varied from –1.42 m (Silver birch) to +1.37 m (European larch and Silver birch).

The deviations of the residuals from the predicted values of the heights are shown in Figure 4, where the darker points indicate a larger number of measurements. The ellipse shows a normal two-dimensional distribution with a variability of 95%. This xy-distribution is symmetric about

Table 3.
Descriptive characteristics of model residuals

Species	Scots pine	European beech	Black alder	European larch	Silver birch	Oak	Norway spruce	Silver fir
Mean	–0.0096	0.0030	–0.0037	–0.0169	–0.0243	–0.0111	0.0021	0.0105
Variance	0.2243	0.2186	0.1498	0.5005	0.5054	0.2831	0.3462	0.2628
n	13563	3074	10044	2707	973	4407	6443	2015
t-value	–2.0904	0.5892	–0.4179	–0.7927	–0.5635	–0.9698	0.7538	–0.3639
p-value	0.0183*	0.7221	0.3380	0.2140	0.2866	0.1661	0.7745	0.3580
Dickey-Fuller	–23.54	–14.805	–20.578	–17.398	–11.476	–16.813	–17.81	–12.115
Lag order	23	14	21	13	9	16	18	12
p-value	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}	<0.01^{HA}

*statistical significance at $\alpha=0.05$; ^{HA} statistical significance at $\alpha=0.05$, adopted alternative hypothesis: stationary

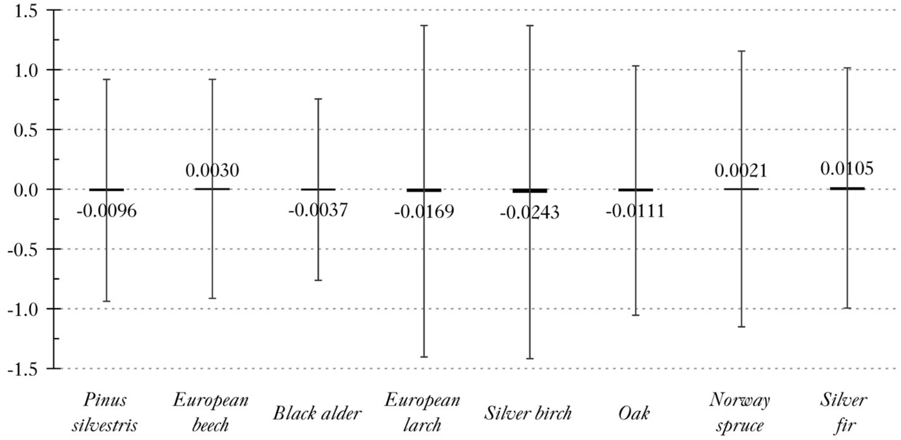

Fig. 3.
Residuals average (BIAS of models) and its deviations (95% confidence interval) in meter

the OX-axis, so it shows no trend in the data beyond the range of data variability. In conclusion, the residuals of the model had no or very little bias (mean equal to zero) and constant variance. However, despite the very good performance of the model and an explanation of the variance of more than 99%, the value of the Ljung-Box statistic still indicated the presence of a slight autocorrelation of the data (Table 4). Although the residuals for European beech and Oak were not statistically significant, neither autocorrelation nor partial autocorrelation was found. For the other species, the residuals showed statistically significant data correlation ranging from –0.08 for the partial autocorrelation function to 0.18 for the autocorrelation function observed in the case of Silver fir.

Discussion

The research described in this paper has resulted in new height growth models for the main forest forming tree species in Poland. The models developed are compatible, *i.e.* they can be used as both height growth and site index models. It was possible to obtain polymorphic growth trajectories with different asymptotes because the growth function used to develop the models was derived using the GADA method. The growth function used in the modelling derived by Cieszewski (2001) has been repeatedly shown to be robust for modelling the growth in stand height of the forest forming species analysed. A very high proportion of explained variance confirms its adequacy. With the additional inclusion of autocorrelation, the proportion of explained variance exceeded 99.25%, and the mean absolute error (MAE) ranged from 0.22 to 0.54 metres, with errors distributed symmetrically around zero.

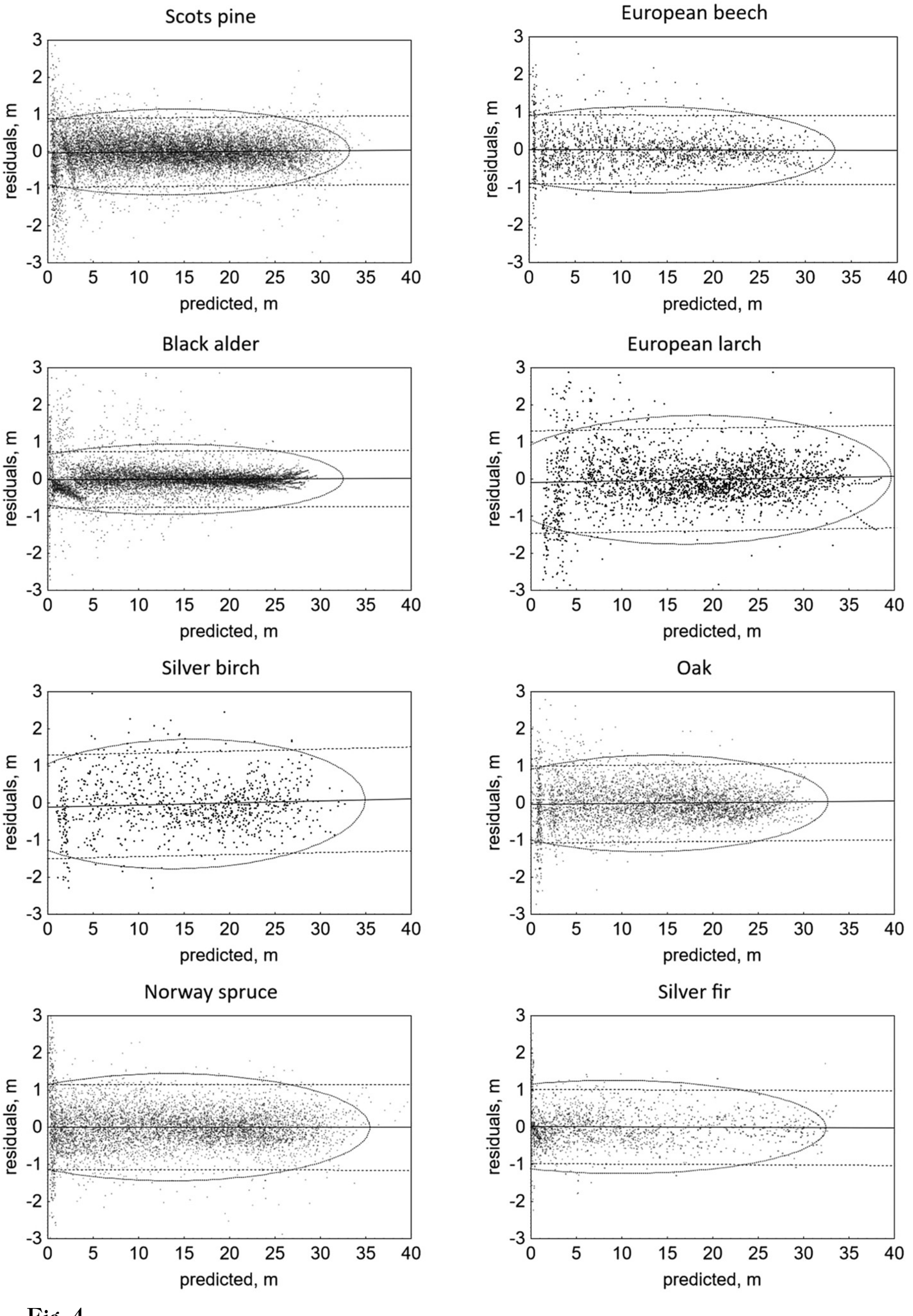

Fig. 4.

Residuals average (BIAS of models) and its deviations (95% confidence interval) in meter

Table 4.
Results of the Ljung-Box Q-test for residuals

Species	Q – statistics	df	*p*-value
Scots pine	49.112	1	<0.001
European beech	11.9121	1	<0.001
Black alder	147.571	1	<0.001
European larch	45.135	1	<0.001
Silver birch	9.2055	1	<0.001
Oak	8.9044	1	<0.001
Norway spruce	58.309	1	<0.001
Silver fir	7.9417	1	<0.001

Compared to the tools still used in Polish forestry practice, such as yield tables, the developed models provide a better predictive ability, which has also been reported by many other studies (Socha and Orzeł, 2011, 2013; Socha *et al.*, 2015, 2020b). The inadequacy of yield tables had been pointed out much earlier by practising foresters and scientists. Consequently, an attempt to solve this problem by developing new growth models was made by Bruchwald *et al.* (1999, 2003, 2011), which developed site index models for the most important tree species in Poland. However, the developed models used a single function describing a growth rate, which was then extrapolated to different sites. As a result, the developed models sometimes produce unrealistically high site index values, especially for stands of younger age classes (Socha and Orzeł, 2011).

The models developed in presented research are characterised by growth trajectories that better reflect growth patterns than the tools used in Polish forestry practice, such as yield tables. This is probably due to the more up-to-date research material and the range of sites represented by the growth trajectories obtained from stem analyses. The changing dynamics of forest growth in Europe (Pretzch *et al.*, 2014; Mensah *et al.*, 2021) have contributed to the inadequacy of yield tables developed almost 100 years ago. These changes are mainly due to increasing atmospheric nitrogen deposition and climate change, in particular the lengthening of the growing season, which are considered to be the main factors leading to long-term changes in forest sites (Socha *et al.*, 2021). In central Europe, average anthropogenic nitrogen deposition increased from about 3 kg ha^{-1} in 1940 to 9.5 kg ha^{-1} in 2015 and is considered to be the most important environmental factor influencing the accelerated growth of European forests (Ågren *et al.*, 2008; Etzold *et al.*, 2020).

The use of recent data in the model development, including the most recent height growth data, ensures that the developed models better reflect current growth trends. For the calibration of the growth models we have used the data from the retrospective stem analysis. One of the reasons for this was the lack of permanent, long-term experimental plots, which are considered the best source of data for such kind of analysis. However, when using retrospective height growth analysis, there remains some uncertainty about the accuracy of individual tree growth reconstruction. For example, it is difficult to determine whether a given tree was always a dominant tree or whether it changed its biosocial position as it grew, as is often observed (Raullier *et al.*, 2003). Therefore, the accuracy of height projections derived from the developed models should be verified using data from permanent sample plots or measurements from repeated airborne laser scanning (ALS).

Recent studies indicate that repeated ALS can be a very good source of height growth data. The accuracy of measuring the height of individual trees using ALS is far better than traditionally used field measurement methods (Wang *et al.*, 2019, Jurjević *et al.*, 2020). Thus, data from

ALS measurements can be regarded as a substitute for permanent sample plots and even allow the detection of short-term trends in height increment resulting, for example, from weather conditions (Tymińska-Czabańska *et al*., 2021). The pattern of height growth of stands shows local variation due to site conditions, especially soil and climate (Guerra-Hernández *et al*., 2023). Therefore, there is a need to conduct research on site index models to include parameters related to site characteristics on a local or regional scale. With the increasing proportion of mixed-age stands, the need for age independent site index models is also becoming increasingly important (Solberg *et al*., 2019). The aforementioned issues can be addressed using remote sensing data and in particular ALS (Socha *et al*., 2017b, 2020a; Tymińska-Czabańska *et al*., 2021) or digital aerial photogrammetry (Tompalski *et al*., 2021). Further research is needed in the area of site index modelling, including comparing various data sources such as ALS, stem analysis and permanent sample plots, and exploring alternative approaches to model development.

Conclusions

We have developed new site index models for eight forest tree species in Poland. The use of the dynamic GADA function and the taking into account of the autocorrelation of the residuals allowed a very good fit of the models to the empirical data. For each of the analysed species, the explained variance exceeds 99% of the variation in height growth, and the model residuals do not show any systematic errors. The determination of site productivity is the main purpose of the developed models. However, thanks to the properties of the functions used to develop the models, they can also be used as models for predicting the height growth of stands. This research is a response to the demand for the development of new yield tables (growth models) for the main forest forming tree species in Poland from the State Forests in Poland, the Forest Management Office and other stakeholders. The research presented in this paper is the first stage of the construction of new empirical growth models. The presented site index models will be one of the functions of the developed growth models.

Authors' contribution

Conceptualization – J.S.; Methodology – J.S., P.H., K.L.; Validation – K.L.; Resources, J.S. – Data Curation – J.S.; Writing-Original Draft Preparation – J.S., P. H., K.L., L.T-C., ; Writing-Review & Editing – J.S., P. H., K.L., L.T-C.; Visualization – K.L..; Funding Acquisition – J.S.

All authors have read and agreed to the published version of the manuscript.

Conflict of interest

Authors declare there is no conflict of interest.

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References

Adame, P., Cańellas, I., Roig, S., Del Río, M., 2006. Modelling dominant height growth and site index curves for rebollo oak (*Quercus pyrenaica* Willd.). *Annals of Forest Science*, 63 (8): 929-940. DOI: https://doi.org/10.1051/forest:2006076.

Ågren, G.I., Hyvönen, R., Nilsson, T., 2008. Are Swedish forest soils sinks or sources for CO_2 – model analyses based on forest inventory data. *Biogeochemistry*, 89: 139-149. DOI: https://doi.org/10.1007/s10533-007-9151-x.

Bailey, R.L., Clutter, J.L., 1974. Base-age invariant polymorphic site curves. *Forest Science*, 20 (2): 155-159. DOI: https://doi.org/10.1093/forestscience/20.2.155.

Bronisz, K., Mehtätalo, L., 2020. Mixed-effects generalized height-diameter model for young silver birch stands on post-agricultural lands. *Forest Ecology and Management*, 460: 117901. DOI: 10.1016/j.foreco.2020.117901.

Bruchwald, A., Dmyterko, E., Wojtan, R., 2011. Model wzrostu dla modrzewia europejskiego (*Larix decidua* Mill.) wykorzystujący cechy taksacyjne drzewostanu. [A growth model for European larch (*Larix decidua* Mill.) based on stand characteristics]. *Leśne Prace Badawcze*, 72 (1): 77-81. DOI: https://doi.org/10.2478/v10111-011-0009-6.

Bruchwald, A., Dudek, A., Michalak, K., Zasada, M., Rymer-Dudzinska, T., Wróblewski, L., 1999. Model wzrostu dla drzewostanów świerkowych. (A growth model for spruce stands). *Sylwan*, 143 (1): 19-31.

Bruchwald, A., Dudzińska, M., Wirowski, M., 2003. Model wzrostu dla olszy czarnej (*Alnus glutinosa* (L.) Gaertn.). [A distance-independent tree growth model for black alder (*Alnus glutinosa* (L.) Gaertn.)] *Sylwan*, 147 (8): 3-10. DOI: https://doi.org/10.26202/sylwan.2003190.

Carmean, W.H., 1972. Site index curves for upland oaks in the Central States. *Forest Science*, 18 (2): 109-120. DOI: https://doi.org/10.1093/forestscience/18.2.109.

Chen, H.Y., Klinka, K., Kabzems, R.D., 1998. Site index, site quality, and foliar nutrients of trembling aspen: Relationships and prediction. *Canadian Journal of Forest Research*, 28 (12): 1743-1755. DOI: https://doi.org/10.1139/x98-154.

Cieszewski, C.J., 2001. Three methods of deriving advanced dynamic site equations demonstrated on inland Douglas-fir site curves. *Canadian Journal of Forest Research*, 31 (1): 165-173. DOI: https://doi.org/10.1139/x00-132.

Cieszewski, C.J., 2003. Developing a well-behaved dynamic site equation using a modified Hossfeld IV function Y3 =(axm)/(c+x m–1), a simplified mixed-model and scant subalpine fir data. *Forest Science*, 49 (4): 539-554. DOI: https://doi.org/10.1093/forestscience/49.4.539.

Cieszewski, C.J., 2004. GADA derivation of dynamic site equations with polymorphism and variable asymptotes from Richards, Weibull and other exponential functions. University of Georgia: PMRC Technical Report 2004-5.

Cieszewski, C.J., Bailey, L., 2000. Generalized algebraic difference approach: theory based derivation of dynamic site equations with polymorphism and variable asymptotes. *Forest Science*, 46 (1): 116-126. DOI: https://doi.org/10.1093/forestscience/46.1.116.

Cieszewski, C.J., Harrison, M., Martin, S.W., 2000. Examples of practical methods for unbiased parameter estimation in self-referencing functions. Proceedings of the First International Conference on Measurements and Quantitative Methods and Management and The 1999 Southern Mensurationists Meeting, Jekyll Island, Athens, GA, USA, 17-18 November 1999, Athens, GA, USA : D.B. Warnell School of Forest Resources, University of Georgia, 207 pp.

Cieszewski, C.J., Strub, M., Zasada, M., 2007. New dynamic site equation that fits best the Schwappach data for Scots pine (*Pinus sylvestris* L.) in Central Europe. *Forest Ecology and Management*, 243: 83-93. DOI: https://doi.org/10.1016/j.foreco.2007.02.025.

Cieszewski, C.J., Zasada, M., 2002. Dynamiczna forma anamorficznego modelu bonitacyjnego dla sosny pospolitej w Polsce. (A dynamic form of the anamorphic bonitation model for Scots pine in Poland.) *Sylwan*, 146 (7): 17-24.

Diéguez-Aranda, U., Grandas-Arias, J.A., Álvarez-González, J.G., von Gadow, K., 2006. Site quality curves for birch stands in north-western Spain. *Silva Fennica*, 40 (4): 631. DOI: https://doi.org/10.14214/sf.319.

Elfving, B., Kiviste, A., 1997. Construction of site index equations for *Pinus sylvestris* L. using permanent plot data in Sweden. *Forest Ecology and Management*, 98 (2): 125-134. DOI: https://doi.org/10.1016/S0378-1127(97)00077-7.

Ercanli, I., Kahriman, A., Yavuz, H., 2014. Dynamic base-age invariant site index models based on generalized algebraic difference approach for mixed Scots pine (*Pinus sylvestris* L.) and Oriental beech (*Fagus orientalis* Lipsky) stands. *Turkish Journal of Agriculture and Forestry*, 38 (1): 134-147. DOI: https://doi.org/10.3906/tar-1212-67.

Etzold, S., Ferretti, M., Reinds, G.J., Solberg, S., Gessler, A., Waldner, P., Schaub, M., Simpson, D., Benham, S., Hansen, K., Ingerslev, M., 2020. Nitrogen deposition is the most important environmental driver of growth of pure, even-aged and managed European forests. *Forest Ecology and Management*, 458: 117762. DOI: https://doi.org/10.1016/j.foreco.2019.117762.

Etzold, S., Ziemińska, K., Rohner, B., Bottero, A., Bose, A.K., Ruehr, N.K., Zingg, A., Rigling, A., 2019. One century of forest monitoring data in Switzerland reveals species-and site-specific trends of climate-induced tree mortality. *Frontiers in Plant Science*, 10: 307. DOI: https://doi.org/10.3389/fpls.2019.00307.

Goelz, J.C.G., Burk, T.E., 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. *Canadian Journal of Forest Research*, 22 (6): 776-784. DOI: https://doi.org/10.1139/x92-106.

Guerra-Hernández, J., Narine, L.L., Pascual A., Gonzalez-Ferreiro, E., Botequim, B., Malambo, L., Neuenschwander, A., Popescu, S.C., Godinho, S., 2023. Aboveground biomass mapping by integrating ICESat-2, SENTINEL-1, SENTINEL-2, ALOS2/PALSAR2, and topographic information in Mediterranean forests. *GIScience and Remote Sensing*, 59 (1): 1509-1533. DOI: https://doi.org/10.1080/15481603.2022.2115599.

Jedliński, W., 1932. Tablice zasobności i przyrostu drzewostanów sosnowych w Polsce. Warszawa, 24 pp.

Jurjević, L., Liang, X., Gašparović, M., Balenović, I., 2020. Is field-measured tree height as reliable as believed – Part II, A comparison study of tree height estimates from conventional field measurement and low-cost close-range remote sensing in a deciduous forest. *ISPRS Journal of Photogrammetry and Remote Sensing*, 169: 227-241. DOI: https://doi.org/10.1016/j.isprsjprs.2020.09.014.

Karlsson, K., 2000. Height growth patterns of Scots pine and Norway spruce in the coastal areas of western Finland. *Forest Ecology and Management*, 135 (1-3): 205-216. DOI: https://doi.org/10.1016/S0378-1127(00)00311-X.

Kayahara, G.J., Klinka, K., Marshall, P.L., 1998. Testing site index-site-factor relationships for predicting Pinus contorta and Picea engelmannii×P. glauca productivity in central British Columbia, Canada. *Forest Ecology and Management*, 110 (1-3): 141-150. DOI: https://doi.org/10.1016/S0378-1127(98)00279-5.

Kiviste, A., Álvarez González, J.G., Rojo-Alboreca, A., Ruiz González, A.D., 2002. Funciones de crecimiento de aplicación en el Ámbito Forestal. Madrid, Spain: Ministerio de Ciencia y Tecnología, Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria, 380 pp.

Lindner, M., Maroschek, M., Netherer, S., Kremer, A., Barbati, A., Lexer, M.J., Marchetti, M., 2010. Climate change impacts, adaptive capacity, and vulnerability of European forest ecosystems. *Forest Ecology and Management*, 259 (4): 698-709. DOI: https://doi.org/10.1016/j.foreco.2009.09.023.

Mensah, A.A., Holmström, E., Petersson, H., Nyström, K., Mason, E.G., Nilsson, U., 2021. The millennium shift: investigating the relationship between environment and growth trends of Norway spruce and Scots pine in northern Europe. *Forest Ecology and Management*, 481: 118727. DOI: https://doi.org/10.1016/j.foreco.2020.118727.

Nunes, L., Patrício, M., Tomé, J., Tomé, M., 2011. Modeling dominant height growth of maritime pine in Portugal using GADA methodology with parameters depending on soil and climate variables. *Annals of Forest Science*, 68 (2): 311-323. DOI: https://doi.org/10.1007/s13595-011-0036-8.

Palahí, M., Tomé, M., Pukkala, T., Trasobares, A., Montero, G., Palahí, M., 2004. Site index model for *Pinus sylvestris* in north-east Spain. *Forest Ecology and Management*, 187 (1): 35-47. DOI: https://doi.org/10.1016/S0378-1127(03)00312-8.

Parresol, B.R., Vissage, J.S., 1998. White pine site index for the Southern Forest Survey (Vol. 10). Asheville: U.S. Department of Agriculture, Forest Service, Southern Research Station, 12 pp.

Pau, M., Gauthier, S., Chavardčs, R.D., Girardin, M.P., Marchand, W., Bergeron, Y., 2022. Site index as a predictor of the effect of climate warming on boreal tree growth. *Global Change Biology*, 28 (5): 1903-1918. DOI: https://doi.org/10.1111/gcb.16030.

Płoński, W., 1937. Tablice zasobności i przyrostu drzewostanów. Sosna. *Prace Instytutu Badawczego Leśnictwa Ser. B*, 4.

Pretzsch, H., Biber, P., Schütze, G., Uhl, E., Rötzer, T., 2014. Forest stand growth dynamics in Central Europe have accelerated since 1870. *Nature Communications*, 5 (1): 4967. DOI: https://doi.org/10.1038/ncomms5967.

R Core Team, 2021. R: A language and environment for statistical computing. Vienna: R Foundation for Statistical Computing. Available: https://www.R-project.org/.

Raulier, F., Lambert, M.C., Pothier, D., Ung, C.H., 2003. Impact of dominant tree dynamics on site index curves. *Forest Ecology and Management*, 184 (1-3): 65-78. DOI: https://doi.org/10.1016/S0378-1127(03)00149-X.

Schwappach, A., 1912. Ertragstafeln der wichtigeren Holzarten. (Growth and yield tables for main forest forming tree species). Michigan: J. Neumann, 74 pp.

Skovsgaard, J.P., Vanclay, J.K., 2008. Forest site productivity: a review of the evolution of dendrometric concepts for even-aged stands. *Forestry*, 81 (1): 13-31. DOI: https://doi.org/10.1093/forestry/cpm041.

Socha, J., Bruchwald, A., Neroj, B., Gruba, P., Wertz, B., Ochał, W., Pierzchalski, M., 2017a. Summary of the research project 'Actual and potential site productivity in Poland for main forest forming tree species'. Unpublished, Report of the research project, Faculty of Forestry, University of Agriculture in Krakow.

Socha, J., Hawryło, P., Stereńczak, K., Miścicki, S., Tymińska-Czabańska, L., Młocek, W., Gruba, P., 2020a. Assessing the sensitivity of site index models developed using bi-temporal airborne laser scanning data to different top height estimates and grid cell sizes. *International Journal of Applied Earth Observation and Geoinformation*, 91: 102129. DOI: https://doi.org/10.1016/j.jag.2020.102129.

Socha, J., Ochał, W., Grabczyński, S., Maj, M., 2015. Modele bonitacyjne dla gatunków lasotwórczych Polski opracowane na podstawie tablic zasobności. (Site index models for forest-forming tree species in Poland developed basing on the yield tables.) *Sylwan*, 159 (8): 639-649. DOI: https://doi.org/10.26202/sylwan.2015011.

Socha, J., Orzeł, S., 2011. Dynamiczne krzywe bonitacyjne dla drzewostanów sosnowych Puszczy Niepołomickiej. (Dynamic site index curves for scots pine stands in Niepołomice Primeval Forest.) *Sylwan*, 155 (5): 301-312. DOI: https://doi.org/10.26202/sylwan.2010078.

Socha, J., Orzeł, S., 2013. Dynamiczne krzywe bonitacyjne dla sosny zwyczajnej (*Pinus sylvestris* L.) z południowej Polski. [Dynamic site index curves for Scots pine (*Pinus sylvestris* L.) in southern Poland.] *Sylwan*, 157 (1): 26-38. DOI: https://doi.org/10.26202/sylwan.2012094.

Socha, J., Pierzchalski, M., Bałazy, R., Ciesielski, M., 2017b. Modelling top height growth and site index using repeated laser scanning data. *Forest Ecology and Management*, 406: 307-3017. DOI: https://doi.org/10.1016/j.foreco.2017.09.039.

Socha, J., Tymińska-Czabańska, L., Grabska, E., Orzeł, S., 2020b. Site index models for main forest-forming tree species in Poland. *Forests*, 11 (3): 301. DOI: https://doi.org/10.3390/f11030301.

Socha, J., Tymińska-Czabańska, L., Bronisz, K., Zięba, S., Hawryło, P., 2021. Regional height growth models for scots pine in Poland. *Scientific reports*, 11 (1): 10330. DOI: https://doi.org/10.1038/s41598-021-89826-9.

Solberg, S., Kvaalen, H., Puliti, S., 2019. Age-independent site index mapping with repeated single-tree airborne laser scanning. *Scandinavian Journal of Forest Research*, 34 (8): 763-770. DOI: https://doi.org/10.1080/02827581.2019.1616814.

Splechtna, B.E., 2001. Height growth and site index models for Pacific silver fir in southwestern British Columbia. *Journal of Ecosystems and Management*, 1 (1): 1-14.

Szymkiewicz, B., 1949. Rozszerzenie tablic zasobności Schwappacha dla sosny o klasę Ia. *Sylwan*, 93 (03-04): 76-93.

Tompalski, P., Coops, N.C., White, J.C., Goodbody, T.R., Hennigar, C.R., Wulder, M.A., Socha, J., Woods, M.E., 2021. Estimating changes in forest attributes and enhancing growth projections: A review of existing approaches and future directions using airborne 3D point cloud data. *Current Forestry Reports*, 7: 1-24. DOI: https://doi.org/10.1007/s40725-021-00135-w.

Tymińska-Czabańska, L., Hawryło, P., Janiec, P., Socha, J., 2022. Tree height, growth rate and stand density determined by ALS drive probability of Scots pine mortality. *Ecological Indicators*, 145: 109643. DOI: https://doi.org/10.1016/j.ecolind.2022.109643.

Tymińska-Czabańska, L., Socha, J., Maj, M., Cywicka, D., Hoang Duong, X.V., 2021. Environmental drivers and age trends in site productivity for oak in southern Poland. *Forests*, 12 (2): 209. DOI: https://doi.org/10.3390/f12020209.

Wang, M., Borders, B.E., Zhao, D., 2008. An empirical comparison of two subject-specific approaches to dominant heights modelling: The dummy variable method and the mixed model method. *Forest Ecology and Management*, 255 (7): 2659-2669. DOI: https://doi.org/10.1016/j.foreco.2008.01.030.

Wang, Y., Lehtomäki, M., Liang, X., Pyörälä, J., Kukko, A., Jaakkola, A., Liu, J., Feng, Z., Chen, R., Hyyppä, J., 2019. Is field-measured tree height as reliable as believed – A comparison study of tree height estimates from field measurement, airborne laser scanning and terrestrial laser scanning in a boreal forest. *ISPRS Journal of Photogrammetry and Remote Sensing*, 147: 132-145. DOI: https://doi.org/10.1016/j.isprsjprs.2018.11.008.

Weiskittel, A., Kuehne, C., McTague, J.P., Oppenheimer, M., 2016. Development and evaluation of an individual tree growth and yield model for the mixed species forest of the Adirondacks Region of New York, USA. *Forest Ecosystems*, 3 (1): 1-17. DOI: https://doi.org/10.1186/s40663-016-0086-3.

STRESZCZENIE

Modele wzrostu wysokości i wskaźnika bonitacji siedliska dla głównych gatunków drzew lasotwórczych w Polsce

Znajomość potencjalnej produkcyjności siedliska jest podstawą strategicznych decyzji w gospodarce leśnej. Produkcyjność siedliska jest powszechnie szacowana za pomocą wskaźnika bonitacji, który jest określany na podstawie wysokości i wieku drzewostanu przy użyciu modeli bonitacyjnych. Modele bonitacyjne są również wykorzystywane do prognozowania wzrostu lasu. Właściwa ocena wskaźnika bonitacji siedliska jest ważna dla gospodarki leśnej, ponieważ jest podstawowym kryterium branym pod uwagę przy podejmowaniu specyficznych dla siedliska i gatunku decyzji gospodarczych dotyczących zabiegów hodowlanych, doboru składu gatunkowego, ustalania planu cięć i wieku rębności. W ostatnim czasie opracowano modele bonitacyjne dla głównych gatunków lasotwórczych Polski, jednak przy opracowywaniu parametrów modeli nie uwzględniono w pełni autokorelacji, a ponadto wystąpiły niedokładności w ich wartościach. W związku z planowanym wdrożeniem modeli do systemu informatycznego Lasów Państwowych i zastosowaniem ich do określania przyrostu miąższości lasów w Polsce, konieczna była weryfikacja istniejących modeli. Celem pracy jest stworzenie ujednoliconych modeli bonitacyjnych dla głównych gatunków lasotwórczych Polski z wykorzystaniem aktualnego podejścia. Wykorzystując dane z 2636 serii wzrostowych (ryc. 1; tab. 1), opracowano nowe modele bonitacyjne dla 8 gatunków drzew leśnych w Polsce (tab. 2). Zastosowanie dynamicznej funkcji GADA oraz uwzględnienie autokorelacji reszt pozwoliło na bardzo dobre dopasowanie modeli do danych empirycznych (tab. 2-4; ryc. 3). Ponadto opracowane modele spełniają główne wymagania stawiane nowoczesnym modelom wskaźnika bonitacji siedliska (ryc. 2, 4): (a) polimorfizm, który pozwala na uwzględnienie ewentualnych różnic we wzorcach wzrostu wynikających ze zmienności warunków siedliskowych; (b) zmienne asymptoty dla różnych siedlisk; (c) równość wskaźnika bonitacji siedliska i wysokości w danym

wieku bazowym oraz (d) możliwość wykorzystania tej samej funkcji do modelowania wzrostu wysokości i wskaźnika bonitacji siedliska. Określenie produkcyjności siedliska jest głównym celem opracowanych modeli, jednak dzięki właściwościom użytych funkcji mogą one być również wykorzystywane do przewidywania wzrostu wysokości drzewostanów. Badania te są odpowiedzią na potrzebę opracowania nowych tablic zasobności (modeli wzrostu) dla głównych gatunków drzew lasotwórczych w Polsce ze strony Lasów Państwowych, Biura Urządzania Lasu i Geodezji Leśnej oraz innych interesariuszy. Badania przedstawione w niniejszym opracowaniu są pierwszym etapem budowy nowych empirycznych modeli wzrostu. Przedstawione modele wskaźników bonitacji siedlisk będą jedną z funkcji opracowanych modeli wzrostu. Dokładność prognoz wysokości uzyskanych z opracowanych modeli powinna zostać zweryfikowana z wykorzystaniem danych ze stałych powierzchni próbnych lub pomiarów z powtarzanego lotniczego skaningu laserowego (ALS). Ostatnie badania wskazują, że może on być bardzo dobrym źródłem danych o wzroście wysokości. Wzorzec wzrostu wysokości wykazuje lokalną zmienność ze względu na warunki siedliskowe, w szczególności glebowe i klimatyczne, dlatego też istnieje potrzeba dalszych badań nad modelami bonitacyjnymi z dodatkowymi parametrami związanymi z charakterystykami siedliska w skali lokalnej lub regionalnej. Wraz z rosnącym odsetkiem drzewostanów różnowiekowych coraz silniej zarysowuje się potrzeba opracowania niezależnych od wieku modeli bonitacyjnych. Powyższe kwestie można rozwiązać poprzez wykorzystanie do budowy modeli bonitacyjnych coraz bardziej dostępnych danych teledetekcyjnych, w szczególności ALS lub cyfrowej fotogrametrii lotniczej.